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PREFACE.

The general utility of a work of this nature ſuperſedes the neceſſity of a Preface. But it may be thought neceſſary to ſay a few words concerning the manner in which the ſubjects are arranged.

In the rules in Vulgar Arithmetic, I have ſomewhat departed from the order generally followed, in order to render that ſcience more intelligible, and eaſy of attainment, to thoſe who chooſe to go regularly through it. Almoſt every writer on Arithmetic follows a different order in the diſpoſition of his work; though every one muſt be ſenſible of the abſurdity of teaching thoſe rules which depend chiefly upon fractions, as Practice, &c. before the learner is acquainted with the doctrine of Vulgar Fractions. It may be obſerved, that any rule in Arithmetic may with equal propriety be attempted, when the learner is ſufficiently ſkilled in the four ſingle rules. The rules of Intereſt are alſo placed under the chapter of Stock-holding; and Tare and Tret in that of Merchants Accompts, being more intimately connected therewith.

In the chapter of Optics, I have endeavoured to render the practical part of that uſeful ſcience as intelligible as poſſible, avoiding all algebraical and mathematical calculations (though neceſſary in complete ſcientific treatiſes upon this ſubject), in order to render it more perſpicuous.

In each of the other ſubjects, I have retained thoſe parts which appeared to me the moſt eſſential, and rejected the ſuperfluous: availing myſelf of the labours of others on each of the ſubjects; though I have not quoted every author; as that would be attended with more difficulty than might be at firſt expected, and partake of a preciſeneſs unneceſſary in the preſent work.

It may be proper to mention, that I have endeavoured to inſert all the modern improvements, in each of the ſubjects, that appeared of any conſiderable utility. The errors unavoidable in works of this nature, abounding with calculations, I have, as far as they occurred to me, corrected in this edition. The candour with which the former edition of this work was received, is at leaſt ſome argument in favour of its utility, as alſo conſolation to its author; and tends to alleviate an afflicting diſpenſation of Divine Providence: and render tolerable an uninterrupted confinement to his room, which has now continued upwards of eighteen years.

Middle Temple,
1802.

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THE

ACCOMPLISHED TUTOR.

CHAP. I.

OF ENGLISH GRAMMAR.

SECT. I.

OF THE DIVISION AND SOUND OF LETTERS.

IN the early ages of antiquity, before alphabets were invented, mankind, ſenſible of their want of ſome means of recording hiſtorical events and ſcientifical diſcoveries, had recourſe to various arts for theſe purpoſes; the firſt of which was painting. That partiality for pictures, ſo evident in all ages and countries, afforded the ancients a method of perpetuating their tranſactions. To commemorate that one man had killed another, they painted the figure of a dead man, with another man ſtanding over him, having an hoſtile weapon in his hand. On the firſt diſcovery of America, this was the only kind of writing uſed by the Mexicans.

The firſt improvement made by our anceſtors in the art of writing (if it might then be called an art) was by the introduction of hieroglyphical characters. Theſe conſiſted of certain ſymbols, which were made to repreſent certain inviſible objects, to which, in ſome particulars, ſuch ſymbols were ſuppoſed to bear ſome reſemblance. An eye was the ſymbol of knowledge; a circle, of eternity, as having neither beginning nor end. The figures of animals were alſo much employed in this kind of writing, on account of ſome quality with which they were ſuppoſed to be endowed, and in which they reſembled the object ſignified. Thus, imprudence was repreſented by a fly; wiſdom by an ant; and victory by a hawk. Theſe hieroglyphics flouriſhed moſt in ancient Egypt (as did all other learning at that time), where the knowledge of theſe characters was reduced into a regular art: and many ſpecimens of them are ſtill extant in relics of Egyptian antiquities. Hieroglyphics, though an improvement upon the former mode of writing, was a very imperfect one, and often confuſed and perplexed its moſt ſkilful profeſſors.

In a few ſucceeding ages, hieroglyphics gave place to ſimple arbitrary marks, which were introduced to repreſent objects, without having the leaſt reſemblance or affinity to the objects repreſented. The Chineſe ſtill uſe characters of this nature: they have no alphabet of letters, but every ſingle mark or character ſignifies one perfect idea or object. The number of theſe characters are therefore great;—near ſeventy thouſand. To be perfectly acquainted with them, conſtitutes the buſineſs of a whole life; which muſt be an inſurmountable obſtacle to the improvement of ſcience. Our common figures, 1, 2, 3, 4, &c. afford us an example of this ſort of writing; where each figure or character conveys the idea of the number for which it ſtands as clearly and intelligibly as the words themſelves, one, two, three, &c. But when marks or characters come to be uſed for all our ideas, in excluſion to an alphabet of letters, they then, from their number, become inconvenient.

The next improvement in the art of writing was by the invention of ſigns or marks, which ſtood, not directly, for the objects themſelves, but for the *words* or *names* whereby they were diſtinguiſhed. This was an *alphabet of ſyllables*. An alphabet of this kind is ſtill in uſe in Æthiopia, and ſome countries of India.

But the noble and ſublime diſcovery of an *alphabet of letters* ſuperſeded every other improvement in this art. Who was the firſt in this invention is uncertain. An alphabet of letters was, however, brought into Greece by Cadmus, the Phœnician, who was contemporary with king David. This alphabet conſiſted of only ſixteen letters: the reſt were added afterwards, as ſigns for proper ſounds were found to be wanting. The Phœnician, Hebrew, Greek, and Roman alphabets are ſo much alike in the figures and names of the letters, as plainly to evince they were originally derived from the ſame.

By the uſe of the alphabet, we are now, therefore, enabled to expreſs our ideas with the ſame clearneſs and preciſion, as in converſation.

The Engliſh alphabet conſiſts of twenty-ſix letters: A, a; B, b; C, c; D, d; E, e; F, f; G, g; H, h; I, i; J, j; K, k; L, l; M, m; N, n; O, o; P, p; Q, q; R, r; S, ſ, s; T, t; U, u; V, v; W, w; X, x; Y, y; Z, z; and is divided into vowels and conſonants; mutes and ſemi-vowels.

The names of the twenty-ſix letters are as follow: a, bee, cee, dee, e, ef, gee, aitch, i, ja, ka, el, em, en, o, pee, cue, ar, eſs, tee, u, vee, double u, ex, y, zad.

The vowels are ſix in number, viz. *a, e, i, o, u, y*; all the reſt are conſonants.

The mutes are thoſe letters which are begun, when they are ſpelled, by a conſonant: as, *b, bee; c, cee; d, dee*, &c.; thoſe which are begun with a vowel are called ſemi-vowels: as, *l, el; m, em; n, en; r, ar; s, eſs*, &c.; *l, m, n, r*, are alſo called liquids.

When two vowels meet together, they are called a diphthong: of theſe there are thirteen, viz. *ai, ei, oi, ui, au, eu, ou, ee, oo, ea, eo, oa*, and *ie*.

When three vowels meet, they are called a triphthong: as in the word *beauty*.

With regard to the ſound of theſe double vowels, there are no rules, that can be given, which will hold good in all caſes, as words are ſounded according to the caprice or affectation of the age, or the ſpeakers: a knowledge of them muſt, therefore, be acquired by experience and obſervation.

And concerning the ſound of ſingle letters, the following rules are all that can ſafely be depended upon.

C is pronounced hard like *k*, before *a*, *o*, *u*; and ſoft like *s*, before *e*, *i*, and *y*.

G is alſo ſounded hard before *a*, *o*, *u*; ſometimes hard, and ſometimes ſoft, before *i* and *y*; and generally ſoft before *e*.

E is moſtly ſilent at the end of a word; but in that caſe it lengthens the foregoing vowel; as, *hid*, *hide*; and that ſometimes in the middle of a word; as *ungrateful*. But ſometimes it only ſoftens a preceding *g*, as in *lodge*, *judge*.

H is only an aſpiration of the breath, and ſometimes at the beginning of a word is not ſounded at all: as, an *hour*, an *honeſt* man.

W is either a vowel or a diphthong; its proper ſound is the ſame as *u* in the Italian, *ou* in the French, or *oo* in the Engliſh. Sometimes it is not ſounded at all after *o*, ſometimes like *au*.

X is a double conſonant, compoſed of a hard *c* or *k*, and *s*; and at the beginning of a word moſtly ſounded like *z*.

Y has exactly the ſame ſound as *i*; and is only a ſubſtitute for it at the end of a word, or before *i*: as, *cry*, *flying*. It is a perfect vowel; and when uſed as a conſonant at the beginning of a word, it anſwers to the ancient Saxon *i*: as, *yew*, *iw*; *young*, *iong*.

Z is a double conſonant; it ſounds as much coarſer and thicker than *s*, as *v* does than *f*.

J and *v*, though confounded by ſome old writers with *i* and *u*, are entirely different letters; the former having the ſound of a ſoft *g*, and the latter that of a coarſer *f*. The former is called *ja*, and the latter *vee*.

SECT.

SECT. II.

OF SYLLABLES.

A SYLLABLE is a conſtituent part of a word, or a whole word; it conſiſts of one or more letters, and is formed by a ſingle impulſe of the voice.

Spelling is the art of rightly naming the letters ſingly that conſtitute a word, and dividing them into ſyllables.

The art of ſpelling perfectly is not to be acquired (particularly in the Engliſh) but by practice: but a few rules for the dividing of ſyllables may poſſibly be of ſervice.

1. A ſyllable in the beginning, or middle, of a word ends in a vowel, except ſuch vowel be followed by *x*, or two or more conſonants: as in *re-li-gi-on.*

2. When two or more conſonants follow a vowel, which is pronounced ſhort, they muſt be ſeparated; and one, at leaſt, always belongs to the preceding ſyllable: as in *ab-ſti-nence.*

3. When two or more conſonants follow a vowel, which is pronounced long, they ſometimes belong to the following ſyllable: as, *di-greſs.*

4. A particle, though placed immediately before a vowel, is ſeldom divided: as, *un-e-qual.*

5. A mute with a liquid following are ſeldom divided; but a liquid or a mute, with a mute following, are moſtly divided.

6. When *le* or *re* follow a mute they are never divided.

Theſe are the fundamental rules for the dividing of ſyllables: but ſome grammarians recommend them to be divided as they are founded in a juſt pronunciation.

SECT. III.

OF THE NINE PARTS OF SPEECH.

WHEN mankind had arrived at ſome perfection in the art of writing, they ſoon diſcovered the propriety of reducing language into different forms of words; or, as they are commonly called, parts of ſpeech.

In Engliſh we have nine forms of words, or parts of ſpeech.

1. The Article—placed before nouns to help to determine their ſignification: as, *a* man, *the* man.

2. The Substantive, or noun—which is the name of any object whatever, of which we have any idea: as, a *man*, a *horſe*, a *ſpirit*, *grief*, *love*.

3. The Pronoun—uſed as a ſubſtitute for the noun: as, *he*, *ſhe*, *it*.

4. The Adjective—added to the noun to denote its quality: as, an *honeſt* man, a *white* horſe, an *evil* ſpirit.

5. The Verb—which ſignifies a ſtate of exiſtence, acting, or ſuffering: as, "the centinels *ſlept*, the enemy *entered*, and the fort *was taken*."

6. The Adverb—uſed to qualify or enforce the meaning of other words. It is added ſometimes to a verb: as, he reads *well*. Sometimes to an adjective: as, "an *exceeding* high mountain." Sometimes to another adverb: as, "*moſt divinely* fair."

7. The Preposition—placed between words to connect them together: as, "the path *to* fame lies *through* the road *to* danger."

8. The Conjunction—uſed to connect ſentences, as prepoſitions connect words: as, "fear God *and* honour the King."

9. The Interjection—uſed to expreſs the ſurpriſe or affection of the ſpeaker: as, *O! alas!*

Theſe are the nine parts of ſpeech in the Engliſh tongue; every word in which is reducible to one of theſe parts.

The ARTICLE is placed before ſubſtantives to ſhew how far their ſignification extends.

There are in Engliſh only two articles; *a* and *the*. *A* is called the indefinite, and *the* the definite article.

The indefinite article *a* is changed into *an* when the next word begins with a vowel, or a ſilent *h* before the vowel; except ſuch vowel be *y* or *w*. This article, as its name imports, is uſed in an indefinite and undetermined ſenſe: as, *a* *man*, means one man, without determining who in particular; leaving that to be explained, if neceſſary, by the other parts of the ſentence.

This article can, therefore, be joined to ſubſtantives in the ſingular number only, except it come before the adjectives *few* and *many*: as, *a few days*, *a (great) many men*; or before the complete numerical adjectives, *a dozen*, *a ſcore*, *an hundred*, *a thouſand*, *a million*, *a myriad*: as, *a dozen pound*, *a ſcore of eggs*, *an hundred men*, &c.

The definite article *the* determines which particular thing is meant; or of many, which they are: this is therefore employed both with ſingular and plural nouns: as, "*the ſtork in the heavens*."

It is alſo uſed ſometimes before adverbs in the comparative and ſuperlative degrees; as, *the more*, *the better*, *the leaſt* (of all).

The SUBSTANTIVE is the name of any object whatever of which we have any idea.

There are two ſorts of ſubſtantives, Proper and Common. Proper names are ſuch as belong to individual objects, whether animate or inanimate; but not to every individual or object of the ſame ſort or ſpecies: thus, *John* is the name of a man, but not the name of every man; *Chance* may be the name of a dog, but it is not the name of every dog; *London* the name of a city, but not of every city. Theſe are therefore proper names; but *man*, *dog*, and *city* are common names: for *man* is the common name of every man; *dog*, the common name of every dog; and *city*, the common name of every city.

Proper names have no articles nor plural number, unleſs by a metaphor: as when a cruel tyrant is called *a Nero*; or when a common name is underſtood, as the (ſhip) *Royal George*; or when there are many of the ſame name, as *the* twelve *Cæſars*.

There are three attributes belonging to the Engliſh noun, viz. number, gender, and caſe.

The number is either ſingular or plural. The ſingular number expreſſes one object: as, a *king*: the plural, two or more, as, *kings*.

The plural number is moſtly formed, in Engliſh, by the addition of *s* or *es*: as, *boy*, *boys*; *goat*, *goats*; *fox*, *foxes*.

Some change the *f* into *v*: as, *wife*, *wives*; *leaf*, *leaves*.

Some plurals end in *en*: as, *ox*, *oxen*; *child*, *children*; *man*, *men*; *brother*, *brethren*. In the latter, and ſome others, the *a* in the firſt ſyllable is changed into *e*. This form is the remains of the Teutonic language, and the following of the Saxon; *louſe*, *lice*; *mouſe*, *mice*; *tooth*, *teeth*.

Some nouns ending in *y*, change it into *i*: as, *city*, *cities*; *gallery*, *galleries*.

Some nouns have no plural: as, *corn*, *gold*, *pitch*, *ſloth*, &c. And others no ſingular numbers: as, *annals*, *bellows*, *ſciſſars*, *lungs*, &c. The words *ſheep* and *deer* have no variation of number.

All ſubſtantives in Engliſh, whether common or proper, are conſidered with relation to gender, as being either of the maſculine, feminine, or neuter gender. That is, either of the male or female ſex, or neither.

The maſculine or feminine genders are applied to the names of animals only, and ſuch whoſe ſex is obvious: as, *man*, *Thomas*, *ox*, which are the maſculine gender: *Mary*, *woman*, *hen*, *cow*, which are the feminine gender. Thoſe whoſe ſex is not evident, with all inanimate objects, are of the neuter gender; as, *oyſter*, *worm*, *houſe*, *tree*; except in poetry, and the elegant ſpecies of compoſition, where inanimate objects, and the human paſſions, are perſonified into men and women, and conſequently have their genders.

A few ſubſtantives are marked by their terminations: as, *prince*, *princeſs*; *lion*, *lioneſs*; *actor*, *actreſs*.

The chief uſe of gender is to agree with the pronoun.

In Engliſh there are three different caſes of the noun: the nominative, poſſeſſive, and objective caſe.

The nominative caſe is the ſimple name of the noun, without relation to other objects: as, "*John*."

The poſſeſſive ſignifies the relation of poſſeſſion: as, "*John's* book," and was formerly written "*Johnis* book," and not "*John his* book," as is vulgarly written. It may alſo be expreſſed by inverting the phraſe, and inſerting the prepoſition: as, "the *book of John*."

When the ſign of the poſſeſſive caſe is to be added to a phraſe of ſeveral words, it is generally joined to the laſt word: as, "the Emperor of *Morocco's* dominions." To the plural noun ending in *s*, the *s* is not added: as, the "*arrows'* heads."

The objective caſe, though founded in grammar, is formed as the nominative, and expreſſes the object of a ſentence: as, "man loves *ſociety*."

The PRONOUN is a word uſed as a ſubſtitute for the noun, and to prevent its repetition: as, "John loves *his* book." Here the pronoun *his* ſupplies the place of the noun *John*, and prevents the repetition of it.

In a pronoun there are perſon, number, gender, and caſe.

There are three perſons in the pronoun. Firſt, the perſon who ſpeaks may mean himſelf: as, *I*, *me*; theſe are therefore called pronouns of the firſt perſon:—or he may ſpeak of the perſon to whom he addreſſes himſelf: as, *thou*, *thee*; theſe are pronouns of the ſecond perſon:—or he may intend ſome other perſon or thing: as, *he*, *ſhe*, or *it*; theſe are pronouns of the third perſon. Each of theſe perſons have the ſingular and plural numbers, and the three variations of caſe. The third perſon has alſo the variations of gender.

The Pronouns declined according to their Variations of Perſons, Numbers, Caſes, and Genders.

	Singular Number.			*Plural Number.*		
	Nominative.	Poſſeſſive.	Objective.	Nominative.	Poſſeſſive.	Objective.
	Firſt Perſon.			*Firſt Perſon.*		
	I,	mine,	me.	We,	ours,	us.
	Second Perſon.			*Second Perſon.*		
	Thou,	thine,	thee.	Ye or you,	yours,	you.
Genders.	*Third Perſon.*			*Third Perſon.*		
Maſ.	He,	his,	him.	They,	theirs,	them.
Fem.	She,	hers,	her.			
Neu.	It,	its,	it.			

Thou, in all its caſes, generally gives place to *you*, except in a very ſolemn ſtyle, and in addreſſes to the Deity.

As the perſonal pronoun is a ſubſtitute for the noun, ſo it has the ſame nature in grammatical conſtruction, and is capable of forming a ſentence without the aid of the ſubſtantive: as, "*I* that ſpeak unto thee am *he* [Chriſt]."—John, iv. 26.

But there are other pronouns, of the nature of adjectives, and, like adjectives, require ſome ſubſtantive to be joined with them, which is either expreſſed or underſtood. Thoſe are therefore called Pronominal Adjectives. Of thoſe there are four ſorts; poſſeſſives, definitives, relatives, and diſtributives.

The poſſeſſives are *my*, *thy*, *our*, *your*, *her*, *their*; and are entirely different words from the poſſeſſive pronouns, *mine*, *thine*, &c. In theſe the ſubſtantive is ſometimes underſtood without being expreſſed: as, "*the ſhame be yours*;" whereas, to thoſe the ſubſtantive ſhould be added: as, "*thy people ſhall be my people*, and *thy God my God*."—Ruth, i. 16.

The definitives are, *this*, *that*, *other*, *any*, *ſame*, *one*, *none*. Theſe define, or limit, the meaning of the ſubſtantive to which they refer, or are joined. The three firſt have the plural number; as, *theſe*, *thoſe*, *others*. *This* refers to the latter term or ſentence, *that* to the former: as, "*theſe* are definitives, *thoſe* (mentioned before) are poſſeſſives; *other* is uſed in the plural form only, when the ſubſtantive belonging to it is not expreſſed, as is ſometimes the caſe in definitives; *other* and *one* have alſo the poſſeſſive caſe; as, "this is *others'* property," and "it ſtartles *one*'s apprehenſion;" *one* is alſo uſed in an indefinite ſenſe, as, "*one* thinks." Theſe are all the variations definitives admit of.

The relatives are, *who*, *which*, *that*. Theſe refer to ſome ſubſtantive or pronoun going before, which is therefore called the antecedent: as, "I am *the Lord that* maketh all things, *that* ſtretcheth forth the heavens alone."—Iſaiah, xliv. 24. They alſo connect the following and foregoing parts of a ſentence together. *Who* is varied to expreſs the three caſes: as, *who*, *whoſe*, *whom*; theſe relatives refer to all the three perſons, whereas the other pronominal adjectives belong only to the third perſon. *Who*, *which*, *what*, are called interrogatives, when they are uſed in a queſtion.

Each, *every*, *either*, are called diſtributives, becauſe they relate to objects or perſons taken ſeparately.

Beſides the foregoing pronominal adjectives, there are two others: *own*, and *ſelf*, in the plural *ſelves*. Both of them expreſs emphaſis, or oppoſition, and are joined to the poſſeſſives, and ſo form a compound pronominal adjective; as, *my own*, *thyſelf*, *yourſelves*.

Ourſelf, the plural pronominal adjective, with the ſingular ſubſtantive, belongs to the regal ſtyle.

All ſubſtantives belong to the third perſon, except when an addreſs is made to a perſon, then the ſubſtantive is of the ſecond perſon.

An **ADJECTIVE** is a word added to a ſubſtantive, to expreſs the quality of it, or ſome other property: as, *a good man*, *an hundred pounds*, *a burning mountain*.

Adjectives have no variation, except the three degrees of compariſon: poſitive, comparative, and ſuperlative.

Moſt qualities conſiſt of different degrees, or of *more* or *leſs*. Thus, when a quality is ſimply expreſſed, without reference to a greater or leſs degree of the ſame, it is called the poſitive degree: as, *large*, *ſhort*. When it is expreſſed with relation to a leſs degree, it is called the comparative degree: as, *larger*, *ſhorter*. When it is expreſſed as being of the higheſt degree in its quality, it is called the ſuperlative: as, *largeſt*, *ſhorteſt*.

There are two ways of forming the comparative and ſuperlative degrees; the firſt is by adding *r* or *er* to the poſitive degree, which forms the comparative; and by adding *ſt* or *eſ*, to the poſitive, and ſo forming the ſuperlative degree; as in the laſt example. Moſt monoſyllables are compared in this manner; and diſſyllables ending in *y*: as, *happy*; and *le*, when they are followed by a mute: as, *able*; or when they are accented on the laſt ſyllable: as, *genteel*. But others, and words of more than two ſyllables, ſeldom admit of theſe terminations; but are compared according to the following rule.

The ſecond method of forming the comparative and ſuperlative degrees of compariſon, is by placing the adverbs *more* or *moſt* before the adjective of the poſitive degree: as, *experienced*, *more experienced*, *moſt experienced*.

The ſuperlative is, in a few words, formed by adding the abverb *moſt* to the end of them: as, *nethermoſt*, *foremoſt*, *uppermoſt*, *undermoſt*, &c.

The four following adjectives are irregular in the formation of the degrees of compariſon in moſt languages: *good*, *better*, *beſt*; *bad*, *worſe*, *worſt*; *little*, *leſs*, *leaſt*; *much* or *many* *more*, *moſt*.

Double comparatives and ſuperlatives are improper in grammar: as, the *more greater*, or the *moſt greateſt*; except the *moſt higheſt*, a phraſe uſed in the Pſalms of David, and is properly applicable to the Supreme Being only.

The VERB is a word which ſignifies an action, or a ſtate or condition of being; and is either active, paſſive, or neuter.

A verb active expreſſes an action in its natural order, that is, when the agent is placed firſt, and the object laſt; as, "*John loves his book.*" Here *John* is called the agent, becauſe he performs the action, which is *love*, and which is the verb; and *book* is the object, upon which the action love is exerciſed.

A verb paſſive expreſſes an action, but in an inverted order: that is, when the object is placed firſt, and the agent laſt: as, "the book *is loved* by John." Here may be ſeen the uſe of the caſes of nouns and pronouns: for when verb is active, the agent is placed firſt, and is in the nominative caſe: as, "*I* love *thee*." When the verb is paſſive, the object being placed firſt, muſt be in the nominative caſe, and the agent being laſt muſt be in the objective caſe, accompanied with a prepoſition; and the verb aſſumes a different form: as, "*thou art loved by me*." Thus an active verb may be tranſformed into a paſſive, or a paſſive verb into an active verb.

A verb neuter expreſſes a ſtate, or condition, of being only, and conſequently has no object to be acted upon; but an agent only: as, *I walk, I ſleep, I dreamed, he fell, ſhe aroſe.* The verb neuter is called alſo the intranſitive verb, and the verb active tranſitive.

Many verbs are uſed in Engliſh both in an active and neuter ſignification; their form demonſtrating of which kind they are.

The verb is varied, firſt, to correſpond with the three different perſons of the pronoun; ſecondly, to agree with the number ſingular or plural; thirdly, to expreſs the three principal gradations of time,—preſent, paſt, and future; laſtly, to ſhew the mode or manner in which the action or ſtate of being is expreſſed.

In a verb are, therefore, theſe four attributes; perſon, number, time, and mode.

To agree with the three perſonal pronouns, though of the ſame number, the verb is varied: as, *I love, thou loveſt, he loveth or loves.*

Alſo to agree with the two different numbers of the ſame perſon: as, thou *loveſt*, ye *love*; he *loveth*, they *love*.

Likewiſe, to expreſs the different times in which the action is repreſented: as, I *love*, I *loved*, I have *loved*. To expreſs the different times, other verbs are often uſed, called auxiliaries, as will be ſeen hereafter.

The verb is moreover varied according to the different *manner* of expreſſing the action or being; which variations are called its modes; of which there are generally reckoned four, beſides the participle: the indicative, imperative, ſubjunctive, and infinitive.

When a circumſtance is ſimply declared, or when a queſtion is aſked, the verb employed on theſe occaſions is of the indicative mode: as, I *love*, he *loves*, *loveſt* thou? When any thing is commanded, or ſolicited, or requeſted to be done, be it to a ſuperior or inferior, the verb employed in the ſentence is of the imperative mode: as, *love* thou, "O Thou my voice *inſpire!*"—Pope. When any thing is expreſſed in a doubtful manner, or under a condition, or ſuppoſition, or the like; or when it is ſubjoined as the end or deſign; or when it is expreſſed in the form of a wiſh; it is, in any of theſe caſes, and in many others, the ſubjunctive mode of the verb: as, *if* I love, "*though* he *ſlay* me, yet will I put my truſt in him."—Job, xiii. 15. "Long *may* he *live*." The verb in this mode generally depends on ſome other verb, and has a conjunction before it; as, "*Though* after my ſkin, worms *deſtroy* this body, yet in my fleſh ſhall I *ſee* God."—Job, xix. 26.

Here it muſt be noted, that when an application is made, of the nature of a requeſt, the verb is of the imperative mode: as, "Lord *ſave* me."—Mat. xiv. When it is of the nature of a wiſh, the verb is of the ſubjunctive mode: as, "O King *live* for ever."—Dan. iii. When the verb is expreſſed without any reference to perſon or number, it is called the infinitive mode: as, *to love;* and when it is expreſſed in ſuch a form, that it may be joined to an adjective, it is called the participle, and has ſomewhat of the nature of an adjective: as, *loving*, *loved*.

To expreſs the different times of the verb, in Engliſh, it is often neceſſary to make uſe of other verbs, called therefore auxiliaries, or helpers; of which there are nine: *be*, *have*, *do*, *let*, *may*, *muſt*, *can*, *ſhall*, *will*. The two firſt are chiefly uſed for forming the different times of the verb: it is, therefore, neceſſary to know how to decline them according to perſon, number, time, and mode.

TO HAVE.

INDICATIVE MODE.—*Preſent Time.*

	Singular Number.	*Plural Number.*
Perſons.	1. I have,	We have.
	2. Thou haſt,	Ye have.
	3. He hath, or has;	They have.

Paſt Time.

	Singular Number.		*Plural Number.*	
Perſons.	1. I had,		We	had.
	2. Thou hadſt,		Ye	
	3. He had;		They	

Future Time.

Perſons.	1. I ſhall, or will	have;	We	ſhall, or will have.
	2. Thou ſhalt, or wilt		Ye	
	3. He ſhall, or will		They	

IMPERATIVE MODE.

Perſons.	1. Let me have.	Let us have.
	2. Have thou, or do thou have.	Have ye, or do ye have.
	3. Let him have.	Let them have.

SUBJUNCTIVE MODE.—*Preſent Time.*

Perſons.	1. If, or though, I	have;	If, or though, We	have.
	2. If, or though, Thou		If, or though, Ye	
	3. If, or though, He		If, or though, They	

INFINITIVE MODE.

Preſent, To have;—*Paſt*, To have had.

PARTICIPLE.

Preſent, Having;—*Perfect*, Had;—*Paſt*, Having had.

As the plural pronoun *you* is moſtly employed inſtead of the ſingular *thou*, it muſt, therefore, have the plural verb; as, *you have*, *you had*, *you were;* and not *you haſt*, *you hadſt*, *you was*. Which laſt is an erroneous ſoleciſm, being the plural pronoun of the ſecond perſon, placed in agreement with the firſt or third perſon ſingular of the verb; and which error writers of the firſt eminence have committed.

In the third perſon ſingular, preſent time, indicative mode, the termination of the verb is generally in *s*, inſtead of *th* or *eth*, in the polite and familiar ſtyle: as, he *has*, inſtead of he *hath*; he *loves*, for he *loveth*. But in the very ſolemn and ſerious ſtyle the termination is in *th*.

TO BE.

INDICATIVE MODE.—*Preſent Time.*

	Singular Number.		*Plural Number.*	
Perſons.	1. I am,		We	are.
	2. Thou art,		Ye	
	3. He is;		They	

Paſt Time.

Perſons.	1. I was,		We	were.
	2. Thou waſt,		Ye	
	3. He was;		They	

Future Time.

Perſons.	1. I ſhall, or will,	be;	We	ſhall or will be.
	2. Thou ſhalt, or wilt,		Ye	
	3. He ſhall, or will,		They	

IMPERATIVE MODE.

Perſons.	1. Let me be,	Let us be.
	2. Be thou, or do thou be,	Be ye, or do ye be.
	3. Let him be;	Let them be.

SUBJUNCTIVE MODE.—*Preſent Time.*

Perſons.	1. If I	be;	If we	be.
	2. If thou		If ye	
	3. If he		If they	

Paſt Time.

Perſons.	1. I were,	We	were.
	2. Thou wert,	Ye	
	3. He were;	They	

INFINITIVE MODE.

Preſent, To be;—*Paſt*, To have been.

PARTICIPLE.

Preſent, Being;—*Perfect*, Been;—*Paſt*, Having been.

The

The Variation of the Verb active according to Perſon, Number, Time, and Mode.

TO LOVE.

INDICATIVE MODE.—*Preſent Time.*

	Singular Number.		*Plural Number.*	
Perſons.	1. I love,		We	love.
	2. Thou loveſt,		Ye	
	3. He loveth, or loves;		They	

Paſt Time.

Perſons.	1. I loved,		We	loved.
	2. Thou lovedſt,		Ye	
	3. He loved;		They	

Future Time.

Perſons.	1. I ſhall, or will,	love;	We	ſhall or will love.
	2. Thou ſhalt, or wilt,		Ye	
	3. He ſhall, or will,		They	

IMPERATIVE MODE.

Perſons.	1. Let me love,	Let us love.
	2. Love thou, or do thou love,	Love ye, or do ye love.
	3. Let him love;	Let them love.

SUBJUNCTIVE MODE.—*Preſent Time.*

Perſons.	1. I	love;	We	love.
	2. Thou		Ye	
	3. He		They	

Alſo,

Perſons.	1. I may	love;	We	may love; and have loved.
	2. Thou may		Ye	
	3. He may		They	

Paſt Time.

Perſons.	1. I might	love;	We	might love; and have loved.
	2. Thou mighteſt		Ye	
	3. He might		They	

Alſo,

Perſons.	1. I could, ſhould, or would	love; and have loved;	We	could, ſhould, or would love; and have loved.
	2. Thou couldeſt, ſhouldeſt, or wouldeſt		Ye	
	3. He could, ſhould, or would		They	

INFINITIVE MODE.

Preſent, To love;—*Paſt*, To have loved.

PARTICIPLES.

Preſent, Loving;—*Perfect*, Loved;—*Paſt*, Having loved.

In the plural number of the ſubjunctive mode, the imperfect and perfect times are put together. The verb in the preſent time, and the auxiliary of the preſent and paſt imperfect times of this mode, frequently have a future ſignification; as, "*if* he *arrive* (hereafter) we ſhall do well:" "if he *ſhould*, or *would* come (to-morrow), *I might*, *would*, *could*, or *ſhould* ſpeak to him." The auxiliaries *ſhould* and *would*, in the paſt imperfect, are alſo uſed to expreſs the three ſpecies of time, preſent, paſt, and future: as, "it *is* my deſire that he *ſhould* or *would* come (now or to-morrow), and it *was* my deſire that he *ſhould* or *would* come (yeſterday)." Therefore, in this mode, the time of the verb is known moſtly by the nature of the ſentence.

The three grand diviſions of time, into preſent, paſt, and future, are called indefinite, or undeterminate time; but each of theſe are ſubdivided into imperfect and perfect time, which ſubdiviſion is called definite or determined time.

Definite or determined Time.

Active Verb.	Paſſive Verb.	Neuter Verb.
	Preſent Imperfect.	
I am (now) loving,	I am loved,	I am entering.
	Preſent Perfect.	
I have (now) loved,	I have been loved,	I have entered.
	Paſt Imperfect.	
I was (then) loving,	I was loved,	I was entering.
	Paſt Perfect.	
I had (then) loved,	I had been loved,	I had entered.
	Future Imperfect.	
I ſhall (then) be loving,	I ſhall be loved,	I ſhall be entering.
	Future Perfect.	
I ſhall (then) have loved,	I ſhall have been loved,	I ſhall have entered.

It is not neceſſary to give the variations of the definite times, as they are formed only by the proper variations of the auxiliaries, joined to the preſent or perfect participle.

In the formation of theſe definite times, the active and neuter verbs are alike; the paſſive verb differs from the active, only in having the perfect participle inſtead of the preſent in the imperfect times; and having the perfect participle *been* in the perfect times.

The paſſive verb is formed through all its variations of perſon, number, time, and mode, by adding the perfect, or paſſive participle (both which are the ſame, and in all regular verbs the ſame as the indefinite paſt time active) to the auxiliary verb *to be*, through all its variations: as, *I am loved*, *I was loved*, *I ſhall be loved*, *thou art loved*, &c.; the object is placed before the verb, and is in the nominative caſe, and the agent follows the verb, and is in the objective caſe, accompanied with a prepoſition: as, *I am loved by him*. Here the pronoun *him* is the agent, as he performs the action, viz. *loves*, and is placed in the objective caſe; and the perſonal pronoun *I* is the object, and is placed firſt, and is in the nominative caſe.

The neuter verb is formed through all its variations like the active verb; but as it does not expreſs any action or paſſion, but only a ſtate or condition of being, it, conſequently, can have no object to be acted upon: it therefore has only an agent, and that always in the nominative caſe: as, *I am*, *I ſleep*. But in ſome inſtances the neuter verb has a paſſive form; this is in ſome verbs which ſignify a ſort of motion, or change of place or condition: as, *I am riſen*, *I was fallen*, *I am come*, *I was gone*. Theſe verbs partake ſomewhat of the nature of the paſſive verb, though they have ſtill the neuter ſignification. The auxiliary *am*, *was*, defines the time of the verb.

The two principal auxiliaries have already been declined through all their variations. The auxiliary *to have*, through its ſeveral variations, is placed only before the perfect participle; *to be* is alſo placed before the preſent and perfect participle. The other auxiliaries are placed before the verb, or another auxiliary in its original form.

The nature of the other auxiliaries ſhould be juſt mentioned. *Do*, paſt time *did*, expreſſes the action, or the time of it, with peculiar force. It is much uſed in interrogative and negative ſentences. It alſo ſupplies the place of a verb, and renders the repetition of it unneceſſary; as, "go and *do* thou likewiſe." *Let* expreſſes permiſſion, but its chief uſe is in forming the imperative mode, which it does,

when to	a ſuperior,	by	petition.
	an equal,		perſuaſion.
	an inferior,		command.

The *poſſibility* of performing an action depends upon the power of the agent; and is expreſſed,

when	*abſolute,*	by the auxiliary	*can.*
	conditional,		*could.*

The *liberty* of performing an action depends upon a freedom from all hindrances; and is expreſſed,

when	*abſolute,*	by the auxiliary	*may.*
	conditional,		*might.*

When the agent expreſſes the *reſolution of his own will*, to perform an action; it is expreſſed,

if	*abſolute,*	by the auxiliary	*will*
	conditional,		*would.*

The *neceſſity* of performing an action from ſome *external obligation*, whether it be *natural* or *moral*, and what we call *duty;* is expreſſed,

if	*abſolute,*	by the auxiliary	*muſt, ought, ſhall.*
	conditional,		*muſt, ought, ſhould.*

Some of the auxiliaries vary their import according to the perſon with which they are joined; thus *will*, in the firſt perſon, both ſingular and plural, promiſes or threatens; in the ſecond and third perſons only foretels; *ſhall*, on the contrary, in the firſt only foretels; in the ſecond or third, promiſes, commands, or threatens. But this rule regards explicative ſentences only; for in interrogative ſentences generally the reverſe takes place: thus, I *will* write, we *will* write, they *ſhall* write, you *ſhall* write; theſe promiſe, command, or threaten: but, I *ſhall* write, we *ſhall* write, they *will* write, ye *will* write, expreſs event only, and ſimply foretel. The verb *to will* is quite a different word from the auxiliary *will*, and is formed regularly: as, I *will*, thou *willeſt*, he *willeth*, or *wills*.

The auxiliaries *could*, *ſhould*, *would*, *may*, *might*, are uſed in forming the ſubjunctive mode. The auxiliary *muſt* has no variation.

When two or more verbs or auxiliaries, or both, are joined together, the firſt only of them is varied according to perſon, number, time, and mode.

Before we proceed to irregular verbs, it may be neceſſary to ſay a few words concerning, what are called, by grammarians, contracted verbs.

Verbs which end in *ch*, *ck*, *p*, *x*, *ll*, *ſs*, in forming the paſt time active, and the participle perfect or paſſive, often change the *d* or *ed* (which all regular verbs end in, in theſe forms) into *t*: as, *fetched*, *fetcht*; *packed*, *packt*; *ſnapped*, *ſnapt*; *mixed*, *mixt*; omitting alſo one of the double conſonants; *dwelled*, *dwelt*; *preſſed*, *preſt*.

Some verbs that end in *ve*, beſides the contraction, change the *ve* into *f*: as, *bereave*, *bereft*; *leave*, *left*.

Moſt contracted verbs have the entire as well as the contracted form; and the entire form is always to be preferred to the contracted; which latter is only a corruption of the verb, and an error in the rules of grammar, introduced into converſation for the ſake of a more agreeable ſound. This is the reaſon of the contraction of the verb of the ſecond perſon ſingular, which was originally written *lovedeſt*, *turnedeſt*; but is now contracted into *lovedſt*, *turnedſt*. And the third perſon, which was formerly *loveth*, *turneth*, is now in moſt ſtyles written *loves*, *turns*.

IRREGULAR VERBS are next to be conſidered. In all regular verbs the indefinite paſt time active, and the participle perfect, or paſſive, are formed by adding to the verb *ed*, or *d* only when the verb ends in *e*: as, *turn*, *turned*; *love*, *loved*. Verbs which vary from this rule are called irregular verbs.

The Engliſh language being compounded of the Saxon, and Norman French, muſt neceſſarily partake of the nature of both; but the formation of all our verbs is derived from the Saxon.

All our irregular verbs are monoſyllables, except the compounded ones; and are generally the ſame verbs which are irregular in the Saxon.

There are three claſſes of irregular verbs. The firſt claſs conſiſts of thoſe verbs which are become irregular by contraction, and either have the preſent and paſt time active, and participle perfect, or paſſive, all exactly alike; or elſe vary in the formation of the paſt time, and participle, from the preſent, by ſhortening the diphthong, or changing the *d* at the end of the verb into *t*. The ſecond claſs of irregular verbs are thoſe that in the paſt time and participle end in *ght*, and change the vowel or diphthong of the preſent time into *au* or *ou*. Irregulars of the third claſs form the paſt time by changing the vowel or diphthong of the preſent; and form the participle, by adding the termination *en*, with generally the change of the vowel or diphthong.

Irregular Verbs of the Firſt Claſs, which have the Preſent, Paſt Time, and Participle alike.

Beat,
burſt,
caſt,
coſt,
cut,
heat*,
hit,
hurt,
knit,
let,
lift*,
light*,
put,
quit*,
read,
rent,
rid,
ſet,
ſhed,
ſhred,
ſhut,
ſlit,
ſplit,
ſpread,
thruſt,
wet*.

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* The verbs marked thus *, throughout the three claſſes of irregulars, have the regular as well as the irregular form in uſe.

The two firſt have alſo *beaten* and *burſten* in the participle; and thus they ſometimes belong to the irregulars of the third claſs. The verb *light* is pronounced ſhort in the paſt time and participle; as, I have *lit* the candle; but the regular form is preferable—*lighted*. The verb *read* is alſo pronounced ſhort in the paſt time and participle: as, I have *red*, for *read*.

The following vary in the form of the paſt time and participle from the preſent: lead, led; ſweat, ſwet*; meet, met; bleed, bled; breed, bred; feed, fed; ſpeed, ſped; bend, bent*; lend, lent; rend, rent; ſend, ſent; ſpend, ſpent; build, built*; geld, gelt; gild, gilt*; gird, girt; loſe, loſt. The following are formed by contraction: have, had; make, made; flee, fled; ſhoe, ſhod. And ſome change the vowel alſo; as, ſell, ſold; tell, told; clothe, clad*; ſtand, ſtood; dare, durſt.

Irregular Verbs of the Second Claſs.

Bring,	brought.
buy,	bought.
catch,	caught.
fight,	fought.
teach,	taught.
think,	thought.
ſeek,	ſought.
work,	wrought.

Irregular Verbs of the Third Claſs.

	Preſent Time.	Paſt Time.	Participle.
a changed into *e*.	Fall,	fell,	fallen.
a into *o*.	awake,	awoke,	(awaked).
a into *oo*.	forſake,	forſook,	forſaken.
	ſhake,	ſhook,	ſhaken.
	take,	took,	taken.
aw into *ew*.	draw,	drew,	drawn.
ay into *ew*.	ſlay,	ſlew,	ſlain.
e into *a* or *o*.	get,	gat or got,	gotten.
	help,	(helped),	holpen*.
	melt,	(melted),	molten*.
	ſwell,	(ſwelled),	ſwollen.
ea into *a* or *o*.	eat,	ate,	eaten.
	bear,	bare or bore,	borne.
	break,	brake or broke.	broken.

	Preſent Time.	Paſt Time.	Participle.
ea into *a* or *o*.	cleave,	clave or clove*,	cloven or cleft.
	ſpeak,	ſpake or ſpoke,	ſpoken.
	ſwear,	ſware or ſwore,	ſworn.
	tear,	tare or tore,	torn.
	wear,	ware or wore,	worn.
	heave,	hove*,	hoven*.
	ſhear,	ſhore,	ſhorn.
	ſteal,	ſtole,	ſtolen or ſtoln.
	tread,	trode,	trodden.
	weave,	wove,	woven.
ee into *o*.	creep,	crope*,	crept*.
	freeze,	froze,	frozen.
	ſeethe,	ſod,	ſodden.
ee into *aw*.	ſee,	ſaw,	ſeen.
i long into *i* ſhort.	bite,	bit,	bitten.
	chide,	chid,	chidden.
	hide,	hid,	hidden.
	ſlide,	ſlid,	ſlidden.
i long into *o*.	abide,	abode,	*no participle.*
	climb,	clomb,	(climbed).
	drive,	drove,	driven.
	ride,	rode,	ridden.
	riſe,	roſe,	riſen.
	ſhine,	ſhone*,	(ſhined).
	ſhrive,	ſhrove,	ſhriven.
	ſmite,	ſmote,	ſmitten.
	ſtride,	ſtrode,	ſtridden.
	ſtrive,	ſtrove*,	ſtriven.
	thrive,	throve,	thriven.
	write,	wrote,	written.
i long into *u*.	ſtrike,	ſtruck,	ſtrucken or ſtricken.
i ſhort into *a*.	bid,	bade,	bidden.
	give,	gave,	given.
	ſit,	ſat,	ſitten.
	ſpit,	ſpat,	ſpitten.
i ſhort into *u*.	dig,	dug*,	(digged).
ie into *ay*.	lie,	lay,	lien or lain.
o into *e*.	hold,	held,	holden.
o into *i*.	do,	did,	done, *from doen.*
oo into *o*.	chooſe,	choſe,	choſen.
ow into *ew*.	blow,	blew,	blown.
	crow,	crew,	(crowed).
	grow,	grew,	grown.
	know,	knew,	known.
	throw,	threw,	thrown.
y into *ew*.	fly,	flew,	flown.

The following verbs have the participle now formed, without the termination *en;* except two of them, viz. *drunken* and *bounden;* though this was undoubtedly their original form.

	Preſent Time.	Paſt Time.	Participle.
i ſhort into or *u*.	Begin,	began,	begun.
	cling,	clang or clung,	clung.
	drink,	drank,	drunk or drunken.
	fling,	flung,	flung.
	ring,	rang or rung,	rung.
	ſhrink,	ſhrank or ſhrunk,	ſhrunk.
	ſing,	ſang or ſung,	ſung.
	ſink,	ſank or ſunk,	ſunk.
	ſling,	ſlang or ſlung,	ſlung.
	ſlink,	ſlunk,	ſlunk.
	ſpin,	ſpan or ſpun,	ſpun.
	ſpring,	ſprang or ſprung,	ſprung.
	ſting,	ſtung,	ſtung.
	ſtink,	ſtank or ſtunk,	ſtunk.
	ſtring,	ſtrung,	ſtrung.
	ſwim,	ſwam or ſwum,	ſwum.
	ſwing,	ſwung,	ſwung.
	wring,	wrung,	wrung.
i long into *ou*.	bind,	bound,	bound or bounden.
	find,	found,	found.
	grind,	ground,	ground.
	wind,	wound,	wound.
a into *u*.	hang,	hung*,	hung*.
oo into *o*.	ſhoot,	ſhot,	ſhot.
i into *u*.	ſtick,	ſtuck,	ſtuck.
o into *a*.	come,	came,	come.
u into *a*.	run,	ran,	run.
i into *o*.	win,	won,	won.

The following are irregular in the participle only; and even then they do not change the vowel:

Lade,	(laded),	laden.
rive,	(rived),	riven.
ſhave,	(ſhaved),	ſhaven.
ſhow,	(ſhowed),	ſhown.
wreath,	(wreathed),	wreathen.
writhe,	(writhed),	writhen.

The following alſo are irregular in the participle only, which they form in the ſame manner:

Bake,
fold,
grave,
hew,
load,
mow,
owe,
ſaw,
ſhape,
ſhew,
ſow,
ſtraw,
ſtrew or ſtrow,
waſh,
wax.

Theſe have alſo the regular as well as the irregular form; and the regular form ſhould always be preferred.

Beſides the irregular verbs, there are alſo others called defective verbs; which are not only irregular in their forms, but alſo wanting in ſome of their forms; ſome have no paſt time, ſome no participle, and others neither..

Moſt of the auxiliary verbs are of this claſs.

Preſent Time.	Paſt Time.	Participle.
Am,	was,	been.
can,	could,	*no participle.*
go,	went,	gone.
may,	might,	*no participle.*
muſt,	*no paſt,*	*no participle.*
quoth,	quoth,	*no participle.*
ſhall,	ſhould,	*no participle.*
weet, wit, or wot,	wot,	*no participle.*
will,	would,	*no participle.*
wis,	wiſt,	*no participle.*

The whole number of verbs in the Engliſh language, regular and irregular, ſimple and compounded, taken together, amount to about 4300: whereof fifty-two are irregulars of the firſt claſs, eight irregulars of the ſecond claſs, and 107 irregulars of the third claſs; which, with the ten defective verbs, make 177: all the reſt are regular verbs, and have the paſt time active, and participle perfect or paſſive, formed alike, and ending in *d* or *ed*, as was before obſerved. Of the irregular verbs there are not ſo many as 100, which have different forms for the paſt time active, and the participle. This has given occaſion to introduce a great corruption, by confounding the different forms of the paſt time and participle with each other, in irregular verbs. Thus, the participle is ſometimes uſed for the paſt time: as, *he run*, for *he ran; he drunk*, for *he drank:* but the paſt time is very frequently uſed inſtead of the participle: as, I *have rode*, for I *have ridden;* I have *ſat*, for *ſitten;* I *have got*, for *gotten*. This error is frequently committed by our beſt writers, and the vulgar tranſlation of the Old and New Teſtament, which is the beſt ſtandard of the Engliſh grammar, is not free from this fault: *held* is there often uſed for *holden;* *hid* ſometimes for *hidden;* and *bid* for *bidden;* and *begot* is twice uſed for *begotten*. This error is, however, an enormous ſoleciſm; and the impropriety of it will appear by the abuſe of ſome verbs which have not been ſo corrupted: as, I have *ſaw*, for I have *ſeen;* I have *did*, for I have *done;* I have *went*, for I have *gone;* which are not a whit more ungrammatical than any other verbs which have the paſt time, inſtead of the participle, after the auxiliary *have*.

In the formation of the preſent participle, it is to be obſerved, that if the verb end in a ſingle conſonant, following a ſingle vowel, and accented on the laſt ſyllable, if it conſiſts of more than one ſyllable, it doubles the laſt conſonant in forming the preſent participle; alſo in every other form of the verb in which a ſyllable is added: as, *cut*, *cutting*, *regret*, *regretting*, *regretteth*, *regretted*. Verbs which end in *e* omit the *e* in the preſent participle: as, *increaſe*, *increaſing*: all other verbs form the preſent participle by barely adding *ing* to the verb in its original form: as, *turn*, *turning*.

The ADVERB, as its name imports, is added to the verb, and alſo to the adjective, to expreſs ſome modification, or other circumſtance of the action expreſſed by the verb, or the quality expreſſed by the adjective: as, the time, he reads *now;*—diſtance, the countries lie *wide apart;*—relation, they are *cloſely* united;—quantity, an *exceeding* high mountain;—quality, to live *ſoberly;*—compariſon, they are much *alike;*—doubt, *poſſibly*, *perhaps;*—affirmation, *yes*, *certainly;*—negation, *no;*—demonſtration, *evidently;*—interrogation, *how*, *what;*—manner, *well*, *ill;*—order, *regularly;*—place, *here*, *there;*—motion, *ſlow*, *ſwift*.

The adverb in Engliſh has no variation, except a few, which have the degrees of compariſon; and in theſe the

SECT. IV.

OF SYNTAX.

SYNTAX is the right ordering and framing of words, in order to form ſentences with grammatical propriety. And for this purpoſe, *words are ſaid to govern, or agree with each other.*

When a word governs another, it cauſes that particular word which it governs, to be in ſuch a particular number, gender, caſe, perſon, time, or mode.

When a word agrees with another, it is in that particular mode, number, caſe, &c. which is required by the word that governs it. Thus: "And ſhe ſhall bring forth a Son, and thou ſhalt call his name Jeſus; for he ſhall ſave his people from their ſins."—Matt. i. 21. In this ſentence *ſhe* is a pronoun of the third perſon, ſingular, feminine gender, and ought to agree with the ſubject foregoing, namely, the Virgin Mary;—*ſhall bring*, the future time of the active verb *bring* (referring to the time of the birth of the child), the third-perſon, ſingular number, to agree with the pronoun *ſhe*, and indicative mode, as it ſimply declares the event;—*forth*, a prepoſition added to the verb *bring*, and which alters its meaning from *bring*, to *bring forth*, which ſignifies to bear or produce;—*a*, the indefinite article;—*Son*, noun ſubſtantive, ſingular number, maſculine gender, objective caſe;—*and*, a conjunction copulative, connecting the following and foregoing ſentences together;—*thou*, the pronoun of the ſecond perſon, ſingular, and agent of this ſentence;—*ſhalt call*, the future time of the active verb *call*, ſecond perſon, ſingular, being governed by *thou*, indicative mode, as it only foretels or declares the name of the child, but does not command it;—*his*, a poſſeſſive pronoun, third perſon, ſingular, maſculine gender, governed by the noun *Son*;—*name*, a ſubſtantive common, and the object of the ſentence;—*Jeſus*, a ſubſtantive proper, maſculine gender, agreeing with the noun *Son*, and nominative caſe;—*for*, a copulative conjunction;—*he*, a pronoun, third perſon, ſingular, maſculine gender, nominative caſe, being a ſubſtitute for the noun *Jeſus*, and governed by it, and the agent of this ſentence;—*ſhall ſave*, the future time of the active verb (agreeing in time with the other verbs in the ſentence, which are all future), third perſon, ſingular, governed by the pronoun *he*, indicative mode;—*his*, a poſſeſſive pronoun;—*people*, a plural noun, neuter gender, objective caſe;—*from*, a prepoſition, ſhewing the relation between the nouns *people* and *ſins*;—*their*, a poſſeſſive pronominal adjective, and as ſuch joined to the plural noun *ſins*, objective caſe.

SENTENCES are either ſimple or compounded.

A ſimple ſentence hath but one ſubject, or agent, and one verb in the indicative, imperative, or ſubjunctive mode; and conſiſts of three parts, if the verb be active; the agent, the attribute, and the object: as was ſeen page 12.

A compound ſentence conſiſts of two or more ſimple ſentences united together by the aid of conjunctions, as hath been ſhewn before: or by relatives, as will be ſeen hereafter.

As language and ſtyle is only an aſſemblage of ſentences, too much attention can hardly be beſtowed upon their conſtruction. We will therefore take a view of the rules of Engliſh Syntax, as they regard the ſeveral parts of ſpeech reſpectively.

The ARTICLE, as hath been ſeen, if definite, is placed before both the ſingular and plural noun: as, *the man*, *the men*: the indefinite article *a* is placed before the ſingular noun only: as, *a man*.

The SUBSTANTIVE governs both the pronoun and the verb: for if the ſubſtantive be plural, it requires both the pronoun and verb to be plural alſo; and if ſingular, they muſt alſo be ſingular: as, "nothing has ſo much expoſed *men* of learning to contempt and ridicule, as *their* ignorance of *things* which *are* known to all but *themſelves*."—Johnſon's

Rambler. Example of the ſingular: "*Seneca ſpeaks* in the natural and genuine language of a man of honour, when *he declares*, that were there no God to ſee or puniſh vice, *he would* not commit it."—Guardian.

There are ſome nouns called nouns of multitude, which ſignify many, and have the pronoun and verb agreeing with them, either in the ſingular or plural number; but if they convey a plural idea, the verb and pronoun ſhould be plural likewiſe; if they convey a ſingular idea, the verb and pronoun ſhould be ſingular: as, "the *aſſembly* of the wicked *have* encloſed me"—Pſalm xxii. 16. inſtead of *hath* encloſed me. "My *people is* fooliſh"—Jer. iv. 22. inſtead of *are fooliſh*.

Two or more ſingular nouns, joined by copulative conjunctions, have the verbs, nouns, and pronouns agreeing with them in the plural number: as, "Shakeſpeare and Milton *were* the moſt eminent *poets* of the Engliſh nation."

But ſometimes the verb follows in the ſingular number, and refers to each of the preceding nouns taken ſeparately; as, "*ſand*, and *ſalt*, and *a maſs of iron is* eaſier to bear than a man without underſtanding."

Nouns of number, weight, and meaſure, are often uſed in the ſingular form, when they are joined to numeral adjectives; though they denote plurality: as, an hundred *thouſand*, inſtead of *thouſands*; an hundred *pound* weight; ſix *foot*; forty *fathom*.

Nouns, whether of the maſculine, feminine, or neuter gender, always govern the ſame gender in the pronoun: as, "I have a *friend*, *who*, becauſe *he* knows *his* own fidelity and uſefulneſs, is never willing to ſink into a companion." "I have a *wife*, whoſe beauty firſt ſeduced me, and whoſe wit confirmed *her* conqueſt."—Johnſon's Rambler.

Every noun in the nominative caſe belongs to ſome verb, either expreſſed or underſtood, except the caſe abſolute, as will be ſeen hereafter; and when an addreſs is made to a perſon, called the vocative caſe. Thus in the anſwer to this queſtion, Who conquered the Perſians?—*Alexander*; that is, Alexander *conquered them*.

Every noun in the poſſeſſive caſe has alſo ſome noun belonging to it: as, St. Paul's, that is, St. Paul's *Cathedral*; St. James's, St. James's *Palace*.

The noun has no different form in Engliſh for the objective caſe, though this caſe be founded in nature and grammar: as, "*books* can never teach the uſe of *books*."

The PRONOUN, being a ſubſtitute for the noun, muſt conſequently have the ſame nature, with regard to the government and agreement of nouns and verbs.

When two or more pronouns of the ſingular number are joined together, to make the plural pronoun agree with them in perſon, the ſecond perſon is preferred before the third, and the firſt perſon before both: as, *he* and *you* did as *you* were commanded; *I*, *thou*, and *he* loſt *our* characters by it.

The neuter pronoun *it* is employed in a threefold ſenſe:—firſt, it expreſſes the ſubject of any diſcourſe: as, *it* ſo happened; who is *it*?—ſecondly, the ſtate of any perſon or thing: as, how is *it* with you?—thirdly, the thing that is the cauſe of any event: as, *it* was I that did it; *it* is theſe that corrupt the mind.

PRONOMINAL ADJECTIVES have ſome ſubſtantive belonging to them, either expreſſed or underſtood.

The definitives *this*, *that*, in the plural *theſe*, *thoſe*, muſt always agree with their ſubſtantive in number: thus, "By *this means* thou ſhalt have no portion on this ſide the river." Ezra, iv. 16. "I have wept not *this* forty *years*." It ſhould be *theſe* means; *theſe* forty years. Again, "*They are theſe kind* of gods which Horace mentions in his allegorical veſſel."—Addiſon's Dialogues, ii. on Medals. Here it ſhould be *theſe kinds*, or *this kind*.

The relatives *who*, *which*, *that*, have no variation of gender, and therefore muſt agree with their antecedent in this reſpect. For every relative muſt have a noun or pronoun, to which it refers, either expreſſed or underſtood; which is therefore called its antecedent: as, "*Who* ſteals my purſe, ſteals traſh."—Shakeſpeare. That is, the *man who* ſteals my purſe, ſteals traſh.

Who is applied to perſons only, and is either maſculine or feminine; *which* is now applied to things only, and irrational animals; *that* is uſed both for perſons and things, but it ſhould be confined to the latter, particularly in the ſolemn ſtyle; *what* ſtands for both the relative and the antecedent: as, this is *what* was ſpoken of before; that is, the *thing which* was ſpoken of before.

The relative is the nominative caſe of the verb, when no other nominative comes between it and the verb; but if another nominative comes between it and the verb, the relative is governed according to the ſenſe of the ſentence, as, "the God *who* made me, *whoſe* I am, and *whom* I ought to ſerve." In the different members of this ſentence, the relative in uſed in a different ſenſe; in the firſt member it ſtands for the nominative caſe of the verb, having no other nominative caſe between it and the verb; in the ſecond member it ſtands for the poſſeſſive caſe; in the third member it repreſents the object.

Every relative is of the ſame perſon with the antecedent to which it refers, and the verb muſt therefore agree with it: as, "*I that* (who) ſpeak unto thee *am he*." John, iv. 22. "*Thou that* (who) dwelleſt betwen the Cherubims." Pſalm lxxx. 1.

The relative is often omitted, and underſtood, without being expreſſed; as, "the God I ſerve;" that is, the God *whom* I ſerve. The relative ſhould ſeldom be omitted even in the familiar ſtyle, and never in the ſerious and ſolemn ſtyles.

The proper uſe of the relative conſiſts in the property of preſenting the antecedent to the mind of the hearer or reader, without any ambiguity.

The diſtributive pronominal adjectives *each*, *every*, *either*, always agree with the nouns, pronouns, and verbs in the ſingular number only; as, "The king of Iſrael, and Jehoſaphat the king of Judah, *ſat each* (king) on *his* throne, having (both) put on their robes."—1 Kings, xxii. 10. "*Every one ought* to cheriſh and encourage in *himſelf* this modeſty and aſſurance I have here mentioned."—Spectator. *Each* ſignifies ſies *both* taken diſtinctly or ſeparately; *either* ſignifies *only the one* or *the other*, taken disjunctively.

The ADJECTIVE, having no variation of number or gender, muſt agree with its ſubſtantive.

The adjective is always placed immediately before the noun: as, a *good man*; except in the following inſtances: firſt when ſomething depends upon the adjective: as, "feed me with *food convenient* for me." Here the relative and the verb are underſtood: as, food *that is* convenient.—Secondly, when the adjective is emphatical: as, George the *Third*; St. John the *Divine*.—Thirdly, when the verb *to be*, or any auxiliary belonging to it, is placed between the adjective and the noun: as, "How *beautiful are* the tabernacles of the Lord of Hoſts!" *happy ſhall* he be.—Fourthly, when two or more adjectives belong to the ſame noun: as, "a general *brave* and *ſkilful*."

The article is placed moſtly before the adjective, except the adjectives *all*, *ſuch*, and *many*: as, *all the* men; *ſuch a* man; *many a* man. Or when an adjective is joined to the adverbs *ſo*, *as*, *how*: as, not *ſo large a* concern; *as good a* man; *how fine a* ſight is this!

Every adjective has ſome ſubſtantive belonging to it, either expreſſed or underſtood: as, "the *twelve*," for the twelve *apoſtles*.

Sometimes the adjective is uſed as a ſubſtantive, and has an adjective joined to it: as, the *greateſt evil*, the *chief good*. At other times the ſubſtantive becomes an adjective, and has a ſubſtantive added to it, and linked to it by a mark of conjunction: as, *animal-food*, *milk-diet*.

The VERB, or attribute of a ſentence, always agrees with the agent or nominative caſe.

For every verb, except the participle or the infinitive mode, hath a nominative caſe belonging to it, either expreſſed or underſtood: as, "*awake*, *ariſe*, or *be* for ever fallen;" that is, *awake ye*, *ariſe ye*, &c. And when the verb is active, it hath, moreover, an objective caſe: as, "God made *man*."

The nominative caſe in Engliſh is placed before the verb, and the objective caſe after the verb: except, firſt, when the agent or object is expreſſed by a pronoun, in which caſe they are often reverſed in their order: as, "*Whom* ye ignorantly *worſhip, him declare I* unto you."—Secondly, when the verb is neuter, the nominative caſe is ſometimes placed after it: as, "preſently entered the *troops*."—Thirdly, when the adverb *there* is connected with the neuter verb: as, "*there* was a *man* in the land of Uz." Job, i. 1.—Fourthly, when the relatives are uſed, though in the objective caſe, they are always placed before the verb, as are alſo their compounds, *whoever*, *whichever*, &c. as, "*whomſoever* you *chooſe*;" "this is he *whom* you *ſeek*."—Fifthly, when a ſentence is interrogative, or when a queſtion is aſked, the nominative follows the principal verb, or the auxiliary: as, *loveſt thou* me? or *doſt thou* love me?—Sixthly, when the imperative mode is employed, or when any thing is commanded or requeſted to be done, the nominative caſe follows the verb or auxiliary: as, *come thou* here, or *do thou* come here; or the auxiliary *let* with the objective caſe is uſed: as, *let me* go; *let us* be gone.—Seventhly, when the conjunctions *if* or *though*, are omitted, the nominative caſe ſometimes follows the auxiliary or the verb, and the verb is in the ſubjunctive mode: as, "*had he* done this he had been right;" "*ſing ſhe* ever ſo ſweetly."

The neuter verb expreſſing no action, but only a ſtate or condition of being, it conſequently can have no objective caſe expreſſing the object of an action. Whenever a noun follows a neuter verb, it either expreſſes the ſame idea with the verb, or ſome circumſtance of the verb; a prepoſition being underſtood: as, *to fight a duel*; *to ride a mile*; that is, the ſpace of a mile.

When the pronoun follows the neuter verb *to be*, it ſhould always be in the nominative caſe: as, it *was I*; it *was he*. Except the verb be in the infinitive mode; as, it was thought *to be him*. Thus, "*Whom* ſay ye that I *am*"—Matt. xiv. is improper; it ought to be, *who* ſay ye, &c.

When a verb immediately follows another verb, the laſt verb ſhould be in the infinitive mode: as, boys *love to play*: but the following verbs admit of other verbs following ~~them~~ without being in the infinitive mode: *bid, dare, need, make,* incorrect *ſee, hear, feel,* and all the auxiliaries, except *be* and *have,* which have already been ſpoken of.

The infinitive mode has the nature of a ſubſtantive, expreſſing the action itſelf in the abſtract, without regard to any perſon as an agent. Thus the infinitive is employed as a ſubſtantive in the nominative caſe: as, *to teach* is a maſter's duty; and in the objective caſe: as, mankind love *to learn*. [illegible]

The prepoſition *for*, placed before the infinitive mode, is now become obſolete.

The participle has the nature of an adjective, as the infinitive mode has that of a ſubſtantive; and the participle is ſometimes joined to a ſubſtantive, like an adjective, merely to denote its quality; and admits of the degrees of compariſon: as, *a learned man, a more learned man, a moſt learned man:* and the preſent participle is varied in the ſame mander: as, *a loving father, a more loving father, a moſt loving father.*

When the participle has an article before it, and the prepoſition *of* after it, it then has the nature of a ſubſtantive, expreſſing the action itſelf which the verb ſignifies: as, "by *the preaching of* repentance," and not by preaching of repentance, as is erroneouſly written in the Collect for St. John the Baptiſt. For when the participle has the article before it, the prepoſition ſhould always follow it; and when the prepoſition follows it, the article ſhould precede it. But the participle may have a prepoſition *before* it; which anſwers to what is in Latin called the gerund: as, "Juſtice conſiſts *in puniſhing* the guilty; and *in protecting* the innocent."

The caſe abſolute is, when a ſentence is formed without a finite verb, by the aid of the participle, or infinitive mode. This caſe is in Engliſh always the nominative; and the adverbs of time, *when, while, after,* &c. which ſhould otherwiſe be inſerted, are omitted: as, "the doors *being* ſhut, Jesus ſtood in the midſt;" inſtead of, *when* the doors *were* ſhut, &c.

The infinitive mode is often made abſolute, when it ſupplies the place of the conjunction *that* with the ſubjunctive mode: as, "*to begin* with the firſt;" "*to conclude*;" that is, *that I may begin* with the firſt; *that I may conclude.*

The participle is alſo made abſolute in the ſame manner as the infinitive mode; as, "this, ſtrictly *ſpeaking*, is the ſenſe;" that is, this, *if I may ſpeak* ſtrictly, is the ſenſe.

ADVERBS have no government. They ſhould always be placed as near as poſſible to the word which they modify.

Their place is generally before adjectives, after active and neuter verbs, and ſometimes between the auxiliary and the verb: as, "He was a *very* wiſe prince; he governed *mildly*; and was *reverentially* beloved by his ſubjects."

When two negatives occur in one ſentence, they deſtroy one another, and are equal to an affirmative: as,

> *Nor* did they *not* perceive the evil plight
> In which they were, or the ſharp pains *not* feel.
>
> MILTON, P. L. i. 335.

There is no adverb ſo liable to be miſplaced as the adverb *only*: moſt of our claſſical writers have erred in placing this adverb at too great diſtance from the word which it modifies: thus, it is commonly ſaid, "they *only* fought *one hour*;" whereas it ſhould be, they fought *only* one hour.

PREPOSITIONS have a government of caſe, and in Engliſh they always require the objective caſe after them: as, *with her*, *from me*, *to him*, *by us*, &c.

When the prepoſition governs a relative, it ſhould be placed before the relative; as, "this is he *of whom* it is written;" though ſome writers have placed the prepoſition at the end of the ſentence, and ſeparate from the relative, which is always ungraceful, and can hardly be reconciled to the rules of grammar; as, "Horace is an author *whom* I am much delighted *with*."

Many of our beſt writers have committed errors in the uſe of the prepoſition, ſome omitting it entirely when it ſhould be uſed, others uſing one prepoſition for another: as, *into* for *in*, *to*, or *unto*; *for* for *of*; *of* for *on*, &c. as may be ſeen by an attentive peruſal of the claſſical writers in the Engliſh tongue.

The noun moſtly requires after it the ſame prepoſition as the verb from which it is formed would do: as, "the wiſeſt princes need not think it any diminution *to* (of) their greatneſs, or derogation *to* (from) their ſufficiency, to rely upon counſel."—Bacon, Eſſay 20. Here the nouns *diminution* and *derogation*, being formed from the verbs *diminiſh* and *derogate*, require the prepoſitions *of* and *from*.

The prepoſitions *to* and *for* are often underſtood, without being expreſſed, before the objective caſe of the pronoun: as, get *me* a place; pay *him* the money you owe *them*; that is, get *for* me a place; pay *to* him the money you owe *to* them." This is a relic of the Saxon; in which language, thoſe pronouns which are in the objective caſe in Engliſh, are in the dative caſe; and conſequently have the prepoſitions *to* and *for* underſtood. *In* or *on* is alſo often omitted before nouns expreſſing time: as, laſt week; next year; to-morrow: that is, *in* laſt week; *in* next year; *on* to-morrow.

The prepoſition ſubjoined to the adverbs *here*, *there*, *where*: as, *herewith* or *hereof*, *therefore*, *therewith*, *wherewith*, *whereupon*, &c. have the conſtruction or nature of a pronoun; but they are now almoſt obſolete, except in the very ſolemn ſtyle only.

CONJUNCTIONS have only a government of mode. Some require the indicative, others the ſubjunctive mode.

The following govern the ſubjunctive: firſt, hypothetical conjunctions: as, "*if* thou *be* the Son of God." Matt. iv. 3. —Secondly, conditional conjunctions: as, "*though* he *ſlay* me, yet will I put my truſt in him." Job, xiii.—Thirdly, diſtributive conjunctions; "*whether* it *were* I, *or* (whether it were) they, ſo we preach." 1 Cor. xv. 11.—Fourthly, conceſſive conjunctions; "*Unleſs* he *waſh* his fleſh." Lev. xii. 6.—Fifthly, exceptive conjunctions; "no power, *except* it *were* given from above."—Sixthly, the conjunctions *leſt* and *that*, following a verb in the imperative mode; as, "let him that ſtandeth take heed *leſt* he *fall*." 1 Cor. x. 12. "Take heed *that* thou *ſpeak* not to Jacob." Gen. xxxi. 24.

Here it muſt be noted, that all the foregoing conjunctions may be uſed, and often are, with the indicative mode; when the circumſtance expreſſed is of a more abſolute and certain nature. It is the circumſtance being of a doubtful nature, or expreſſed under a condition, or ſuppoſition, or in the form of a wiſh, that determines the verb to be in the ſubjunctive mode.

Other conjunctions of a more poſitive nature govern the indicative mode.

There are ſome conjunctions which have other conjunctions belonging to them, and anſwering to them: as, *although* (though), *yet*, *nevertheleſs*; *whether—or*; *neither* or *nor—nor*; *either—or*; *as—as*; *as—ſo*; *ſo—as*. *As—as* expreſs a compariſon of equality; "*as* good *as* another;" that is, equal in ſome or every quality; *as—ſo* expreſs a compariſon ſometimes of equality; "*as* he is, *ſo* ſhall we be;" ſometimes a compariſon of quality; "*as* the one dieth, *ſo* dieth the other;" that is, in like manner; *ſo—as* expreſs both quality and quantity: as, "Pope was not *ſo* ſublime a poet *as* Milton, nor *ſo* great a man *as* Johnſon;" *neither*, *nor*, and *not*, *nor*, expreſs a double negative, as in the foregoing example; *either—or*, a double diſtributive, as, *either* chooſe ye this *or* that.

In comparative ſentences, or when different qualities are compared, the caſe of the latter noun or pronoun is governed by the verb or prepoſition, which are ſometimes not expreſſed: as, thou art a greater man than *he*; that is, than *he is*; you ſhew him more favour than *me*; that is, than (*you ſhew*) me. Thus, by ſupplying that part of the ſentence that is underſtood, the caſe of the latter noun or pronoun will be underſtood.

INTERJECTIONS in Engliſh have no government, nor any particular place in a ſentence; they are uſed only to expreſs the ſpeaker's affection; but when they are uſed too frequently, they rather expoſe his affectation.

Theſe are the rules of Engliſh ſyntax, in the conſtruction of ſentences. But there are alſo phraſes, conſiſting of two or more words, employed in the formation of ſentences: of which the following are the moſt common:

1ſt Phraſe: when a ſubſtantive or pronoun is placed before a verb active, paſſive, or neuter: as, *I* love; *he* is loved; *thou* art.

2d Phraſe: when the ſubſtantive follows the verb neuter, or paſſive; or when the ſubſtantive following the verb intends the ſame thing as the ſubſtantive before the verb: as, I am *he;* Milton is eſteemed a *claſſic.*

3d Phraſe: when the adjective follows a verb neuter, or paſſive: as, life is *ſhort;* exerciſe *is eſteemed wholeſome.*

4th Phraſe: when the ſubſtantive follows a verb active; as, to build *a houſe.*

5th Phraſe: when a verb follows another verb: as, *they* deſire *to die.*

6th Phraſe: when one thing is ſaid to belong to another: as, Milton's poems; or the poems of Milton.

7th Phraſe: when a ſubſtantive is added to another, to explain it more fully: as, St. John the Baptiſt; King George; Robert Brown, &c. Here the latter noun is ſaid to be in appoſition to the former.

8th Phraſe: when the adjective or participle is placed before the noun: as, a *fine* day; a *loving* friend.

9th Phraſe: when the adjective is placed before a verb in the infinitive mode: as, *worthy* to live.

10th Phraſe: when an adverb is uſed with an adjective, or a verb: as, he *writes quick;* he is *very juſt.*

11th Phraſe: when a ſubſtantive with a prepoſition before it, is added to a verb or an adjective: as, he acts *with prudence;* good *for nothing.*

12th Phraſe: when the quality of a ſubject is compared with that of another ſubject; the poſitive adjective having after it the conjunction *as*; the comparative the conjunction *than*; the ſuperlative the prepoſition *of*: as, as tall *as* you; taller *than* he; talleſt *of* all.

SECT. V.

OF PUNCTUATION.

PUNCTUATION is the art of making the ſeveral points, uſed in ſentences, to expreſs the degrees of connexion between ſentences and their parts; and to expreſs the ſtops, or pauſes, as they are expreſſed in a juſt and accurate pronunciation.

Notwithſtanding the different pauſes in ſentences, and degrees of connexion between them, admit of great variety, yet we have but four points by which to expreſs them.

Thus we are often obliged to expreſs pauſes of the ſame quantity, on different occaſions, by different points; but more frequently to expreſs pauſes of different quantities by the ſame points.

The doctrine of punctuation muſt therefore be very imperfect: few rules can be given that will hold good in all caſes; but much muſt be left to the judgment of the writer.

Grammarians have followed the diviſion of the Rhetoricians, who divide all the pauſes in writing or diſcourſe into the four following:

The Period, marked thus - (.)
The Colon, marked thus - (:)
The Semicolon, marked thus - (;)
The Comma, marked thus - (,)

The Period marks a whole ſentence, either ſimple or compound, making a full and perfect ſenſe, and not connected in conſtruction with another ſentence.

The Colon marks the greateſt diviſion of a ſentence, and is a member thereof; containing a perfect ſenſe, but not a perfect ſentence.

The Semicolon is a leſs conſtructive part of a ſentence that the colon, and does not form a perfect ſenſe, but holds a middle place between the colon and the comma, being a greater pauſe than the latter, and leſs than the former.

The Comma marks the leaſt conſtructive parts of a ſentence; or it marks a ſimple ſentence.

The preciſe quantity of time required at each of theſe pauſes or points is uncertain; as the ſame compoſition may be rehearſed in a longer or ſhorter time; but the proportional quantity of time of the points is as follows: the period is a pauſe of double the quantity of time of the colon; the colon is double of the ſemicolon; the ſemicolon is double of the comma.

In order to diſcover the proper uſe of theſe points, we muſt conſider the nature of a ſentence, as divided into its conſtructive parts, and the degrees of connexion between theſe parts: as alſo the nature of an imperfect phraſe.

An imperfect phraſe contains no aſſertion, or does not amount to a propoſition or ſentence, as was ſeen in *page* 41.

A ſimple ſentence, as was before hinted, conſiſts of an agent, or ſubject, an attribute, and (if the verb be active or paſſive) an object: or it conſiſts of one agent or ſubject, and one finite verb; that is, a verb in the indicative, imperative, or ſubjunctive mode: as, "God made man;" here *God* is the agent or ſubject, as he performs the action, viz. *made man*; the verb *made* is the attribute which always expreſſes the action; and *man* is the object upon which the action is exerciſed; that is, the action of creation.

But the ſubject, or agent, and the attribute, and the object, may each of them be accompanied with ſeveral circum-

ſtances

ſtances or characteriſtics, called therefore their adjuncts; as the motive, place, time, manner, cauſe, and the like: and theſe may be connected either immediately, or mediately, to the parts of the ſentence to which they belong.

If the ſeveral adjuncts are related to the parts of a ſentence in a different manner, they are then only ſo many imperfect phraſes, and the ſentence is ſimple.

But if the ſeveral adjuncts belong to the parts of a ſentence in the ſame manner, they then become ſo many ſimple ſentences; and the ſentence is then compound.

For a compound ſentence conſiſts of two or more ſimple ſentences connected together; or it hath more than one ſubject, and one finite verb.

Thus, if ſeveral ſubjects belong in the ſame manner to one verb, or ſeveral verbs belong in the ſame manner to one ſubject, the ſubjects and verbs are to be accounted equal in number: for every ſubject, except the caſe abſolute, and the vocative caſe, muſt have its verb: and every finite verb its ſubject; and generally has its point of diſtinction.

Examples of the Uſe of the Comma.

"This ſingle conſideration will be ſufficient to extinguiſh all envy in inferior natures." In this ſentence *conſideration* is the ſubject or agent, *extinguiſh* the attribute or verb, *envy* the object: each of which is connected with its adjuncts. The ſubject does not mean any conſideration indefinitely, but a particular conſideration, treated of in the former part of the Eſſay (namely, the conſideration of the progreſs of a human ſoul towards perfection in knowledge), and here defined by the adjunct, *this ſingle* (conſideration). The attribute or verb is alſo connected with its adjuncts; immediately with *envy*, as the object of the action; and mediately by the intervention of the word *envy* .. *inferior natures*, the ſubject in which envy is extinguiſhed; the adverb *ſufficient* is the adjunct of the verb, denoting its power. It is to be obſerved, that each of theſe adjuncts belong to the verb in a different manner; *envy* belongs to it as the object; *inferior natures* as the ſubject in which the object is extinguiſhed; and *ſufficient* as the power of the verb to produce the effect. The adjuncts are, therefore, only ſo many imperfect phraſes; and the ſentence is ſimple, and admits of no points to diſtinguiſh it into parts.

"Methinks this ſingle conſideration, of the progreſs of a finite ſpirit to perfection, will be ſufficient to extinguiſh all envy in inferior natures, and all contempt in ſuperior." Here two new ſentences are introduced; the one inſerted in the middle of the former ſentence, and the other added to the end of it. The former of them, *of the progreſs of a finite ſpirit to perfection*, is connected with the attribute in the ſame manner as the agent, or *this ſingle conſideration*, for they both expreſs the ſame idea; and the verb may be joined with either of them, and the ſentence have the ſame ſenſe: as, "the progreſs of a finite ſpirit to perfection, will be ſufficient to extinguiſh all envy in inferior natures." The latter ſentence, *and all contempt in ſuperior*, is connected exactly in the ſame manner with the attribute as the object in the firſt ſentence is, though it does not convey the ſame idea; and may be made the object of the ſentence in the ſame manner as the former ſentence was made the agent; as, "Methinks this ſingle conſideration will be ſufficient to extinguiſh all contempt in ſuperior." (beings) The firſt of theſe newly inſerted ſentences may therefore be conſidered as only another agent; and the latter as another object to the verb, in the firſt-mentioned ſentence. They are therefore ſo many ſimple ſentences, and ſhould each be diſtinguiſhed by a comma; and the whole is a compounded ſentence.

And again, "A wiſe man will deſire no more than what he may get juſtly, uſe ſoberly, diſtribute cheerfully, and live upon contentedly." In this ſentence, the phraſes, *get juſtly, uſe ſoberly, diſtribute cheerfully, and live upon contentedly*, are each connected with the ſubject, he, in the ſame manner, and in effect, form ſo many diſtinct ſentences; as, "a wiſe man will deſire no more than what *he may get juſtly* : a wiſe man will deſire no more than what *he may uſe ſoberly* : a wiſe man will deſire no more than what *he may diſtribute chearfully* : a wiſe man will deſire no more than what *he may live upon contentedly*. They muſt each of them, therefore, be diſtinguiſhed by a comma. They are ſo many ſimple ſentences, and the whole is a compound ſentence.

As ſentences themſelves are divided into ſimple and compound; ſo the members of ſentences may be divided into ſimple and compound members: for whole ſentences, whether ſimple or compound, may become members of other ſentences, by means of a connexion.

The following are marked by a Comma:

Firſt, ſimple members of ſentences, cloſely connected together in one compound member or ſentence: as in the foregoing example: except, firſt, when the members are ſhort in comparative ſentences: as, in the former part of the laſt example, " a wiſe man will deſire no more than what he may get juſtly." Theſe are, in fact, two ſimple ſentences *compared* by the conjunction *than;* but, being ſhort, they are not ſeparated by a comma. Secondly, when two ſimple members or ſentences are cloſely connected by a relative, and the ſubject of the antecedent is confined to a particular ſenſe: as, " the man who is poſſeſſed of this excellent frame of mind, is not only eaſy in his thoughts, but a perfect maſter of all the powers and faculties of his ſoul."—Spectator. In this ſentence, *the man*, is connected to the following ſentences by the relative *who;* which reſtrains the idea of the antecedent to the ſenſe here mentioned.

Secondly, the caſe abſolute: as, " the doors being ſhut, Jeſus ſtood in the midſt."

Thirdly, nouns in appoſition, when conſiſting of many terms: as, " Alexander, the great, cruel, and unjuſt."

Fourthly, the vocative caſe, or when an addreſs is made to a perſon: as, " This ſaid, he formed thee, Adam: thee, O man."—Milton.

Fifthly, when there are more than two nouns, or adjectives, connected by copulatives or disjunctives; or when there are only two, if the conjunction be underſtood: as, "Raptures, tranſports, and ecſtacies are the rewards which they confer: ſighs and tears, prayers and broken hearts, are the offerings which are paid to them."—Spectator.

Sixthly, a circumſtance of importance, though only an imperfect phraſe: as, "the principle may be defective or faulty; but the conſequences it produces are ſo good, *that*, for the benefit of mankind, it ought not to be extinguiſhed." Spectator.

Laſtly, the participle with ſomething depending on it: as,

> "Now morn, her roſy ſteps in th' eaſtern clime
> *Advancing*, ſow'd the earth with orient pearl." MILTON.

The Semicolon is uſed when a ſentence, or member of a ſentence, requires a greater pauſe than a comma, yet neither forms a perfect ſenſe, nor a perfect ſentence; but is followed by ſome other member, or ſentence, with which it is cloſely connected: as,

"To look upon the ſoul as going from ſtrength to ſtrength, to conſider that ſhe is to ſhine for ever with new acceſſions of glory, and brighten to all eternity; that ſhe will be ſtill adding virtue to virtue, and knowledge to knowledge; carries in it ſomething wonderfully agreeable to that ambition which is natural to the mind of man."—Spectator.

This compound ſentence is divided into three principal parts by the ſemicolon; each part requires a greater pauſe than a comma; but neither of them expreſſes perfect ſenſe, or forms a perfect ſentence, being cloſely connected in ſenſe with each other.

The Colon is uſed when a ſentence, whether ſimple or compound, requires a greater pauſe than a ſemicolon: it always forms a perfect ſenſe, and would by itſelf form a complete ſentence, but is followed by another member, or ſentence; making the ſenſe more full and complete: as, “Were all books reduced to their quinteſſence, many a bulky author would make his appearance in a penny paper: there would be ſcarce any ſuch thing in nature as a folio: the works of an age would be contained on a few ſhelves: not to mention millions of volumes, that would be utterly annihilated.”—Spectator, No. 124.

This ſentence is divided into four parts by the colon: the firſt and laſt parts are compound members, divided by the comma; the ſecond and third are ſimple members.

Each of theſe parts contains perfect ſenſe, and would alſo form a complete ſentence, if the other parts had not been joined to it.

The colon is alſo uſed when a ſemicolon goes before, and a greater pauſe is required; though the ſentence be not complete; alſo when a ſpeech or an example is introduced.

The PERIOD is uſed when a ſentence is ſo far finiſhed as not to be connected in conſtruction with the following ſentence; and marks both a perfect ſenſe, and a perfect ſentence: as,

“Man, conſidered in his preſent ſtate, ſeems only ſent into the world to propagate his kind. He provides himſelf with a ſucceſſor, and immediately quits his poſt to make room for him.”—Spectator.

Theſe are two perfect ſentences, divided from each other by the period, and ſubdivided into ſimple ſentences by the comma. The firſt of theſe two ſentences repreſents the *apparent* end of man's miſſion into this preſent world; namely, to propagate his kind: the ſecond ſhews the *real conſequence* of his reſidence here; begetting a ſucceſſor, and making room for him. They are entirely diſtinct ſentences; unconnected in conſtruction with each other, and wanting nothing to make each perfect both in ſenſe and ſentence.

Beſide the four foregoing points uſed to expreſs the pauſes in diſcourſe; there are three others, which alſo affect the modulation of the voice: namely,

The Note of Interrogation, marked thus - - (?)
The Note of Admiration, or Exclamation, marked thus (!)
The Parentheſis, marked thus - - - ( )

The Note of Interrogation is uſed when a queſtion is aſked: as, "Would an infinitely wiſe Being make ſuch glorious creatures for ſo mean a purpoſe? Can he delight in the production of ſuch abortive intelligences, ſuch ſhort-lived reaſonable beings? Would he give us talents that are not to be exerted? capacities that are never to be gratified?" Spect. Theſe are all interrogative ſentences, and, as ſuch, are marked by the interrogation point.

The Note of Admiration, or Exclamation, is uſed to mark the ſpeaker's admiration, and any ſudden paſſion: as,

> "And ſee!
> 'Tis come, the glorious morn! the ſecond birth
> Of Heaven and earth!" THOMSON'S WINTER.

Both theſe points mark an elevation of the voice. The length of time required by them as pauſes is uncertain, and may be equal to that of a ſemicolon, colon, or a period.

The Parenthesis is uſed to encloſe a word or ſentence, inſerted in another ſentence; which is not neceſſary to the ſenſe, nor does it at all affect the conſtruction: as, "Pompey on the other ſide (that hardly ever ſpake in public without a bluſh) had a wonderful ſweetneſs of nature." This point marks a moderate depreſſion of the voice, with a pauſe ſomewhat greater than a comma.

There are alſo other marks uſed in writing, which, though they do not affect the voice as pauſes, yet are often neceſſary to the ſenſe: the principal of which marks we ſhall juſt mention.

The Apostrophe, marked thus ('), is only a comma placed at the head of a letter, and ſignifies either that ſome letter or letters are left out for a quicker pronunciation; as, *I'll*, for *I will*; *he don't*, for *he does not*: or elſe it marks the poſſeſſive caſe: as, John's book.

The Accent (´) is placed over a vowel to denote that the word is accented on that ſyllable; as, *Com-pre-hén-ſion.*

The **Breve** (˘), placed over a vowel to ſignify it ſhould be ſounded ſhort: as, *Pĭty*.

The **Circumflex** (^), placed over ſome vowel to ſhew that it is to be pronounced long: as, *diſpûte*.

The **Caret**, of the ſame form with the circumflex, but placed beneath the line, to ſhew that ſomething is omitted which is inſerted over the line: as, *this* $\overset{\text{is}}{\wedge}$ *he*.

The **Dialysis** (¨), or two points placed over two vowels in a word, to ſhew they are to be parted, not being a diphthong: as, *Duët*.

The **Hyphen**, or note of conjunction (-), being a ſtraight line, uſed to connect compound words together: as, *Heart-breaking*; *Book-keeper*. And when inſerted at the end of a line, it ſhews the word is divided according to its ſyllables: as, *diſ-tinguiſh*.

The **Index** (☞), or hand, pointing to ſomething remarkable.

The **Asterism** (*), or Star, directing to ſome note in the margin, or at the bottom of the page. Several of them together denote ſomething defective, or improper to be repeated.

The **Obelisk** (†), is uſed to refer to a note like the foregoing. And in ſome dictionaries it denotes the word to be obſolete.

The **Paragraph** (¶) marks a diviſion of ſentences under one head. It is uſed chiefly in the vulgar tranſlation of the Bible.

The **Section** (§) ſhews a greater diviſion than the paragraph.

The **Crotchets** ([ ]) include a word or ſentence explanatory of what was mentioned before.

The **Quotation** (“—”) is a double comma (the firſt reverſed), and placed at the beginning and end of a ſentence, or ſentences, taken from another author.

A PRAXIS;

Or Examples of Grammatical Reſolution.

JOHN, IV.

1 When therefore the Lord knew how the Phariſees had heard that Jeſus made and baptized more diſciples than John;

2 (Though Jeſus himſelf baptized not, but his diſciples;)

3 He left Judæa, and departed again into Galilee.

4 And he muſt needs go through Samaria.

5 Then cometh he to a city of Samaria, which is called Sychar, near to the parcel of ground that Jacob gave to his ſon Joſeph.

6 Now Jacob's well was there. Jeſus therefore being wearied with his journey, ſat thus on the well: and it was about the ſixth hour.

7 There cometh a woman of Samaria to draw water: Jeſus ſaith unto her, Give me to drink.

1 *When* is an adverb of time; *therefore,* an adverb with a prepoſition added to it; *the,* the definite article; *Lord,* common noun, ſingular number, maſculine gender, nominative caſe, being the agent of the ſentence; *knew,* the verb neuter, third perſon, ſingular, governed by the noun *Lord,* and agreeing with it, paſt time, indicative mode, and the attribute of the ſentence; *how,* an adverb of manner; *the,* definite article; *Phariſees,* noun common, plural number, nominative caſe; *had heard,* verb neuter with the auxiliary *had,* plural, third perſon, paſt time, indicative mode, governed by *Phariſees; that,* a copulative conjunction; *Jeſus,* a proper name, maſculine, nominative caſe, and the agent of the ſentence; *made,* and *baptized,* two verbs active, third perſon, ſingular, paſt time, indicative mode, connected with each other by the copulative conjunction *and,* and are the attribute of the ſentence; *more,* an adverb of compariſon; *diſciples,* noun plural, objective caſe, and the object of the ſentence; *than,* comparative con-

junction, comparing the number mentioned in the foregoing ſentence with that of the following: as, *baptized more diſciples, than John* (did); *John*, a proper name.

2 *Though*, a disjunctive conjunction; *Jeſus*, a proper name, as before; *himſelf*, the objective pronoun of the third perſon ſingular, maſculine gender, joined to the ſingular ſubſtantive *ſelf*, to expreſs an excluſive emphaſis; *baptized*, verb active, third perſon, ſingular, paſt time, and indicative mode, though it be governed by the conjunction *though*, for it is not of a doubtful, but abſolute nature, declaring that *Jeſus did not baptize any*; it forms the attribute of the ſentence to the agent *Jeſus*; *not*, a negative adverb, added to the verb, and altering its ſenſe, the poſſeſſive pronoun *any* being underſtood; *but*, a disjunctive conjunction, correſponding to the former conjunction *though*, and expreſſing an oppoſition in meaning; *his*, the poſſeſſive pronoun, third perſon, ſingular, maſculine gender; *diſciples*, the plural ſubſtantive, nominative caſe, the auxiliary (did) being underſtood: as, *his diſciples did*. The whole verſe is included in a parentheſis, and is not connected in ſenſe or conſtruction, with the reſt of the chapter.

3 *He*, a pronoun of the third perſon ſingular, maſculine gender, nominative caſe; uſed as a ſubſtitute for the noun *Jeſus*, and governed by it; *left*, a verb active, third perſon, ſingular, governed by the pronoun *he*, paſt time, indicative mode; *Judæa*, a proper name of a city, objective caſe, as it forms the object to the ſentence; *and*, a copulative conjunction; *departed*, a verb neuter, third perſon, ſingular, governed alſo by the pronoun *he*, paſt time, indicative mode; *again*, an adverb; *into*, a prepoſition; *Galilee*, a proper noun, objective caſe, governed by the prepoſition *into*.

4 *And*, a copulative conjunction, expreſſing an addition; *he*, a pronoun as before, agreeing with its antecedent noun *Jeſus*; *muſt*, an auxiliary, denoting neceſſity, and having no variation; *needs*, an adverb; *go*, a neuter verb; *through*, a prepoſition; *Samaria*, a proper name for a country, objective caſe, being governed by the prepoſition *through*.

5 *Then*, an adverb of time; *cometh*, a neuter verb, governed by the pronoun *he* in perſon and number, and placed before it; as is ſometimes the caſe when the agent is a pronoun; indicative mode, and preſent time (improperly, for it ſhould be in the paſt time, as, then *came* he); *he*, a pronoun; *to*, a prepoſition; *a*, the indefinite article; *city*, a common noun, ſingular, neuter gender, objective caſe, being governed by the prepoſition *to*; *of*, a prepoſition; *Samaria*, a proper name, ſingular, objective caſe; *which*, a relative, agreeing with its antecedent *city*; *is*, the neuter verb *to be*, preſent time, ſingular, third perſon, indicative mode; *called*, the perfect participle of the neuter verb *to call*, being placed after the auxiliary *is*; *Sychar*, a proper name of a city, nominative caſe, being placed after the neuter verb, *called*; *near*, an adverb of diſtance; *to*, a prepoſition; *the*, the definite article; *parcel*, a common noun, ſingular, objective caſe, governed by the prepoſition *to*; *of*, a prepoſition; *ground*, ſubſtantive common, objective caſe, governed by the prepoſition *of*; *that*, a relative, agreeing with its antecedent, *the parcel of ground*; *Jacob*, a proper name of the patriarch, maſculine, nominative caſe; *gave*, a verb active, third perſon, ſingular, paſt time, indicative mode, governed by *Jacob*, the ſame as, *Jacob gave that*; *to*, a prepoſition; *his*, a poſſeſſive pronoun, agreeing in perſon, number, and gender, with the noun *Jacob*, and governed by it, and in the poſſeſſive caſe, to ſhew the relation of poſſeſſion between *Jacob* and *ſon*; *ſon* is a common noun, ſingular, maſculine gender, objective caſe, governed by the prepoſition *to*; *Joſeph*, a proper name, put in appoſition to *ſon*; that is, in the ſame caſe, and governed by the ſame prepoſition *to*.

6 *Now*, an adverb; *Jacob's*, a ſubſtantive proper, poſſeſſive caſe; *well*, a common noun, ſingular, nominative caſe, the ſame as, *the well of Jacob*; *was*, the neuter verb *to be*, third perſon ſingular, paſt time, indicative mode; *there*, an adverb of place; *Jeſus*, a proper name; *therefore*, an adverb with a prepoſition, the ſame as *there*; and *for*; it has the conſtruction

of a pronoun; *being*, the preſent participle of the neuter verb *to be*; *wearied*, the perfect participle of the neuter verb *to weary*, being placed after the participle of the auxiliary *to be*, which, as has been ſhewn, in all its forms requires the preſent or perfect participle after it; *with*, a prepoſition; *his*, a poſſeſſive pronoun, third perſon ſingular, maſculine gender, ſtanding for the poſſeſſive caſe of the noun *Jeſus*; *journey*, a ſubſtantive ſingular, objective caſe, governed by the prepoſition *with*; *ſat*, a neuter verb, third perſon, ſingular, paſt time, indicative mode, governed by *Jeſus*; *thus*, an adverb of manner, improperly inſerted in this place, as it refers to the manner of ſitting, or the poſture; whereas no particular poſture or manner of ſitting is mentioned, either before or after this ſentence; *on*, a prepoſition; *the*, the definite article; *well*, a common noun, ſingular, objective caſe, governed by the prepoſition *on*; *and*, a conjunction copulative; *it*, the neuter pronoun, nominative caſe, ſtanding for the time of the day; *was*, the neuter verb *to be*, third perſon, ſingular, paſt time, indicative mode, governed by the neuter pronoun *it*; *about*, an adverb; *the*, the definite article; *ſixth*, an adjective; *hour*, a common ſubſtantive ſingular.

7 *There*, an adverb; *cometh*, the neuter verb, third perſon, ſingular, indicative mode, and preſent time, though it ſhould be the paſt time, as the event here mentioned was evidently paſt, and in the ſeventeenth verſe of this chapter the woman of Samaria is ſpoken of in the paſt time: as, "the woman *anſwered* and ſaid;" *a*, indefinite article; *woman*, a ſubſtantive common, ſingular, feminine gender, nominative caſe, being the agent of the ſentence, though placed after the verb; *of*, a prepoſition; *Samaria*, a ſubſtantive common, ſingular, objective caſe, governed by the prepoſition *of*; *to draw*, the infinitive mode of the verb *draw*; *water*, a common ſubſtantive, ſingular; *Jeſus*, a proper name as before; *ſaith*, a neuter verb, third perſon ſingular, governed by *Jeſus*, and preſent time, improperly for the paſt; as this ſhould alſo be as it ſtands in the tenth verſe of this chapter; "Jeſus *anſwered* and *ſaid*

unto her;" *unto*, a prepoſition; *her*, a pronoun, third perſon ſingular, feminine gender, objective caſe, ſtanding for *the woman of Samaria*; *give*, a verb active, ſecond perſon ſingular, imperative mode; *me*, a pronoun, firſt perſon ſingular, objective caſe, ſtanding for *Jeſus*, and governed by the verb in the imperative mode, the word *water* being underſtood; *to drink*, the infinitive mode of the active verb; it is here made abſolute, ſupplying the place of the conjunction *that* with the ſubjunctive mode; as, give to me *that I may drink*.

SECT. VI.

OF ELOCUTION.

ELOCUTION is the art of reading or ſpeaking with propriety and elegance; or delivering our words in a juſt and graceful manner, untainted with pedantry or affectation, and uncorrupted with any provincial ſound.

Few people need be told the neceſſity of a graceful elocution. All will allow, that what a man has occaſion daily to do, ſhould be done well; yet ſo little attention has ſometimes been paid to this accompliſhment, even by thoſe from whom (from their profeſſion as public ſpeakers) we have been led to expect a perfect model of the art, as hath tended greatly to eclipſe all their other merits, however great; while others, of inferior attainments, by the help of a tolerably good ſtyle, and a juſt elocution, have riſen to a conſiderable eminence.

A graceful elocution is to a good ſtyle, what a good ſtyle is to the ſubject matter of a diſcourſe, an eſſential ornament. For if the ſubject of a diſcourſe be ever ſo intereſting, and the ſpeaker's knowledge ever ſo profound, without a correct ſtyle, the diſcourſe muſt ſuffer greatly in its reputation; and though the ſpeaker's abilities be great, and the ſtyle good, with a bad elocution, it will fare little better. So great an effect have theſe exterior accompliſhments upon public taſte.

The firſt rule in Elocution, as in Rhetoric, is *to follow nature*. All other rules, without a ſtrict regard to this, will only produce the affected declaimer, and not the graceful ſpeaker. The excellency of this rule, and the affectation of ſome modern examples of eloquence, have induced ſome to diſregard all other rules as ſuperfluous; and to imagine that this rule alone, in a perſon of good ſenſe, and cultivated taſte, is ſufficient to form a public ſpeaker. This, however, is an error. But it is neceſſary to obſerve the effects of nature, in order to diſtinguiſh between them, and thoſe of arbitrary cuſtom, or falſe taſte; and to obſerve the various ways by which nature expreſſes the ſeveral paſſions, emotions, and perceptions of the human mind: in order to which, much aſſiſtance may certainly be derived from the following particular rules:

1. *A diſtinct and deliberate articulation* is the firſt particular rule. This conſiſts in giving a clear and full utterance to all the ſounds, both ſimple and complex. For this purpoſe, the nature of thoſe ſounds ſhould be well underſtood, and much pains taken to correct any faults in the pronunciation. Sometimes a fault in articulation is aſcribed to a natural defect in ſome of the organs of ſpeech, when it is the effect of bad example, or inattention only. Many of theſe faults concern the ſounding of the conſonants. Some cannot pronounce the letter *l*, others the ſimple ſounds, *r*, *s*, *th*, *ſh*; others omit the aſpirate *h*. Theſe may be corrected by reading ſentences choſen for that purpoſe, wherein theſe ſounds frequently occur; and by conſtantly guarding againſt them in converſation.

The other fault in articulation conſiſts in the confuſion and cluttering of words together. Moſt people, who have not learned the art of ſpeaking, have a habit of uttering their words ſo rapidly and indiſtinctly, that it is impoſſible for them to give any emphaſis to their words, or natural tone to their ſentences; though they may pronounce every word plainly.

The moſt effectual means to conquer this habit are, to read aloud paſſages choſen for that purpoſe, which abound with long and unuſual words; and to read much ſlower than the ſenſe and juſt ſpeaking would require. Aim at nothing higher till you can read diſtinctly and deliberately.

2. *Let every word be pronounced with propriety.* This is a moſt neceſſary rule, as errors in pronunciation are more frequently committed, and leſs guarded againſt than any others. Faults in pronunciation are innumerable. Some of them are the following: omitting the aſpirate *h* where it ſhould be uſed: as, *our* for *hour*; and uſing it where there ſhould be none; as, *hegg* for *egg*: confounding and interchanging the *v* and *w*; pronouncing the diphthong *ou* like *au*, or *oo*; and the vowel *i* like *oi*, or *e*; and cluttering ſeveral conſonants together, without regarding the vowels. Theſe errors, and many others too numerous to mention here, muſt all be carefully guarded againſt, by every perſon who would be thought a correct ſpeaker.

It is difficult to point out any ſtandard by which the propriety of pronunciation is to be determined. Profound ſcholars generally make the etymology of the word the rule of pronunciation, and thus bring upon themſelves the charge of pedantry and affectation, by a delivery altogether novel to the reſt of mankind. While, on the other hand, the mere man of the world, notwithſtanding all his other accompliſhments, often retains ſo much of his provincial dialect, as juſtly to exclude him from the honour of being the pattern of a juſt pronunciation. This excellency is perhaps to be found in thoſe only who unite thoſe two characters, and who, to the correctneſs of true learning, add the elegance and eaſe of genteel life. *An attention to ſuch models, an intercourſe with the polite world, and a regard to the etymology of the words themſelves*, are the beſt rules that can be given, to correct any errors in pronunciation, and to guard againſt any peculiarities of provincial ſound: which every perſon ſhould endeavour to avoid, who is ſuppoſed to have ſeen too much of the world, to retain the peculiarities of the diſtrict in which he was born.

3. *Let every word of more than one ſyllable be pronounced with its proper* ACCENT. Many writers have laid it down as a fundamental rule, that the accent of words in the Engliſh language ſhould be thrown as far back as poſſible from the end of the word. A very erroneous rule! and one which has no foundation in the Engliſh language; for many words in the Engliſh have the accent on the laſt ſyllable, and the generality of them have it on the ſecond or third ſyllable from the end, but a very few on the fourth ſyllable from the end of the word. The beſt rule that can be given for this precept, is the foregoing; namely, a ſtrict attention to the beſt examples; always obſerving that the accent is not to be determined by any rules of quantity, but by the number and nature of the ſimple ſounds.

4. *The pronunciation ſhould be bold and forcible.* Many public ſpeakers deliver their diſcourſe with ſuch a faint and feeble utterance, and ſeem ſo wholly unconcerned with their ſubject, that their words may be ſaid rather to drop from their lips than to be delivered by them. This ſeldom fails of producing the ſame unconcern and indifference among their auditory. Inanimation is a principal fault: a ſpeaker without energy is little better than a lifeleſs ſtatue.

To remedy this evil, inure yourſelf, when reading, to take in as much breath as you can with eaſe contain, and to expel it with vehemence, in uttering thoſe ſounds which require an emphatical pronunciation; read aloud in the open air, with all the energy you can command; preſerve your body in an erect attitude; let all the ſounds, both vowels and conſonants, have a free, full, and bold utterance. Practiſe theſe rules with perſeverance, till you are maſter of a bold and manly delivery.

But in acquiring an energy of ſpeech, the following caution is neceſſary, namely, that the ſpeaker avoid the oppoſite extreme, of running into a paſſionate and diſtracting vociferation. This exiſts among thoſe ſpeakers, who value nothing above the admiration of the vulgar. Theſe, as Shakeſpeare has juſtly obſerved, "offend the judicious hearer to the ſoul, by tearing a paſſion to rags, to very tatters, to ſplit the ears of the groundlings." Cicero compares ſuch ſpeakers to cripples, who get on horſeback becauſe they cannot walk;—they bellow, becauſe they cannot ſpeak.

5. *Learn to acquire a graceful variety, both in the height and tone of the voice.* It is owing to the neglect of this rule, that we daily hear ſuch languid and unaffecting diſcourſes. Speakers, who deliver their ſentiments, or read their compoſitions in a dull, flat, inſipid manner, whatever other merits they may poſſeſs, ſeldom convince or affect their auditors. This ſame unmeaning mode of delivery, ſo univerſally prevalent, is called a *monotony*; and conſiſts in one unvaried tone of delivery. Moſt people, in familiar converſation, can deliver their thoughts with ſome variety, both in the tone and the height of their voice, which they generally modify according to the nature of their diſcourſe; yet the ſame perſons, when required to read any compoſition, immediately aſſume a dry, uniform manner of delivery; as though reading and converſation were to be performed in quite a different manner from each other; whereas, the ſame ſentences, whether delivered ſpontaneouſly by the ſpeaker, or read by the lecturer, ſhould always be delivered with the ſame tone, and on the ſame key, notwithſtanding all that has been ſaid by ſuperficial critics to the contrary.

A perſon of a moderate capacity will, therefore, be able to alter the tone, as well as the height of his voice, as it may be neceſſary. Different ſpecies of ſpeaking require different modifications of the voice. Different characters, in different ſituations, ſpeak in very different keys. The vagrant when he begs, the ſoldier when he gives the word of command, the watchman when he announces the hour of the night, the ſenator when he harangues, the lover when he whiſpers his tender tale, differ as much in the key as in the tone in which they ſpeak. The ſame perſon alſo, in the ſame ſituation, may is neceſſary than previouſly to enter into the meaning and ſpirit of every ſentence; and to expreſs it as nearly as poſſible to the manner in which we expreſs ourſelves in converſation; for, in familiar diſcourſe, we generally expreſs ourſelves emphatically, and place the emphaſis properly. As to artificial helps, ſuch as diſtinguiſhing words by particular characters, they generally miſlead rather than aſſiſt the reader, by confining him to a particular word or ſentence, which he generally overſtrains.

7. *Learn to acquire a juſt variety of pauſe and cadence.* The pauſes conſiſt of thoſe certain ſtops, which are uſed both in reading and converſation, and which are always neceſſary to the ſenſe. The cadence is that fall of the voice which is generally directed to be made before every full ſtop.

There are principally two faults committed with regard to pauſes. The firſt, and one of the worſt faults a ſpeaker can have, is to make no other pauſes than what are neceſſary for breathing. Such ſpeakers or readers make no diſtinction between a good a ſpeaker and quick one; and ſeem to conſider reading quick the ſame thing as reading well. But, without pauſes, the ſenſe muſt always appear confuſed and obſcure, the meaning often be miſunderſtood, and conſequently the energy of the piece wholly loſt.

The next fault conſiſts in a too mechanical attention to the ſtops or pauſes uſed in printing. Theſe, though very proper in writing, are often unnatural in ſpeaking; for, as was obſerved in Punctuation, the doctrine of points is very imperfect; the variety of pauſes required in diſcourſe is ſo great. A ſtrict regard, therefore, to theſe points has been one chief cauſe of monotony, by uſing the reader to an uniform ſound at every imperfect break, and an uniform cadence at every period. The uſe of points is, to aſſiſt the reader in diſcerning the grammatical conſtruction; not to direct his pronunciation. In reading, it is often neceſſary to make a pauſe where the grammatical conſtruction requires none, for the ſake of pointing out the ſenſe more ſtrongly, preparing the audience for what is to follow, or enabling the ſpeaker to alter the tone or height of his voice. In doing this, however, it is neceſſary that the tone of the voice be kept up in ſuch a manner, that the hearer may know that the ſenſe is not completed. Mr. GARRICK and Mr. SHERIDAN often practiſed this rule with great ſucceſs.

The principal fault reſpecting cadence conſiſts in an uniform fall of the voice in the word which precedes every full pauſe. This, though ſometimes proper, yet when ſtrictly adhered to, ſeldom fails of deſtroying all grace and energy of ſpeech. The tones and height of the voice, at the cloſe of a ſentence, ſhould be infinitely diverſified, according to the nature of the diſcourſe, and the particular meaning of the ſentence: thus, in plain narrative, argumentative diſcourſes, interrogative ſentences, and when the laſt word of a ſentence is emphatical, it is always neceſſary to cloſe the ſentence with an elevation of the voice. This will appear obvious, if we do but conſider the manner in which we relate a ſtory, ſupport an argument, or propoſe a queſtion, in converſation. But when the ſenſe of the ſentence does not require the laſt ſound to be emphatical, an eaſy fall of the voice, ſufficient to ſhew that the ſenſe is finiſhed, will be proper. And in pathetic pieces of the plaintive, tender, or ſolemn kind, the tone of the paſſion will often require a ſtill lower cadence of the voice. The beſt method to correct an uniform cadence is frequently to read ſelect ſentences, where the ſenſe is pointed and emphatical; and argumentative pieces, where frequent interrogatories occur; obſerving at the ſame time, the manner in which they ſhould be delivered by a juſt ſpeaker in familiar converſation.

8. The laſt, though not the leaſt, rule to form a graceful ſpeaker, is *to expreſs the paſſions and emotions of the mind by tones, looks, and geſtures, as well as by words.* To expreſs anger, fear, grief, joy, love, or any other active paſſion which ariſes in our minds, nature teaches us to make uſe of certain tones, looks, and geſtures. We naturally diſcover our feelings

by the manner in which we utter our words, the form of the countenance, and other well-known ſigns. And even when we deliver our thoughts on any leſs intereſting ſubject, and when the more violent emotions are not excited, ſome kind of feeling accompanies our words, which ſhould always have its proper external expreſſion. Theſe exterior expreſſions, are the effect of nature alone, and every judicious imitation of it in a ſpeaker will always appear graceful and pleaſing; none can deſerve the appellation of a graceful ſpeaker, till, to a diſtinct articulation, a perfect command of voice, and a true emphaſis, he can add the various expreſſions of the paſſions and emotions of the mind.

It is impoſſible to lay down any ſet of rules by which the ſpeaker may form theſe outward expreſſions, as they muſt be as various as the paſſions they indicate; the following general direction is the only one that can be depended upon:—

In acquiring this accompliſhment, as all others in this art, the fundamental rule is, *to follow nature*. Obſerve in what manner the ſeveral emotions, or paſſions, are expreſſed in real life, or by thoſe who have with great labour and taſte acquired the happy power of imitating nature; and either follow the great original herſelf, or the beſt copies to be met with, always obſerving the caution given by Shakeſpeare:— "*O'erſtep not the modeſty of nature.*"

More is to be learned by a ſtrict attention to good examples than is perhaps imagined. Example has as much the advantage of precept, as practice has of theory; and many, who have arrived to great perfection in this art, have candidly acknowledged it to be more the effect of imitation, than any theoretical rules.

In order to apply the foregoing rules to practice it is abſolutely neceſſary to go through a regular courſe of exerciſes; beginning with ſuch as are moſt eaſy, and proceeding by ſlow advances to thoſe that are more difficult. For if the reader cannot deliver with propriety the plain ſentiments of ſimple narrative, and didactic pieces, how can it be expected he ſhould do juſtice to the ſublime deſcriptions of poetry, or the animated language of the paſſions. The ſtudent ſhould perform theſe exerciſes, at firſt, with a view of correcting his prevailing faults only, whatever they be, whether they regard the articulation, command of voice, emphaſis, or cadence; and content himſelf with reading with this particular intention, before he aims at any thing higher. This may be irkſome and diſagreeable; it will require much time and patience; but it is the only way to ſucceed. It is recorded of Demoſthenes, the greateſt orator the world ever produced, that he, though not formed by nature either to pleaſe or to perſuade, ſtruggled with, and ſurmounted, the moſt formidable impediments. He ſhut himſelf up in a cave, that he might ſtudy with leſs diſtraction: he declaimed by the ſea-ſhore, that he might be uſed to the noiſe of a tumultuous aſſembly; and with pebbles in his mouth, to correct a defect in his ſpeech: he practiſed at home, with a naked ſword hanging over his ſhoulder, that he might check an ungraceful motion to which he was ſubject. Thus, this great pattern of perfection, both in elocution and eloquence, affords the moſt encouraging example to every ſtudent; ſince it ſhews how far art and application could avail, in acquiring an excellence which nature appeared willing to have denied.

When the ſtudent, by a ſtrict attention to what has been delivered, has perfectly corrected any ungraceful habits, he may then, and not before, begin to recite ſome of the higher ſpecies of compoſition; ſuch as, ſublime poetry, paſſionate orations, declamations, and the like; with a view to expreſs the paſſions, feelings, and ſympathies of the author. For this intention, he ſhould daily read aloud, and by himſelf; and as often as convenient, under the direction of ſome ingenious perſon. He ſhould alſo, if poſſible, recite compoſitions from memory. A few ſelect ſentences, treaſured up in the memory, and frequently repeated, has ſeveral advantages attending it. It enables the ſpeaker to enter more fully into the meaning and ſpirit of the piece, by obliging him to dwell upon the ideas which he is to expreſs. It gives him a previous knowledge of the ſeveral inflexions, emphaſes, and tones, which the words require. And by not having his eye confined to the book, it relieves him from the ſchool-boy habit of holding down his head, and, conſequently, reading in a different key and tone from that of converſation; and leaves him at full liberty to expreſs his own feelings, by all the varieties of countenance and geſture.

It generally requires ſome time, and many and frequent exerciſes, before the ſtudent can be brought to conſider reading in the ſame light as converſation; and be perſuaded that it ſhould be conducted in the ſame manner. There is, more or leſs, in all people (except in a very few accompliſhed ſpeakers), an artificial uniformity, which always diſtinguiſhes reading from converſation: the fixed poſture, the bending of the head, and the attentive look at the book, which are requiſite, are all deſtructive of that eaſe, freedom, and variety of both expreſſion and action, neceſſary to a juſt elocution.

It would ſuperſede the neceſſity of moſt of the foregoing rules, if public ſpeakers would deliver their diſcourſes from immediate conception, or from memory. But if this be too much to be expected, eſpecially from preachers of divinity, who have ſo much to compoſe, and are ſo frequently called upon to ſpeak in public; it is, however, very neceſſary, that they ſhould make themſelves ſo well acquainted with their diſcourſe, that they may be able to take in a conſiderable portion with a ſingle glance of the eye.

After the ſtudent has acquired a juſt and natural elocution, it is, perhaps, not the leaſt difficult part of this art to apply this accompliſhment to the purpoſes of real life. Many can deliver their diſcourſes in a graceful manner in private, or before a few ſelect friends, who, when required to ſpeak in a public capacity, at the bar, from the pulpit, or in the ſenate, generally betray either a timid baſhfulneſs, or an impertinent aſſurance. The former is apt to lead the ſpeaker into an awkward uniformity, the latter into a diſguſting affectation;

the former ariſes from an humble diffidence of the ſpeaker's own abilities, and reſpect for the underſtandings of his hearers; the latter is the effect of conceit, and a contempt for the opinions of his auditors: the former may ſoon be overcome by a ſucceſsful attention to the foregoing rules; the latter can ſeldom be ſubdued, till, by repeated diſappointments, the raſh adventurer is convinced, that he made a falſe eſtimate of his own value.

We ſhall cloſe this ſection with Hamlet's inſtructions to the players, taken from Shakeſpeare; which has always been conſidered as containing an important leſſon on elocution, and may exemplify moſt of the foregoing rules.

"Speak the ſpeech, I pray you, as I pronounced it to you, trippingly on the tongue. But if you mouth it, as many of our players do, I had as lieve the town-crier had ſpoke my lines. And do not ſaw the air too much with your hand, thus; but uſe all gently: for in the very torrent, tempeſt, and, as I may ſay, whirlwind of your paſſion, you muſt acquire and beget a temperance that may give it ſmoothneſs. O! it offends me to the ſoul to hear a robuſtious periwig-pated fellow tear a paſſion to tatters, to very rags, to ſplit the ears of the groundlings, who (for the moſt part) are capable of nothing but inexplicable dumb ſhows, and noiſe: I could have ſuch a fellow whipp'd for o'erdoing termagant; it out-herods Herod. Pray you, avoid it.

"Be not too tame neither; but let your own diſcretion be your tutor. Suit the action to the word, the word to the action, with this ſpecial obſervance, that you o'erſtep not the modeſty of nature; for any thing ſo overdone is from the purpoſe of playing; whoſe end, both at the firſt and now, was, and is, to hold, as 't were, the mirror up to nature; to ſhew virtue her own feature, ſcorn her own image, and the very age and body of the time, his form and preſſure. Now this overdone, or come tardy off, though it make the unſkilful laugh, cannot but make the judicious grieve: the cenſure of one of which muſt, in your allowance, o'erweigh

a whole

a whole theatre of others. Oh! there be players that I have ſeen play, and heard others praiſe, and that highly (not to ſpeak it profanely), that neither having the accent of Chriſtians, nor the gait of Chriſtian, Pagan, nor Man, have ſo ſtrutted and bellowed, that I have thought ſome of nature's journeymen had made men, and not made them well; they imitated humanity ſo abominably.

"O, reform it altogether. And let thoſe that play your clowns, ſpeak no more than is ſet down for them; for there be of them, that will themſelves laugh, to ſet on ſome quantity of barren ſpectators to laugh too; though in the mean time, ſome neceſſary queſtion of the play be then to be conſidered: —that 's villainous; and ſhews a moſt pitiful ambition in the fool that uſes it."

CHAP. II.

OF PENMANSHIP.

SECT. I.

RULES FOR THE ATTAINMENT OF THE ART OF WRITING.

THERE is no part of literature acquired with leſs difficulty than the art of writing. Few people, be their capacities ever ſo mean, are incapable of learning this. Hence we ſee ſo many, who, though ignorant of the more early parts of ſcience, ſuch as Engliſh grammar, and even ſpelling good Engliſh, yet can write a tolerably good hand. This is a glaring fault; for the more correct the penmanſhip, the more does it diſplay the orthographical and grammatical errors. I therefore adviſe all thoſe who may have occaſion to write much, to make themſelves perfectly acquainted with what has been delivered in the former chapter concerning Engliſh grammar. I truſt it need not be mentioned, that they ſhould render themſelves perfect in ſpelling; every one knows the neceſſity of this, and the ridicule and contempt which only one or two words wrongly ſpelled bring upon the writer.

I ſhall proceed to give a few directions, by the help of which an inexperienced perſon may qualify himſelf in this art; and without which, though perhaps deemed ſuperfluous by ſome, a work of this kind might poſſibly appear deficient.

It is neceſſary that the learner be provided with the implements requiſite for writing; a good pen, and good free ink; without which it is impoſſible to write a fair copy; a round or flat ruler (the round one is uſed for diſpatch, and the flat one for ſureneſs), a leaden plummet, or black lead pencil, to rule the lines, without which the learner will never write ſtraight; and ſome pounce, or gum ſandrack powder, to rub the paper with, if it be too thin to bear the ink, and when bold hands are to be written, as large text, German text, or the like; alſo when a word or ſentence is ſcratched out with the penknife, in which caſe, the place muſt firſt be rubbed ſmooth with the haft of the knife, or a piece of clean paper, and then rubbed with the pounce, to enable it to bear the ink. A quarto-ſized copy-book is the moſt proper, as each page will contain a copy of ten or a dozen lines, which will be ſufficient to write at one time.

Being provided with theſe implements, the learner may proceed to practice. The lines ſhould be ruled ſtraight and even, and at the ſame diſtance from each other, at the dotted lines, marked No. 1, in the plate. The diſtance between every two lines of writing ſhould be about twice or three times the diſtance which there is between the two pencil lines that belong to the ſame line of writing; though this is often more or leſs, according to the caprice of the writer.

The pen muſt be held in the right hand, between the thumb and the fore and middle fingers. The middle finger muſt be placed on the back of the pen, oppoſite to the upper part of the cut or cradle of the pen, and the fore-finger cloſe to it, and both held ſtraight. The thumb muſt be placed againſt the oppoſite ſide of the pen, called the belly of the quill, and muſt be bent a little in the joint. The top of the pen ſhould point towards the right ſhoulder. The elbow ſhould be drawn in towards the body, but not too cloſe. The arm may reſt lightly on the edge of the deſk, or table, between the elbow and the wriſt; but the ſtomach ſhould not preſs againſt the deſk. The pen muſt be held very lightly; for if it be griped hard, the learner will never acquire an eaſe and expedition in writing.

The learner, having acquired a juſt habit of holding the pen, may copy the ſmall letters *i*, *c*, *o*; having his lines ruled according to the dotted lines in the plate. He may next copy the other ſmall letters; taking care to be perfect in each, before he proceeds to the next. Then the capital letters; ruling his copy in ſquares, according to the pattern in the plate; and alſo join-hand copies, as ſoon as he can make the capitals. When the learner can write an indifferent good round-hand text, he may proceed to the ſmall round-hand and running hand; in writing which laſt, the pen ſhould never be taken off the paper till the word is finiſhed.

The learner ſhould imitate the beſt copies. Copper-plate copies are to be preferred to thoſe written by the pen, as being more correct. Thoſe ſmall letters which have tops or ſtems, as *b*, *d*, *f*, *h*, *k*, *l*, *ſ*, muſt all be of the ſame height, and thoſe with tails, as *f*, *g*, *j*, *p*, *q*, *ſ*, *y*, muſt be all of the ſame length. A due diſtance muſt be obſerved between the words, and between the letters of the ſame words. The capitals muſt be all of the ſame ſize. The upright ſtrokes, or thoſe that are formed by the upright ſtroke of the pen, muſt be fine or hair ſtrokes; and the downright ſtrokes muſt be fuller and blacker; but a conſtant attention to the copy would in a great meaſure ſuperſede the neceſſity of moſt of the foregoing rules. The learner ſhould not ſit long at one time, leſt he grow tired of learning; in which caſe he will not improve; nor be ambitious of writing faſt; for five or ſix lines, well written, will improve the learner more than fifty lines, written in the ſame time, without attention to their correctneſs.

When the learner has arrived to ſome proficiency in writing, it is requiſite that he know how to make and mend his pen. This is ſooner to be acquired by an attentive obſervance of thoſe who can make a pen well than by any verbal directions; the following rules, however, may be of ſervice. Being provided with a good gooſe quill (thoſe called ſeconds are the beſt), ſcrape the ſcurf from it, with the back of the penknife, ſcraping the back of the quill moſt, that the ſlit may be clear; then cut the quill half through, near the end, on the back part; and cut the other ſide of the quill quite through, near half an inch from the end. The quill will now appear forked; next cut away a very ſhort ſlit in the back notch, where the ſlit of the pen is to be; and putting the peg of the penknife haft, or the end of another quill, under this ſlit, and holding the nail of the left thumb pretty hard on the back of the quill, as high as it is intended the ſlit ſhall go, with a quick ſudden force rend up the ſlit; it muſt be very ſudden and quick, that the ſlit may be clear and cloſe; for if the ſlit be clear and cloſe, that it cannot be ſeen through, it is done well. Then, by ſeveral neceſſary cuts, the quill is to be brought into the form of a pen; and having brought it to a fine point on each ſide of the ſlit, it is to be nibbed in this manner; place the inſide of the nib on the left thumb nail, holding the pen between the fore and middle fingers of the left hand, and enter the knife at the extremity of the nib, and cut it through a little ſloping; then, with an almoſt downright cut of the knife, cut off the nib; laſtly, by other proper cuts, the pen is to be brought into an handſome form. But the nib is not afterwards to be mended by any cutting or ſcraping, for that cauſes a roughneſs, and abſolutely ſpoils it. If the nib, therefore, be altered or mended with the knife after it is nibbed, it muſt be nibbed again, as before directed. And obſerve that the breadth of the nib muſt be equal to the breadth of the downright ſtrokes of the letters.

Copies for Round Hand Text.

Avoid bad company.
Be wiſe betimes.
Care deſtroys the body.
Do the things that are juſt.
Expect to recieve as you give.
Frequent good company.
God is perfect in his works.
Hours fly ſwift away.
Innocency need not fear.
Join experience to theory.
Keep faith with all men.
Learn in the time of youth.
Money corrupts many.
No taſk is too hard to learn.
Opportunities are ſlighted.
Provide againſt poverty.
Quiet men have quiet minds.
Remember your duty.
Sin produces ſhame.
Time and tide ſtay for none.
Value a good conſcience.
Underſtand your trade.
Wiſdom is valuable.
Xerxes wept at mortality.
Yield patiently to fate.
Zeal is ſometimes proper.

Copies for Round Hand.

All letters even at head and tail muſt ſtand:
Bear light your pen, and keep a ſteady hand.
Carefully ſtrive in each line to excel;
Do every thing, that is to be done, well.
Excel in each new line, in every part:
Faults for the future ſhun, by rules of art.
Gripe not your pen, but hold it very ſlight,
Hold in your elbow, have a left hand light.
In all your writing at the copy look;
Join all your letters by a fine hair-ſtroke:
Keep free from faults, and blots, your copy-book.
Learn the command of hand by frequent uſe,
Much practice will good penmanſhip produce.
Never ſtrive to write too faſt at firſt;
Of all a learner's faults, this is the worſt.
Practice alone can produce expedition;
Quick writing is moſt learners' vain ambition.

Rule your lines ſtraight, and make them very fine;
Set ſtems of letters fair above the line,
The tops above the ſtems, the tails below,
Uſe pounce to paper, if the ink go through.
View well your copy, ſee how much you mend;
Wipe clean your pen, your taſk being at an end.
Your ſpelling mind, write each word true and well;
Zealouſly ſtrive good writers to excel.

A Receipt for making black Ink.

To one quart of ſoft water, put four ounces of freſh blue galls of Aleppo, bruiſed pretty ſmall; two ounces of copperas; and two ounces of gum-Arabic. Bottle it up, and ſhake it once a day, and in three or four weeks it will be fit for uſe.

The green peelings of walnuts, ſoaked in the water before the ink is made (if they be in ſeaſon), will render it the ſtronger, and more beautiful.

A Receipt for red Ink.

Simmer three pints of ſtale beer, or vinegar, with four ounces of ground Brazil wood, for an hour; then ſtrain it through a flannel, and bottle it up for uſe.

Or a little gum-water and vermillion will make a curious red ink for preſent purpoſe.

SECT. II.

OF SECRET WRITING.

SECRET writing may be performed ſeveral ways. Former ages were very fertile in inventions of this kind; and, by theſe means, intelligence has been obtained by countries, from others, with whom they were in a ſtate of hoſtility; and that not unfrequently in modern times. It may alſo ſerve individual purpoſes, where ſecrecy is required.

There are principally three ways of writing, ſo as not to be read by any, but thoſe who can diſcover the manner in which it is written. Firſt, writing in cipher, which requires great ingenuity, and of which, my limits will not permit me to ſpeak. Secondly, ſubſtituting other arbitrary marks or characters, for words or letters, than the words or letters themſelves. And, thirdly, writing with ſome ink or liquid which will not appear legible, till rendered ſo by ſome mechanical operation.

The ſecond method, of ſubſtituting one character for another, is eaſily performed; as any perſon might make an alphabet of his own, conſiſting of twenty-ſix characters, each of which might ſtand for ſome one letter of the Engliſh alphabet; and thus the writing would be unintelligible to any but thoſe who have the key or index. Or, the numerical figures may be uſed to the ſame purpoſe: as for example *a* may be repreſented by 1; *b*, by 2; *c*, by 3; &c. as follows:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26
a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, ſ, t, v, u, w, x, y, z.

According to this index, the following ſentence, *riches gain friends*, will be written thus: 18, 9, 3, 8, 5, 19. 7, 1, 9, 14. 6, 18, 9, 5, 14, 4, 19. Or the figures, to correſpond to the letters, may be placed in any other order. Or the letters in the alphabet may be tranſpoſed. Certain conſonants may be ſubſtituted inſtead of the vowels, and the vowels inſtead of the conſonants: as, inſtead of the vowels, *a*, *e*, *i*, *o*, *u*, *y*, uſe *l*, *m*, *n*, *p*, *q*, *r*, and *vice verſâ*, reſpectively; then the foregoing ſentence will ſtand thus: *ynchms glni fynmids*. And an infinite number of other ways might be invented, by this mode of ſubſtitutional writing, which the ingenious reader will diſcover; ſuch as, dividing the alphabet into two parts, and tranſpoſing the letters which ſtand in the firſt or ſecond place, in one part, with thoſe in the firſt or ſecond place, in the other part; or, dividing it into three columns, and applying this rule alternately; firſt, to the firſt column, and then to the ſecond: and a number of other ways there are, too numerous to mention.

The third way of writing ſecretly, is, Firſt, by writing with the juice of a lemon, the juice of an onion, urine, or the ſpirits of vitriol, which will not appear legible till it be holden to the fire.

Secondly, by tracing the letters on the back of the paper, after it is written, with a pen dipped in milk; theſe letters, ſo traced, will not appear legible, till the paper be holden to the fire; then they will appear of a bluiſh colour. But, in this manner of writing, the paper ſhould be very thin.

The laſt method I ſhall mention, is, by uſing ſympathetic inks, as they are generally called: there are various preparations under this name. I ſhall mention only two, and which may ſafely be depended upon.

1. If a little green vitriol be diſſolved in water, with a little nitrous acid, the characters written with it will be inviſible, till they are wetted with the following mixture:—

Put two ounces of ſmall Aleppo galls in half a pint of water; when it has ſtood three or four days, pour it off. A pencil, dipped in this mixture, and drawn over the letters, written with the former ink, will render them of a beautiful black.

Or letters written with the latter ink, will be inviſible, till they are wetted with a ſolution of Pruſſian blue in water; and letters written with this ſolution, will alſo be inviſible, till wetted with the above ink of galls and water.

2. Incorporate one ounce of litharge of lead with two ounces of diſtilled vinegar; let it ſtand twenty-four hours, then ſtrain it off, and let it ſettle.

Put one ounce of orpiment, in powder, and two ounces of quick lime, in a quart bottle, with water ſufficient to cover them about an inch above the ingredients. Place the bottle in a moderate heat, for twenty-four hours: then pour it off, and cork it cloſe.

The letters written with the former of theſe inks, will not appear till they be expoſed to the vapours of this latter ink; when they will appear perfectly plain.

SECT. III.

OF EPISTOLARY WRITING, AND SUPERSCRIPTION OF LETTERS.

OF every ſpecies of compoſition, there is none that, in its nature, approaches nearer to familiar converſation (except plain dialogue) then epiſtolary writing. A letter is a direct addreſs from one perſon to another, and ſhould, therefore, contain all the eaſe, elegance, and familiarity of converſation; paying the ſame regard to the nature of the ſubject, and the perſon addreſſed, as in a perſonal application. The principal characteriſtics of a letter are, nature, ſimplicity, ſprightlineſs, and wit. The ſtyle of a letter ſhould be natural; and appear to expreſs the genuine feelings of the mind. It ſhould not indicate the leaſt mark of ſtudy. There ſhould be no formal diviſion of the parts, no laboured introduction, nor pathetic concluſion; but all ſhould appear the ſpontaneous product of the writer's own emotions. It ſhould likewiſe be wholly devoid of any complexity or ambiguity of expreſſion: for this purpoſe, the ſentences ſhould be ſhort, and the ſtyle perſpicuous. It ſhould contain all the vivacity of converſation. And, if the writer be maſter of any wit, a letter (if the ſubject permit) affords as proper a place for the diſplay of it, as any compoſition whatever. A gentle ſatire, a repartee, or a burleſque, may ſometimes be introduced with ſucceſs; nay, it is often expected, in letters on domeſtic ſubjects, between familiar friends.

It is needleſs to give copies of letters on different occaſions, in ſuch a number as is uſually done; the ſubjects of letters being as various as thoſe of converſation: any attempt, therefore, to give ſpecimens of letters, to ſerve for every purpoſe for which the writer may have occaſion, muſt be abſurd. A learner who copies his letter from any precedent, will not be able to expreſs his own thoughts with eaſe and freedom. By being confined to a copy, from which he will find it difficult to depart, his letters will carry an awkward ſtiffneſs and formality; and he will be a long time before he acquires that freedom, and unadorned elegance, always expected in extemporary writing.

To form an epiſtolary writer no more is requiſite than an intimate acquaintance with Engliſh grammar; whereby he will be able to deliver his ſentiments with propriety in converſation. If he poſſeſs this qualification, it will ſuperſede the neceſſity of any artificial helps; but if this be wanting, other aſſiſtances will be of little uſe, except to ſerve to publiſh his deficiency to his correſpondents.

Nevertheleſs, in conformity to general cuſtom, I have added a few examples. It need hardly be mentioned, that the name, by which the perſon is addreſſed, be placed on the left hand ſide of the page, at the top of the letter; and the letter begun juſt under it: the name of the writer at the bottom of the letter, on the right hand; the date of the letter, either on the right hand, at the top of the letter, or on the left, and at the bottom: and the letter ſhould conclude with the name by which the receiver of it is addreſſed at the beginning.

From a Maſter to his Scholar, during the Holidays.

Dear George,

I take the firſt opportunity that has offered, to inquire after your health, and that of your friends. And I expect, that you will regularly anſwer each of my letters, that, during this time of leiſure, I may have an opportunity of obſerving, whether you remember, or have forgotten, the rules I formerly gave you, concerning writing letters. I now, therefore, call upon you to put thoſe rules into practice. But, willing to grant you every indulgence at this time of feſtivity; and leſt your recollection ſhould not be ſo clear, as when in conſtant exerciſe, I ſhall briefly repeat thoſe rules, to which, I hope, you will pay a ſtrict attention.

You remember, no doubt, my firſt direction was, to be very correct and circumſpect in your ſpelling: this is the firſt, and moſt eſſential requiſite in all kinds of writing: and make uſe of no word, of which you do not perfectly underſtand the ſenſe. The vulgar part of the world, in general, are very much addicted to this abſurdity. You will, now, often hear people condemn a work, as ungrammatical, and deficient in the ornaments of ſtyle, though themſelves be unacquainted with the firſt form of grammar, and know not the meaning of a flower in rhetoric.—Avoid repetitions: they always offend the judicious ear, and are ſeldom proper, except when they enforce any particular meaning, or explain it more fully. Parentheſes are always inelegant, and ſhould never be uſed *(but when abſolutely neceſſary)*, as they render the ſentence too complex. Never uſe the long *ſ* in a word, except when two meet, in which caſe it is neceſſary for diſtinction. A letter interlined has a very ungraceful appearance; it is alſo an affront: for it indicates either lazineſs or indifference, or both. Uſe no capital letters, but at the beginning of a ſentence, to proper names, and to the firſt word of every line in poetry. When you wiſh to lay a ſtrong emphaſis on any word, or intend that it ſhould be particularly noticed, it is common to draw a ſtroke under it with the pen, thus; ſuch words, when printed, are put into *Italics;* but when theſe emphatical words are employed too frequently, they loſe their effect, and when uſed improperly, they puzzle the reader. Beware of uſing many monoſyllables, they are inſignificant words; nor uſe many too long words, leſt you exceed comprehenſion. Shun particles, as much as poſſible: be very ſparing of your *ands*, *fors*, and *buts*. Be not fond of inventing new words; there are enough, already, to expreſs all our ideas; and more, I fear, than you will ever fully comprehend. Be attentive to the rules of grammar, and do not jumble the preſent, paſt, and future times of the verb together; as many incorrect writers do; neither confound the genders of the pronoun; nor uſe the ſingular, for the plural verb; which is frequently done: as, *you was*, for *you were*. If the ſentence be conditional, uſe the conditional mode. Let your ſtyle be ſimple and perſpicuous, and your ſentences ſhort: let it be as conciſe as poſſible; for a prolix writer tires the patience of his reader. Obſerve that your points be all placed juſtly, which will add grace and perſpicuity to your writing. Theſe hints I hope will be attended to; let me ſee the effects of them in your next letter; while I remain, with compliments to your father and mother,

Dear George,

Berkſhire, Your ſincere friend, &c. &c.
Jan. 2, 1803.

The Scholar's Anſwer.

Honoured Sir,

I return you my ſincere thanks for the kind attention you ſhew me. It ſhall always be my ſtudy and ambition to follow your inſtructions. I never write a letter to any of my friends, but I pay a particular regard to all the directions you have given me, on the ſubject of letter-writing. I exerciſe myſelf daily, in grammar, arithmetic, or ſome other part of literature; my father inſiſts upon my ſetting apart two hours every morning for that purpoſe. My father and mother deſire their beſt reſpects to you, and return you their kind thanks for the trouble you have taken in your late letter. I remain, with the greateſt reſpect,

Honoured Sir,
Your very much obliged
London, And humble Servant,
Jan. 10, 1803. George * * *.

A Letter from a Father to his Son, at School.

My dear Boy,

I am exceedingly happy to hear of the great progreſs you have made in your learning. I hope you will continue to pay the ſame attention to all your maſter's inſtructions as you have done hitherto. I have herewith ſent you a preſent of a gold watch; which I deſign, in the firſt place, as an encouragement for you to proceed; and in the ſecond place, as a monitor to remind you of the ſwiftneſs and importance of time. This little machine travels with the hours, yet keeps pace with months; it keeps time with minutes, yet does not outrun years. Whenever you look upon this little prompter, let it remind you of your time. Every complete revolution of the minute-hand ſilently informs you, that there has elapſed, what a whole world cannot redeem—*an hour of your precious time.* And, though you may perform in the following hour, or half-hour, the buſineſs that ſhould have been done in the foregoing hour, yet you muſt remember, that one hour is for ever *irrecoverably loſt.* The whole life of man, at the longeſt, is but ſhort; there is no time to loſe. Though you make the beſt uſe of your time poſſible, while young, you will hereafter (if you have any ſerious reflection) chide yourſelf, that you improved your time no better. Suffer no portion of your time, though ever ſo ſmall, to paſs unemployed; but let me give you the advice which Lord Cheſterfield gives his ſon, on this ſubject: "Take care of the minutes, for the hours will take care of themſelves." I have read of an Auguſtine monk, who kept a daily diary of his time, for upwards of thirty years, that he might know how many hours, out of each twenty-four, he had employed in literary and religious exerciſes, how many in ſleep, how many in the neceſſary avocations of the day, and how many he had ſhamefully loſt in ſloth, or unprofitable converſation. And Cato, having ſpent a day in which he had done nothing, thought it a matter of ſuch conſequence, as to be lamented during the reſt of his life.

I do not mean by what I have ſaid, that I would have you always at your book, and allow you no time for recreation. In the hours appointed for play, if you find yourſelf ſo diſpoſed, join in the diverſions of your companions; but if, at any of theſe times, you prefer your book, either to anticipate your future taſks, or to refer to any of thoſe you have gone through, it will better anſwer the purpoſe of your reſidence at ſchool, which is to improve your mind in uſeful knowledge, and not in childiſh ſports; it will redound more to your preſent credit, and contribute infinitely more to your future advantage.

All that I mean by what I have advanced, is, that I would never have you perfectly idle. When you are not at your play, be at your work; and when you are not engaged at your work, you may join in play at the appointed times, but no others. Beware of an inanimate indolence; ſome boys are too great cowards to play, and too great dunces to learn. Hence they ſtand idle ſpectators of all that paſs, and generally, in the iſſue, prove idle men, who are the moſt unprofitable animals in the creation. The Arabians have a proverb, that "an idle perſon is a playfellow for the devil." And it is a maxim of the Chineſe law, that, " for one idle perſon, the whole ſtate ſuffers." Which is as true as it may appear novel to us; for he eats the produce of others' labour, without making an adequate return.

In order to guard againſt idleneſs, never give way to procraſtination, or defer that till to-morrow which may be as well done to-day. Many people poſtone their buſineſs till a future day, merely becauſe it is future; but you muſt remember, that two days ago, yeſterday was a to-morrow; and two days hence, to-morrow will be yeſterday. Never, therefore, defer that to a future time, which may be done at the preſent, without a very ſubſtantial reaſon.

Laſtly, attend to the manner in which every thing is performed, even the moſt trivial matters. It is not ſufficient that a chapter be read, or a copy written, without a fault, but they ſhould be done well, with all the ſkill you are maſter of, if you mean to excel. Whenever you ſpeak, though it be to your own companions, be ſure to expreſs yourſelf in the moſt proper and expreſſive words, paying a ſtrict attention to their grammatical conſtruction. Whenever you read, though it be but a ſign-poſt in the ſtreet, read with propriety and emphaſis. And, whenever you have occaſion to write, though it be only your name, let it be done as correct as poſſible. Theſe directions, conſtantly obſerved, will render you an accompliſhed character, in whatever department of life you may be engaged. I hope the foregoing hints will be ſufficient. Let them be well attended to, and I ſhall, no doubt, ſee the effect of them in your future acquiſitions. Your mother and ſiſter join with me in their love to you.

I am,

Dear Boy,

Your loving Father,

London,

March 20, 1808. THOMAS * * *.

The Son's Anſwer.

C—d—n School, March 25, 1808.

Honoured Father,

I received your letter, with your kind preſent of the watch; for which I return you many thanks. I hope it will always have the effect you deſire, of reminding me of the ſhortneſs and importance of time. I never look at it, but I recollect your alarming letter. I ſhall always ſet a juſt value on time—it is impoſſible to over-value it. I ſhall be particularly attentive to all the commands in your letter: I often decline the diverſions of my ſchoolfellows, and apply to my book, conformably to your deſire. Yeſterday I entered into Decimal Fractions; which having gone through, my maſter informs me, I ſhall begin Algebra. When I have the pleaſure of coming home, next Chriſtmas holidays, I have no doubt, but you will be well ſatisfied that I have made a good uſe of my time. Pray give my duty to my mother, and my kind love to my ſiſter. I am,

Honoured Father,

Your moſt obedient and dutiful Son,

GEORGE ***.

From a young Shopkeeper to a Wholeſale Dealer.

Sir,

By the recommedation of Mr. Chapman, who is my neighbour, I think proper to apply to you for the following articles:—Two dozen pieces of yard-wide calico, ten pieces of the beſt Jaconet muſlin, thirty pieces of ell-wide town printed cottons, &c. &c. I hope you will let them be of the beſt quality of their ſort, and at a reaſonable price; as I intend all our dealings ſhall be for ready money.

Hull, I am, Sir,
March 20, 1808. Yours, &c.

The Anſwer.

Sir, *London, March* 24, 1808.

I have, in anſwer to your favour of the 20th inſtant, ſent you the articles you ordered. Should you think proper to favour me with your orders, I have no doubt but I ſhall give you ample ſatisfaction. I am,

Sir,

Your obliged Servant, &c.

From Wholeſale Dealers to a Retail one.

Sir,

As you have lately been very backward in your remittances, we are under the neceſſity of informing you, that unleſs you ſend us an immediate draft, or order, for 40*l.* which has ſo long been due, we muſt uſe ſuch means as will prove very diſagreeable both to you and ourſelves.

We remain,

Liverpool, Sir,

March 25, 1808. Yours, &c.

The Anſwer.

Gentlemen,

Several ſevere and unexpected loſſes have been the ſole cauſe of my late remiſſneſs. I am very ſenſible of the lenity you have always ſhewn me. I only requeſt your acceptance of my bill, payable in two months; money being at this time very ſcarce, and my debtors very backward in their payments.—Should this be agreeable to you, I ſhall always be very punctual for the future. I am,

Berkſhire, Gentlemen,

March 30, 1808. Your much obliged Servant, &c.

The

The Reply.

Sir, *Liverpool, April* 3, 1808.

As we ſhould be ſorry to diſtreſs you, we are willing to accede to your propoſals; and hope, that, by the time you mention, you will be able to make good your promiſe. We remain,

Sir,

Your humble Servants, &c.

From a Tradeſman to his Friend.

Sir,

Relying upon the ſincerity of your friendſhip, I have taken the liberty to ſolicit your aſſiſtance at this critical juncture. My affairs are greatly embarraſſed; and, unleſs I can procure the ſum of 200*l.* I ſhall inevitably be ruined. If, therefore, you can oblige me with the above ſum, to be paid by inſtalments, at three, ſix, nine, and twelve months, at 50*l.* each payment, you will ſave me from impending bankruptcy, and infinitely oblige,

Sir,

March 27, 1808. Your diſtreſſed Friend, &c.

P. S. You know the value of my ſtock, and alſo my expectations.

The Anſwer.

My dear Friend,

I am very ſorry to hear of your preſent difficulties. I deſire you will call upon me to-morrow, when I hope I ſhall be able to accommodate you with the ſum you want; or, if I cannot furniſh you with the whole, I can let you have the greater part, and the remainder in a few days.

I am,

Weſtminſter, My dear Friend,

March 30, 1808. Yours ſincerely, &c.

It is as neceſſary to know how to direct, or ſubſcribe, as to write a letter. Perſons in high rank, and peculiar ſtations in life, are to be addreſſed in a peculiar manner.

For the Directions or Superſcriptions of Letters.

To the King—To the King's Moſt Excellent Majeſty.
To the Queen—To the Queen's Moſt Excellent Majeſty.
To the Prince of Wales—To His Royal Highneſs, George Prince of Wales.
To the Princeſs—To Her Royal Highneſs, &c.

To the Nobility.

To a Duke—To [His Grace] the Duke of, &c.*
To a Marquis—To [the Moſt Noble] the Marquis of, &c.
To an Earl—To [the Right Honourable] the Earl of, &c.
To a Viſcount—To [the Right Hon.] Lord Viſcount, &c.
To a Lord—To [the Right Honourable] Lord, &c.

The ſons of dukes, marquiſſes, and the eldeſt ſons of earls, have the title of Lord, and Right Honourable; and the title of Lady belongs to their daughters, with the addition of their Chriſtian and ſurnames: as, Lady Caroline Ruſſel, &c.

The eldeſt ſons of dukes, marquiſſes, and earls enjoy the ſecond title of their father, by the courteſy of England.

The children of viſcounts and barons are alſo ſtyled Honourable.

To a Baronet—To [the Honourable] Sir J. H. Bart. &c.
To a Knight—To [the Right Worſhipful] Sir J. H. Knt.
To a Privy Counſellor—To [the Right Honourable] J. H. one of His Majeſty's Moſt Honourable Privy Council.
To an Ambaſſador—To His Excellency, &c.

Alſo plenipotentiaries, foreign governors, the lord lieutenant of Ireland, and lords juſtices of Ireland, and the captain general of His Majeſty's forces, have all the title of Excellency added to their other titles.

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* The words included in crotchets are ſometimes omitted.

The title of Right Honourable is given to no commoner, except members of the Privy Council, the three lord-mayors of London, York, and Dublin, and the lord-provoſt of Edinburgh, during their office only.

To the Clergy.

To an Archbiſhop—To the Moſt Reverend Father in God, &c.; or, To His Grace the Lord Archbiſhop of York, or Canterbury.

To a Biſhop—To the Right Reverend Father in God, &c.; or, To the Lord Biſhop of, &c.

To Deans, Archdeacons, &c.—To the Rev. A. B. Dean of, &c.; or, To the Rev. A. B. Archdeacon of, &c.

To the inferior Clergy—To the Rev. Mr. A. &c.; or, To the Rev. Dr. &c.

To the Military and Navy.

All colonels are ſtyled Honourable; and all inferior officers ſhould have the name of their rank ſet firſt: as, To Major G. C.; To Captain I. H. &c.

All admirals in the navy are ſtyled Honourable; the inferior officers are ſtyled by their rank, as in the army.

To the Judges and Lawyers.

To the Lord High Chancellor of Great Britain—To the Right Honourable J. Lord E. Lord High Chancellor of Great Britain—Lord Preſident of the Council—Lord Privy Seal—One of His Majeſty's Principal Secretaries of State, &c.

To the Maſter of the Rolls—To the Right Honourable Sir W. G. Knt. Maſter of the Rolls.

To a Judge—To the Right Honourable E. Lord E. Lord Chief Juſtice of the King's Bench, or of the Common Pleas.

To the other Judges—To the Right Honourable A. B. Eſq. one of the Juſtices of, &c.; or, To Judge A.

To the Chief Baron—To the Hon. Sir A. M. Lord Chief Baron.

To His Majeſty's Attorney, Solicitor, or Advocate General—To Sir A. B. His Majeſty's Attorney, Solicitor, or Advocate General.

Every judge has the title of Right Honourable, if a Privy Counſellor; otherwiſe only Honourable. All others in the law are ſtyled according to the office they bear; and all barriſters have the title of Eſquire.

To the Lieutenancy and Magiſtracy.

To a Sheriff—To A. B. Eſq. High Sheriff of the County (of York).

To a Recorder—To the Right Worſhipful A. B. Recorder of the City of, &c.

To an Alderman—To the Right Worſhipful A. B. Eſq. Alderman of Tower Ward.

The aldermen and recorder of London are ſtyled Right Worſhipful, and Eſquire; as are alſo all ſheriffs, and mayors of corporations, during their office.

Every gentleman in the commiſſion of the peace has the title of Eſquire, and Worſhipful.

To the Governors of the Crown.

To the Lord Lieutenant of Ireland—To his Excellency A. Lord B. Lord Lieutenant of Ireland.

To the Governor of Dover Caſtle—To the Right Honourable A. Earl of D. Governor of Dover Caſtle, &c.

The ſecond governors of foreign ſettlements, appointed by the king, are called Lieutenant Governors. Thoſe appointed by the proprietors of the ſettlement, as the Eaſt India Company, &c. are called Deputy Governors.

To either Houſe of Parliament, to Commiſſioners, Bodies Corporate, &c.

To the Houſe of Lords—To the Right Honourable the Lords Spiritual and Temporal, in Parliament aſſembled.

To the Houſe of Commons—To the Honourable the Knights, Citizens, and Burgeſſes, in Parliament aſſembled.

To the Treaſury; or Admiralty—To the Right Honourable the Lords Commiſſioners of the Treaſury; or Admiralty.

To the Commiſſioners of the Cuſtoms; or Exciſe—To the Honourable the Commiſſioners of His Majeſty's Cuſtoms; Revenue of the Exciſe, &c.

To any of the Companies of the City of London—To the Maſter, Wardens, and Court of Aſſiſtants of the Worſhipful Company of, &c.

To the Governors of Chriſt's Hoſpital—To the Right Worſhipful the Governors of Chriſt's Hoſpital.

To the Eaſt India Company—To the Honourable Court of Directors of the United Company of Merchants trading to the Eaſt Indies.

To the Inns of Court—To the Honourable Society of the Inner, or Middle Temple; or Gray's Inn, &c.

The governors, truſtees, &c. of hoſpitals, colleges, &c. are ſtyled Worſhipful or Right Worſhipful, or Honourable or Right Honourable, if any of them poſſeſs any of thoſe titles.

It is alſo neceſſary to know how to begin a letter to any of the foregoing characters.

For the Beginning of Letters.

To the King—Sire, or May it pleaſe your Majeſty.

To the Queen—Madam, or May it pleaſe your Majeſty.

To a Prince—Sir, or May it pleaſe your Royal Highneſs.

To a Princeſs—Madam, or May it pleaſe your Royal Highneſs.

To a Duke—My Lord Duke, or May it pleaſe your Grace.

To a Dutcheſs—Madam, or May it pleaſe your Grace.

To a Marquis, Earl, Viſcount, or Baron—My Lord, or May it pleaſe your Lordſhip.

To their Conſorts—Madam, or May it pleaſe your Ladyſhip.

To a Baronet—Sir, or May it pleaſe your Honour.

To a Knight—Sir, or May it pleaſe your Worſhip.

To their Conſorts—Madam, or May it pleaſe your Ladyſhip.

To an Archbiſhop—May it pleaſe your Grace.

To a Biſhop—My Lord, or May it pleaſe your Lordſhip.

To their Conſorts—Madam, or May it pleaſe your Ladyſhip.

To all the other Clergy—Reverend Sir, or Sir.

To an Ambaſſador—Sir, or Your Excellency.

To the Lord Mayor of London, York, or Dublin, and Lord Provoſt of Edinburgh, during their office—My Lord, or May it pleaſe your Lordſhip.

To an Alderman, Recorder, or Sheriff—Sir, or May it pleaſe your Worſhip.

To the Lord Lieutenant of Ireland—My Lord, or May it pleaſe your Lordſhip.

To the Houſe of Lords—My Lords, or May it pleaſe your Lordſhips.

To the Houſe of Commons—May it pleaſe your Honours.

To the Lords of the Treaſury, or Admiralty—My Lords, or May it pleaſe your Lordſhips.

To the Commiſſioners of the Cuſtoms, or Exciſe—Gentlemen, or May it pleaſe your Honours.

To the Benchers of an Inn of Court—Gentlemen, or May it pleaſe your Honours.

To the Directors of the Eaſt India Company—Gentlemen, or May it pleaſe your Honours.

To the Governors of Chriſt's Hoſpital—Gentlemen, or May it pleaſe your Worſhips.

To a Company of the City of London—Gentlemen, or May it pleaſe your Worſhips.

It may not be amiſs, in this place, to inſert a table of precedence, by which the reader will ſee the rank which each individual bears in England, according to his office and occupation. Taken from the Heralds' books.

Table of Precedence.

The King.
Queen.
The King's Children, and Grandchildren.
The King's Brethren.
———— Uncles.
———— Nephews.
Archbiſhop of Canterbury.

Lord Chancellor, or Keeper, if a Baron.
Archbiſhop of York.
Lord Treaſurer,
Lord Preſident of the Council,
Lord Privy Seal,
} if barons
Lord Great Chamberlain,
Lord High Conſtable,
Lord Marſhal,
Lord Admiral,
Lord Steward of the Houſehold,
Lord Chamberlain of the Houſehold,
} above all Peers of their own degree.
Dukes.
Marquiſſes.
Dukes' eldeſt ſons.
Earls.
Marquiſſes' eldeſt Sons.
Dukes' younger Sons.
Viſcounts.
Earls' eldeſt Sons.
Marquiſſes' younger Sons.
Secretary of State, if a Biſhop.
Biſhop of London.
——— Durham.
——— Wincheſter.
Biſhops.
Secretary of State, if a Baron.
Barons.
Speaker of the Houſe of Commons.
Lords Commiſſioners of the Great Seal.
Viſcounts' eldeſt Sons.
Earls' younger Sons.
Barons' eldeſt Sons.
Knights of the Garter.
Privy Counſellors.
Chancellor of the Exchequer.
Chancellor of the Dutchy.
Chief Juſtice of the King's Bench.
Maſter of the Rolls.
Chief Juſtice of the Common Pleas.
Chief Baron of the Exchequer.
Judges and Barons of the Coif.
Knights Bannerets, royal.
Viſcounts' younger Sons.
Barons' younger Sons.
Baronets.
Knights Bannerets.
Knights of the Bath.
Knights Bachelors.
Baronets' eldeſt Sons.
Knights' eldeſt Sons.
Baronets' younger Sons.
Knights' younger Sons.
Colonels.
Serjeants at Law.
Doctors.
Eſquires.
Gentlemen.
Yeomen.
Tradeſmen.
Artificers.
Labourers,

Married women and widows are entitled to the ſame rank among each other as their huſbands would have been entitled to among themſelves; except ſuch rank be merely profeſſional, or official:—and unmarried women to the ſame rank as their eldeſt brothers would bear during the lives of their fathers.

SECT. IV.

THE ART OF STENOGRAPHY, OR SHORT HAND.

Stenography, or Short Hand, is the art of writing in a more expeditious manner than by the common mode, for the purpoſe of taking down a ſpeech, or diſcourſe, as delivered by the ſpeaker.

For this purpoſe, the writer of ſhort hand is permitted to avail himſelf of ſeveral advantages which no other writer is allowed. He has the liberty of inventing an alphabet of his own, conſiſting of certain arbitrary marks, or characters, which, for the ſake of expedition, are of a leſs complex form than thoſe of the other alphabets. He may reject any of the letters in the common alphabet, which are not abſolutely neceſſary for the writer to recollect the ſenſe; or ſubſtitute one and the ſame character to ſerve for two diſtinct letters; as is done here in the ſhort-hand alphabet, where *f* and *v* are uſed indiſcriminately the one for the other, and repreſented by the ſame character; as are alſo the *k* and hard *c*, the *s* and the ſoft *c*. He alſo omits the vowels, except where they are indiſpenſably neceſſary to diſcover the ſenſe. Conſequently, he is not to follow the cuſtomary mode of ſpelling; but is to inſert no more letters in a word than are preciſely neceſſary to expreſs the ſound.

The ſhort-hand writer, beſides theſe advantages, ſometimes makes uſe of ſingle characters to expreſs whole words, and even whole ſentences; but of this hereafter.

The firſt and principal rule in ſhort hand is, *to make uſe of no more letters than are neceſſary to give the reader an idea of the ſound of the word.* For if the writer uſe all the letters that are neceſſary to expreſs the *ſound itſelf,* he will gain but little advantage by it; as he will be obliged to uſe too many letters to be very expeditious.

But it muſt be here obſerved, that this rule muſt not be followed too ſtrictly at firſt, and during the learner's exerciſes; leſt it render his writing too imperfect, and unintelligible to himſelf. He ſhould, therefore, at firſt, content himſelf with uſing every character for every conſonant, till he be perfectly acquainted with them all, and can very readily form them; marking the points for the vowels, as is hereafter directed.

Of the Short-hand Alphabet.

The ſhort-hand alphabet conſiſts of the following conſonants: *b*, *d*, *f* or *v*, *g* or *j*, *k* or hard *c*, *l*, *m*, *n*, *p*, *q*, *r*, *s* or ſoft *c*, and *z*, *t*, *w*, *x*: each of which has its proper character, as ſeen in the plate. The characters ſtanding for theſe letters are to be neatly joined together, to form words; and written in the moſt expeditious manner poſſible, without taking the pen off the paper till the word be finiſhed; and the vowels are to be noted afterwards (if neceſſary) by points.

Though the omiſſion of the vowels may at firſt ſomewhat puzzle the reader, to read even his own writing, yet a little practice will render it perfectly familiar to him; as the chief difficulty, both in writing and reading ſhort hand, ariſes from the novelty of the characters, and from the want of a familiar acquaintance with them.

In order to perfect the learner in the conſtruction of all the characters in the ſhort-hand alphabet, it is abſolutely neceſſary that he frequently copy them. He ſhould, daily, write a copy of the ſhort-hand alphabet; ruling his lines according to the dotted lines in the firſt line of the firſt leſſon in the plate; and writing the ſignification of each character at the end of each line, and the letter it repreſents at the beginning of the line; uſing three or four lines of a quarto copy for each letter.

In the alphabet given, I doubt not but the characters will be found as convenient for practice as any extant. I have endeavoured to make it conſiſt of the moſt ſimple, and, at the ſame time, the moſt diſtinct characters. I ſhall only obſerve, that by the help of this alphabet, and the abbreviations that follow, I have been able to follow the moſt rapid ſpeakers.

This alphabet, though conſiſting of the characters for fifteen conſonants only, will be found quite ſufficient. The *c* being ſupplied by either *k* or *s*, according as it is pronounced, hard or ſoft. The *h* may very well be omitted in ſhort hand, as it is only an aſpiration of the breath; and the ſound of the word (which is all that is minded in this ſpecies of writing) may be diſcovered without it. The *j* is ſupplied by the *g* ſoft, which has the ſame ſound. *V* is alſo ſupplied by *f*, being only a coarſer ſound than that letter. *Y* is repreſented, if neceſſary, by the vowel *i*, having exactly the ſame ſound; and *z* by *s*, from which it differs in ſound but a little, being of a coarſer nature.

The vowels are ſeldom uſed in ſhort-hand; but when it is neceſſary to inſert any of them, they are repreſented by points, as will be ſeen hereafter.

Three of the characters in this alphabet are horizontal figures: viz. *d*, *k*, and *m*; as are alſo the three characters which repreſent the double letters, *ch*, *ſh*, *th*. Seven of them are vertical or lineal, running directly from the top line to the bottom; viz. *f*, *g*, *l*, *n*, *q*, *r*, and *x*. The other five, *b*, *p*, *t*, *s*, and *w*, are inclined characters.

In forming the ſhort-hand characters ſingly, the learner may obſerve (if he wiſhes to have his writing look exact), that the horizontal characters are only half the height of the lineal, and inclined ones. For the two latter ones generally occupy the whole ſpace between the two lines, as ſeen in the plate. But this direction is to be ſtrictly adhered to only while the learner is forming the characters ſingly: when he comes to join the characters together, to form words, it will be often neceſſary to make them leſs or greater than their uſual ſize; ſometimes for convenience, and at other times from neceſſity: as will be ſhewn in its proper place.

Of the B.

This letter, in ſhort hand, is formed by an inclined curve line, or ſmall ſegment of a circle; it ſhould always be formed from the top, which is on the left hand; and univerſally I would adviſe, that all inclined and horizontal letters be formed from the left hand part, as it is the moſt natural, and will be found the moſt expeditious. This letter may alſo be uſed to expreſs the words, *be*, *by*, *but*, and *black*; and, with a ſhort lateral ſtroke joined to the fore part, it repreſents the word *before*; and the ſame ſtroke, when placed in the hinder part, ſignifies the word *behind* *; as ſhewn in the plate. This letter is alſo uſed to denote the inſeparable prepoſitions *be*, *ob*, and *ab*, when belonging to a word: as, *be*-have, *ob*-ſcure, &c. It alſo forms the terminations *ble*, or *able*, both ſingular and plural, and *bly*: as, remark-*able*, move-*ables*, and a-*bly*.

C.

This letter is not diſtinguiſhed by a ſeparate character; for its ſound (except before *h*, which is provided for) is either hard, like *k*; or ſoft, like *s*. It is, therefore, repreſented by the one or other of theſe characters, according to its ſound.

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* When the ſtroke is drawn *through* the character, it ſignifies the words *before* and *behind*; and when the ſtroke is ſet at the left hand point, it ſtands for the word *beginning*.

D.

This letter is the ſegment of a ſmall circle, with its two points downwards; it is begun at the left hand point, as the laſt. When uſed alone, it repreſents the word *and*, *hand*, and *end*. It alſo ſtands for the prepoſitions *de*, *di*, *dis*: as, *de*-bate, *di*-vide, *diſ*-pute. And is uſed for the termination *ed*: as, turn-*ed*.

F, or V.

This letter is formed by a ſtraight, downright or upright ſtroke of the pen (for it may be made either upwards or downwards, as is found moſt convenient to the writer). When it follows a letter that is finiſhed at the bottom, it is formed upwards; but when it ſucceeds a letter that is concluded at the top, it is formed downwards, that the pen be taken off the paper as ſeldom as poſſible. It alſo ſtands for the *v*, which is a letter of nearly the ſame ſound, being only a little coarſer, and is, therefore, not diſtinguiſhed in ſhort hand by a ſeparate character; but if it be neceſſary to diſtinguiſh it at any time from the *f*, it is only making the character a little blacker, or fuller. This letter alſo repreſents the words *for*, *if*, *of*, *off*, and *few*; and the inſeparable prepoſition *for*: as, *for*-get. And the terminations *fy*, *ify*, *ful*, and *fulneſs*: as, de-*fy*, ſanct-*ify*, forget-*ful*, forget-*fulneſs*.

G, or J.

This letter conſiſts of a ſegment of a circle, the ſame ſize as the *d*, but formed in a different direction from that letter; this being in a vertical direction, running from the top line to the bottom one. It is alſo uſed for the *j*, which has partly the ſame ſound as the ſoft *g*. But, when uſed for the *j*, it ſhould be written fainter, particularly towards the bottom. This letter may alſo be begun either at the top or the bottom, as is found moſt convenient to make it join with the foregoing letter. When it ſtands alone, and as a *g*, it repreſents the words *again*, *againſt*, *great*; and as a *j*, it ſignifies the words *judge*, *juſt*, *join*. Joined to other characters, it repreſents the word *grand:* as, *grand*-father; and the terminations *gree* and *join:* as, a-*gree*, con-*join*.

H.

There is no character to diſtinguiſh this letter; for as it is only an aſpiration of the breath, its inſertion is not neceſſary to diſcover the ſound of the word; it is, therefore, wholly omitted.

J.

This is repreſented by *g;* as hath been ſhewn.

K, or C hard.

This letter is formed in the ſame ſhape and ſize as the *d*, but in the quite oppoſite direction; it is always begun at the left hand. When it ſtands alone, it repreſents the words *can*, *could;* and with a ſhort ſtroke drawn under the middle, it ſignifies *cannot*. Uſed as a prepoſition, it ſignifies *com*, *con*, and *contra;* as, *com*-pound, *con*-ceal, and *contra*-ry; as a termination, it repreſents *acle*, *ical*, and *icle;* as, ſpect-*acle*, period-*ical*, and art-*icle*.

L.

This letter is formed of a ſtraight lineal ſtroke, like the *f*, but turned a little at the top, towards the left hand. It is moſtly begun at the top; but, if found neceſſary, may be made from the bottom. By itſelf, it repreſents the words *all*, *always*, *altogether;* joined to other characters, it ſtands for *low*, *latter*, and *late:* as, *low*-ly; *latter*-ly: *late*-ly, and, as a termination, it ſignifies *ly:* as in low-*ly*, latter-*ly*, and late-*ly;* thus, two of thoſe characters repreſent each of theſe words; and when thus uſed, ſhould have its vowel point to the firſt ſyllable.

M.

This letter is repreſented by a ſtraight lateral ſtroke; and is always begun at the left hand. Alone, it repreſents the words *am*, *amongſt*, *my*, *mine*, *me*, *whom;* alſo, *much* or *many;* when made larger than common, it ſignifies the word *more;* with a ſhort ſtroke acroſs the middle, it ſtands for the words *twice as much*, or *twice the number*, or *double the quantity;* with two ſuch ſtrokes, it ſignifies *three times as much*, &c.; and with a dot placed under the middle, it ſtands for *half as much*, &c.; and with two ſuch dots placed under it, it ſignifies the word *moſt*. With the negative ſtroke under it, it ſignifies *not me*, *not ſo much*, &c. Joined with other characters, it repreſents the prepoſitions *magni*, and *mis:* as, *magni*-fy, *mis*-fortune; and as a termination, it ſtands for *ment* or *ments:* as, teſta-*ment*.

N.

This letter is formed of a ſtraight lineal ſtroke, with the top a little turned towards the right hand. It is moſtly begun at the bottom, but it may be formed from the top, if requiſite. When uſed alone, it repreſents the words *an*, *in*, *under;* as a prepoſition, it ſtands for *ante*, *anti*, *in*, *inter*, *under:* as, *ante*-cedent, *anti*-podes, *in*-ter, *inter*-rogation, *under*-mine. As a termination, it ſtands for *ent* and *neſs:* as, garm-*ent*, har-*neſs*.

P.

This letter is formed of an inclined curve line, leaning towards the right hand. It is always begun at the bottom. When uſed alone, it ſtands for the words *upon* and *poor*. As a prepoſition, it repreſents *per*, *pre*, and *pro:* as, *per*-plex, *pre*-ſage, and *pro*-long. As a termination, it is ſeldom or never uſed in Engliſh.

Q.

This letter is formed of a ſtraight ſtroke in a lineal direction, with the top curled quite round towards the left hand. It may be begun either at the top or the bottom, as is found moſt convenient. When it ſtands alone, it ſignifies the words *queſtion* and *quantity;* with a ſhort ſtroke acroſs the middle, it repreſents the phraſe *double the quantity;* and with a dot placed under it, it ſtands for *half the quantity;* with the negative mark under it, it ſtands for *not the quantity*. When joined to other characters, it repreſents the words a *quarter*, and a *quart:* as, a *quarter* of an hour, a *quart* of water. As a termination, it ſtands for *quire:* as, re-*quire*.

R.

This letter is formed like the *g*, but in an oppoſite direction. It may be formed either upwards or downwards, as moſt convenient. When uſed alone, it ſtands for *or* and *other*; when joined to other characters, it repreſents *re*: as, *re*-ſolve. As a termination, it ſtands for *ary*, *ing*, *ings*, and *er*: as, di-*ary*, exceed-*ing*, turn-*er*.

S, ſoft C and Z.

This character is formed of an inclined curve line, and is moſtly begun at the bottom. It repreſents alſo the ſoft *c*, and the *z*: but as the *z* has a coarſer ſound, it may be repreſented by a blacker and fuller character. When uſed alone it ſignifies *as*, *is*, *us*, and *yes*; when it is joined to other characters, it ſtands for *ſatis*, *circum*, *ſuper*, and *ſub*: as, *ſatis*-fy, *circum*-vent, *ſuper*-fine, *ſub*-tract. As a termination, it ſtands for *tion*, *ſelf*, and *ſoever*: as, ora-*tion*, my-*ſelf*, whom-*ſoever*.

T.

This letter is formed of an inclined but ſtraight line, and is moſtly begun at the top. Alone, it ſtands for *the* and *to*. As a prepoſition, it ſtands for *tranſ*: as, *tranſ*-fer. As a termination, it ſignifies *ty* or *ties*: as, beau-*ty*, beau-*ties*.

W.

This letter is formed of a ſtraight but inclined ſtroke, like the former, but in the oppoſite direction. It is moſtly begun at the bottom. By itſelf, it ſtands for *will*, *wilt*, *would*; and with a ſhort ſtroke drawn under the middle it repreſents *will not*, or *would not*. Joined to a following character, as a prepoſition, it ſignifies *with*: as, *with*-out. As a termination, it ſtands for *ward* and *well*: as, out-*ward*, and holy-*well*.

X.

This letter is formed of a ſtraight lineal ſtroke, a little turned at the bottom towards the right hand. It is generally formed downwards, but it may alſo be formed upwards, if found more convenient. Alone it ſtands for *except*, *exerciſe*, and *excellent*. Joined to other characters, it repreſents *ex*, *extra*, and *exceed:* as, *ex*-poſe, *extra*-ordinary, *exceed*-ingly. As a termination, it repreſents *lex*; as, comp-*lex*.

Y.

There is no character appropriated to this letter, being repreſented by the vowel mark for *i*.

Z.

This letter is repreſented by the character which ſtands in common for the ſoft *c*, and *s*; as hath been ſeen.

Beſides the characters appropriated to expreſs the conſonants ſingly, there are three other characters for three double conſonants, viz. *ch*, *ſh*, *th*.

CH.

The character uſed to expreſs theſe two letters, is a ſtraight lateral ſtroke, turned upwards a little, at that end towards the left hand. It is always begun at the left hand point. When it ſtands alone, it ſignifies *each*, *which*, and *ſuch*. As a prepoſition, it ſtands for *arch*: as, *arch*-itect.

SH.

The character appropriated to theſe letters is like the foregoing; but turned downwards at the left hand point, inſtead of upwards, and like that is always begun at the left hand. When it ſtands alone, it repreſents the words *ſhall*, *ſhalt*, and *ſhould;* and with a ſhort ſtroke under the middle, *ſhall*, *ſhould*, or *would not*. As a termination, it ſtands for *ſhip:* as, ward-*ſhip*.

TH.

The character uſed to denote theſe two letters, is alſo a ſtraight lateral ſtroke, with the end turned a little upwards, at the right hand point. This is alſo begun at the left hand. When it ſtands alone, it repreſents the words *they*, *them*, *that*, *this*, and *than*.

There are alſo other characters appropriated to expreſs ſeveral treble conſonants, which frequently occur in words of moſt common uſe.

CHR.

The character uſed for theſe three letters, is two perpendicular lines joined at the top by a curve:—it is moſtly begun at the left hand point; and is uſed in the words *Chr*-iſt; and, with the vowel point for *a*, in the word *char*-mer.

LCH.

This character is formed like the two ſides of a triangle, with the point downwards; and is moſtly begun at the left hand; as the foregoing.

LTR.

This character is formed like the two ſides of a ſquare, with the conjunction of the line, at the bottom, and towards the left hand. It is generally begun, like moſt of the others, at the left hand.

LTH.

This character is formed like that of *lch*, but with the point upwards; and is begun at the left hand point.

MPL.

This character is a double curve, and inclined towards the left hand, at the top; at which part it is begun.

NCH.

This character is juſt the reverſe of that for *chr*, and begun at the left hand.

RCH.

This character is compoſed of the two ſides of a ſquare, joined at the top, towards the right hand.

RTL.

This character is an inclined double curve, leaning towards the right hand, being in the oppoſite direction to that for *mpl*.

RTH.

This character is an inclined line, leaning towards the left hand, where it is begun, and curled quite round at the top.

SHR.

This character is alſo an inclined line, but leaning towards the right hand, and curled round at the bottom.

SKR.

This character is alſo an inclined line, leaning towards the right hand, and curled round at the top.

STR.

This character is formed of two ſtraight lateral lines, joined together at the left hand end by a curve. This letter muſt be begun at the top, and at the right hand.

SPL.

This character conſiſts of the two ſides of a ſquare, joined together at the bottom of the right hand ſide. It is always begun on the left hand.

SPR.

This character is formed of a double curve, and inclined towards the left hand, like that for *mpl*; but the curves are formed in an oppoſite direction.

THR.

This character is an inclined line, leaning towards the left hand, and curled round at the bottom.

Here it muſt be noted, that the characters for thoſe treble conſonants are to be uſed, though the conſonants which they repreſent do not follow each other immediately; as in the words a*lt*a*r*, and a*r*i*th*metic.

When the learner is perfectly acquainted with the conſtruction and full ſignification of each of the foregoing characters, he may then proceed to join theſe characters together, in order to form words; obſerving, that each character be neatly joined with the foregoing one; for which purpoſe, it is often requiſite, that many of the characters be formed leſs than their real ſize; as may be ſeen in the plate.

Of the Vowels.

The vowels, in ſhort hand, are repreſented by dots. In the perpendicular and inclined letters, *a* is repreſented by a dot, placed on the left hand ſide of the letter, and near the top of it: *e*, by a ſimilar dot, near the bottom, on the left hand ſide: *i*, by a dot placed juſt above the letter, like the tittle to a ſmall *i*: *o* is repreſented by a dot placed near the top, but on the right hand ſide of the letter, juſt oppoſite the place of *a*: and *u* is repreſented by a dot near the bottom of the right hand ſide, oppoſite the place of *e*. But, in horizontal characters, the dot for *a* is placed under the letter, and at the left hand point; that for *e*, under the letter alſo, but at the right hand end; the *i*, over the letter, as before; the *o*, over the left hand end of the letter, and oppoſite the place of *a*; the *u*, over the right hand point, oppoſite the place of *e*.

This order of placing the vowels about the character for any conſonant, is to be obſerved only when the vowel immediately follows ſuch conſonant; if the vowel precede the conſonant, juſt the reverſe takes place, in placing the vowels. When this is the caſe, the dots for the *a* and *e* are placed on the right hand ſide of perpendicular and inclined letters, in the place of *o*, and *u*, reſpectively; and *o* and *u* are placed on the left hand where *a* and *e* are placed in the former caſe; that is, when the vowel follows the conſonant. And *i*, or *y*, is always noted by a dot, placed above the conſonant it follows. In horizontal letters the ſame rule obtains, whether the vowel follows or precedes the conſonant.

But the former method of marking the vowels, viz. by ſetting the mark about the conſonant which immediately precedes the vowel, is more natural and perſpicuous; and ſhould always be followed, if poſſible.

To mark the double or treble vowels, when neceſſary, a dot ſhould be ſet in the place of that vowel which is of the neareſt ſound to ſuch double or treble vowel: as for example, to expreſs the *ea* in the word *great*, we uſe the vowel point for *a*: as, *grat*; and to expreſs the triphthong *eau*, in *beau*, we uſe the vowel point for *o*: as, *bo*; but for the vowels in the word *beauty*, we uſe *u*: as, *buty*.

It is neceſſary that the learner, in his firſt exerciſes in this art, make the character for every letter of which each word is compoſed, marking all the points for the vowels. This is neceſſary in order to render every word intelligible at firſt view; for many learners, who can tell the ſignification of every character, when it ſtands alone, are often at a loſs to diſtinguiſh the meaning of their own writing, when the characters are joined together; partly owing to the ſtrange appearance they make to the eye, and partly to the omiſſion of ſome of the letters. I have, therefore, in the firſt leſſon in the plate, inſerted the character for every conſonant, and alſo all the points for the vowels; which method the learner muſt follow, till the compound characters are become quite familiar to him.

When the learner can readily, and at a future time (or at leaſt ſeveral days after it is written), read thoſe compound characters eaſily, he may proceed to avail himſelf of the next advantage this art allows, namely, that of inſerting no more characters in a word than are abſolutely neceſſary to diſcover its ſound. Thus, for the word *univerſe*, *nvs*, with the vowel mark for *u*, will be ſufficient to convey an idea of the ſound, as it ſhews the letters *unvs*. Uſing alſo the ſimple characters of the alphabet for the words, prepoſitions, and terminations, they there ſignify.—Upon this principle, the ſecond leſſon is formed.

But as the chief excellency of ſhort-hand writing conſiſts in expedition, and which the foregoing rules will not ſufficiently favour, there are alſo other methods of abbreviation, by which much time is ſaved. It is not ſufficient that words be expreſſed by an alphabet of very ſimple characters, but they ſhould alſo be expreſſed by as few of thoſe characters as poſſible; or the chief end of writing ſhort hand, which is that of following a ſpeaker, would be fruſtrated. I ſhall, therefore, lay down a few ſimple rules, which, if ſtrictly adhered to, will enable the learner to write as expeditiouſly as is neceſſary.

RULE I.

In writing the conſonants of the ſame word, the end of the foregoing character is always joined to the beginning of the following one: if they be joined in any other manner, as the words *by the* No. 18, in the third leſſon: or if they ſtand ſeparately, they ſignify whole words, and not ſingle letters.

RULE II.

There are many words which frequently occur in writing upon any ſubject whatever, and may eaſily be recollected by the writer, though expreſſed ever ſo imperfectly; ſuch words may be written by only marking the firſt letter of the word, if it be a conſonant; or the vowel point and the firſt conſonant, if it begin with a vowel, as is often done in the third leſſon with the words *modeſty*, *aſſurance*, *baſhful*, and *impudent*, and *baſhfulneſs* and *impudence.*

RULE III.

When words are thus expreſſed by their firſt, or firſt and ſecond letters, it is often neceſſary to diſtinguiſh them, as being either ſubſtantives, adjectives, verbs, or adverbs. If the word be a ſubſtantive, it is marked by a dot placed againſt the end of the character; if an adjective, by a point placed juſt before or above the place where the ſubſtantive point is placed; and if an adverb, by a point juſt behind or below the ſubſtantive mark, that is, on the oppoſite ſide to the adjective point. The places where the three points are or ſhould be ſituated, muſt be in a ſtraight line, which line muſt be at right angles to the end of the character; as ſeen in the plate: (ſee diſtinguiſhing points.) When the ſubſtantive thus expreſſed is plural, the ſubſtantive point is to be made a little longer than uſual. The verb has no point to expreſs it.

RULE IV.

When the ſame word is required to be repeated with a prepoſition, whether it be a ſubſtantive, adjective, verb, or adverb, it is only placing another point after its ſubſtantive, adjective, or adverb point; as in fig. 5, *time after time.*

RULE

RULE V.

When two words are expreſſed by their firſt conſonants, and a prepoſition ſhould be between them, which is omitted, they are to be joined together in the common manner, and a point placed at their point of conjunction; as in fig. 7 and 8, the *love of money*, the *thirſt for gain*.

RULE VI.

When any word begins with any of the prepoſitions, it is to be begun with the mark for ſuch prepoſition; and, in general, the next conſonant will be ſufficient. And when they end with any of the terminations, the firſt, or firſt and ſecond conſonant of the word with ſuch termination, will generally ſuffice to diſcover the ſound. But when prepoſitions and terminations are uſed, as they are joined to the other characters like ſingle letters, the word ſhould always have at leaſt one of its vowel points to prevent confuſion.

RULE VII.

Moſt of thoſe phraſes which conſiſt of a word with a prepoſition, conjunction, or article, before and after it, may be written as one word.

RULE VIII.

When a pronoun follows a prepoſition, they are joined together, as the foregoing. And pronouns may always be expreſſed by their firſt, or firſt and ſecond conſonants, having the vowel point if requiſite.

RULE IX.

To expreſs the definite article *the*, the conſonant mark for *t* is uſed; and to expreſs the article *a*, a ſmall point is placed at the beginning of the following conſonant.

The foregoing rules are all that can be ſafely given; and if ſtrictly attended to, will anſwer every purpoſe that can be

expected

expected from this art. I have avoided arbitrary characters for certain long words and phraſes, as I am convinced they are not adapted to every one's memory; and from the great number of words with which our language abounds, it is impoſſible that a few hundred characters to expreſs ſo many peculiar words can be of much utility to the learner, and particularly as the chief uſe of ſhort hand is to follow a public ſpeaker, many of whom poſſeſs a very extenſive flow of language. Beſides this, the method of ſubſtituting a ſet of characters for many words or phraſes is very burdenſome to the memory, difficult to learn, and is eaſily and ſoon forgotten.

I have thought proper to mark all the vowels by points, according to many of my predeceſſors in this art, though I have differed from all others in the order of placing them. Dr. Mavor, Mr. Angel, Mr. Maſon, and Mr. Gurney, alſo expreſs the vowels by points; but the method of the three latter is to denote the vowels by points placed on the ſame ſide of the letter: thus *a* and *e* are repreſented by points near the top of the character, or the beginning of it; *i* by a point in the middle of the character; and *o* and *u* by points near the bottom or latter end of the character: whereas, in this ſyſtem, the points for *a* and *e* are placed on one ſide the character, and thoſe for *o* and *u* on the other ſide of the character, and the point for *i*, at the top; which muſt give this method a great advantage over the other. By the former method the place of the *a* and *e* cannot always be diſtinguiſhed from each other, nor that of the *o* and *u*: by this method, I will venture to aſſert, ~~they~~ will never be confounded.

Though ~~ſome writers~~ on this ſubject have given different characters for all the vowels, I have given the preference to points: they give the writer infinitely the advantage over the other method, in point of expedition; and by an attentive peruſal of his work after he has written it, and while the ſubject is ſtill freſh in his memory, he may mark all the vowels; which will render it intelligible at any future time.

With regard to the characters for the conſonants, I have endeavoured to render them as ſimple as poſſible in their conſtruction, yet perfectly different from each other. Moſt writers on this art either make their characters too complex, or, if they make them ſimple, form them ſo much alike, that they are eaſily confounded with each other. In ſome modern ſyſtems, the difference between different characters conſiſts only in one being a fine and the other a black ſtroke; which diſtinction, writers of ſhort hand have ſeldom time to make; not to mention the other ſimilarities between the characters, ſuch as, different characters being of the ſame form, but of different ſizes; with many other abſurdities. While, on the other hand, thoſe who have rendered their characters ſufficiently different, have formed them ſo complex, as to give the learner no advantage, in point of facility, above the common round hand. It muſt be allowed by every one, that the more ſimple a character is, the more eaſily it is formed. When a character conſiſts of two lines, whether ſtraight or curve, it requires almoſt twice as much time to form it as to form a character conſiſting of only one of ſuch lines: and characters which conſiſt of two lines joined together at one end, ſo as to form an angle, require ſtill more time to form them. Mr. Willis, who publiſhed a ſyſtem of ſhort hand in the year 1628, has no leſs than thirteen characters in his alphabet, which are formed with angles in them, beſides two which are complete ovals, or circles. And Mr. Mawd, who publiſhed another ſyſtem in 1635, has alſo eleven angled characters, and thoſe for the moſt part taken from Mr. Willis. Mr. Skelton, alſo, who improved upon the two former, has, in his alphabet, publiſhed in 1659, fourteen very complex characters, namely, ten angled ones, three curled at one end, and one complete circle. But Mr. Rich, who publiſhed his ſyſtem in 1669, has only ſeven angled characters, three curled ones, and one entire circle. His ſyſtem was in great repute for many years, and Mr. Locke recommends it in his treatiſe on Education (becauſe there was at that time none better). Yet Mr. Nicholas, who publiſhed his ſyſtem thirty years after Mr. Rich, and evidently took moſt of the characters in his alphabet from that, has nine angled characters, three curled ones, and one circular. Mr. Nicholas, alſo, was the firſt who omitted inſerting regular characters for all the five vowels, though he has uſed one for *y*. For Mr. Willis has diſtinct characters for every one of the vowels; Mr. Mawd has characters for all but *i* and *u*; Mr. Skelton and Mr. Rich have characters for them all.

In the year 1715, Mr. Lane publiſhed a ſyſtem of ſhort hand under his own name; but his alphabet, as well as that of Mr. Addy, publiſhed ſome time before, is taken literally from Mr. Rich; except that Mr. Lane has omitted the character for *c*, which Rich has uſed; and inſerted a character for *v*, which is not in Mr. Rich's.

In 1727, Mr. Weſton publiſhed his ſyſtem, which was taken for the moſt part from Mr. Rich; but conſiſting of more complex characters than any of his predeceſſors, having thirteen angled characters, four curled ones, and a compound circular one.

In the year 1732, Aulay Macaulay, Eſq. publiſhed his treatiſe of ſhort hand, which though more original in the conſtruction of the characters than any preceding one, is, nevertheleſs, very complex, as in forming many of the letters the pen muſt be taken off the paper before the letter can be finiſhed, as is the caſe in the *n*, where the pen muſt be taken off to mark a point to diſtinguiſh it from the *e*, to which letter it otherwiſe would bear an exact reſemblance; the ſame can be ſaid of the character for the *k*, which muſt alſo have its point. And the vowels in his ſyſtem are all expreſſed by characters; which muſt render it ſtill more tedious to the writer.

Mr. Annett ſoon after publiſhed a treatiſe on ſhort hand, taken from the former, but rendered a deal more ſimple, or rather too ſimple. For many of the characters in his alphabet are formed ſo much alike, that it is impoſſible for a writer in haſte to mark the difference: the characters for *a*, *e*, *i*, and *y*, are all formed by a ſtraight lateral ſtroke, and only diſtinguiſhed from each other by their length, which muſt neceſſarily create much confuſion. The characters alſo for *b* and *p* are ſimilar, differing only in length, as are alſo thoſe for *d* and *t*.

In 1753, Mr. Gurney publiſhed his treatiſe of ſhort hand, being an improvement on that of Mr. Maſon, publiſhed near fifty years before, and who was the firſt who made any conſiderable improvements in the conſtruction of the characters ſince Mr. Rich. Mr. Gurney's alphabet, though an improvement on that of Maſon, has, nevertheleſs, eight characters which have angles in them; beſides a curled character, a common round hand *r*, and a circular character with a ſtraight ſtroke before it, like the figure 10.

Mr. Angel, in 1758, publiſhed a ſyſtem of ſhort hand, being an improvement on that of Mr. Gurney, and in the year 1762, Meſſrs. Swaine and Sims publiſhed their ſyſtem, which was alſo an improvement on the ſame author. Each of theſe alphabets contain ſix characters with angles in them, one circular or curled letter, and one like the figure 10.

Soon after this, Mr. Hodgſon publiſhed his treatiſe of ſhort hand, taken chiefly from Mr. Gurney's alphabet, but being a great improvement thereon: his characters are more ſimple, and, at the ſame time, full as different from each other as thoſe of any alphabet that had been made before his time: he has but two characters which have an angle in them, only one curled one, and one like the figure 10; the others are ſimple curves, or ſtraight lines.

The Rev. Mr. Byrom, A. M. publiſhed his univerſal ſhort hand in 1771, which though conſiſting of no angular nor circular characters, has no leſs than nine characters completely curled at one end, which muſt render his ſyſtem very complex and tedious.

In 1774, Mr. Palmer publiſhed a treatiſe on ſhort hand, in which he ſomewhat improved on Mr. Byrom's plan, having only ſeven curled characters in his alphabet.

In 1776, Mr. Williamſon publiſhed his ſyſtem of ſhort hand, in which alphabet there are alſo eight curled characters.

Mr. Blanchard, in 1776, publiſhed a complete ſyſtem of ſhort hand, in which the characters are all ſimple, ſtraight, or curve lines, except one curled character, one circular one, and a croſs one. His alphabet is, therefore, more ſimple and fit for practice than that of any of his predeceſſors, and the letters are ſufficiently different from each other to prevent miſtakes.

Since Mr. Blanchard publiſhed his ſyſtem, ſeveral others have appeared, which I have not room to mention; ſuffice it to ſay, that from that period to the preſent, moſt, if not all the writers on the art have run into the oppoſite extreme to the writers before his time, which is, that of rendering their alphabetical characters *too ſimple.* In many modern alphabets ſome of the characters reſemble each other ſo nearly, that the writer, ſome time after he has written his piece, is often at a loſs to diſcover the ſenſe of the greater part of it. I have met with ſeveral writers who have learned from Mr. Taplin's ſyſtem, and have experienced this difficulty. Mr. Taplin's alphabet has not one angled or compound character, and only one ſmall circular one for the letter *s;* but then he has one character for two or more letters: thus, *b, f, p,* and *v* are all repreſented by a ſtraight ſtroke in the ſame poſition, and diſtinguiſhed from each other only by the length or boldneſs of the character: the ſame may be obſerved of the characters for *g, j, k,* and *x,* which are alſo ſtraight lines in the ſame poſition: and though the ſimilarity between thoſe and other characters in that alphabet may not be ſo apparent when viewed ſingly, yet when ſuch characters come in contact with others, it ſeldom fails to puzzle its writer, except he be very converſant in the ſyſtem.

Mr. Taplin's method of marking the vowels is ſometimes by dots, and ſometimes by the ſituation of one of the conſonants.

I have noticed the alphabetical characters only of the foregoing writers, without adverting to their other characters for double and treble letters, words, and phraſes, as thoſe characters form the ſtandard of each writer's ſyſtem.

Concerning the alphabet I have here the honour to propoſe, I muſt juſt obſerve, that I have avoided all angular, compound, and circular characters; and uſed only one curled one, which is for the letter *q*; and I truſt I have made them ſufficiently diſtinct in their forms. There are ſome angled and compound characters, uſed for the treble letters; but then it ſhould be remembered that each of theſe characters with a vowel point, will very often ſerve for a whole word, and I was afraid, by indulging too great a ſimplicity in theſe characters, I might degenerate into the modern uniformity.

The method of marking the vowels by points, as I have here adviſed, will be found a very great ſaving of time. It will alſo afford thoſe who can ſo far depend upon their memory, an opportunity of entirely omitting them, till the writing be finiſhed; when, by an attentive peruſal, the piece may have all its vowels marked; which will render it intelligible to the writer at a future time, when he has, perhaps, forgotten the ſubject matter.

A PRAXIS,

OF THREE LESSONS.

On Modeſty.

Leſſon I.—I know no two words, that have been more abuſed by the different and wrong interpretations which are put upon them, than theſe two, Modeſty and Aſſurance. To ſay, ſuch a one is a modeſt man, ſometimes indeed paſſes for a good character; but, at preſent, is very often uſed to ſignify a ſheepiſh, awkward fellow, who has neither good breeding, politeneſs, nor any knowledge of the world.

Again, a man of aſſurance, though, at firſt, it only denoted a perſon of a free and open carriage, is now, very uſually, applied to a profligate wretch, who can break through all the rules of decency and morality without a bluſh.

I ſhall endeavour, therefore, in this eſſay, to reſtore theſe words to their true meaning; to prevent the idea of Modeſty from being confounded with that of Sheepiſhneſs, and to hinder Impudence from paſſing for Aſſurance.

Leſſon II.—If I were put to define Modeſty, I would call it the reflection of an ingenuous mind, either when a man has committed an action for which he cenſures himſelf, or fancies that he is expoſed to the cenſure of others.

For this reaſon a man truly modeſt is as much ſo when he is alone as in company, and as ſubject to a bluſh in his cloſet, as when the eyes of multitudes are upon him.

I do not remember to have met with any inſtance of modeſty with which I am ſo well pleaſed, as that celebrated one of the young prince, whoſe father, being a tributary king to the Romans, had ſeveral complaints laid againſt him before the ſenate, as a tyrant and oppreſſor of his ſubjects. The prince went to Rome to defend his father; but coming into the ſenate, and hearing a multitude of crimes proved upon him, was ſo oppreſſed when it came to his turn to ſpeak, that he was unable to utter a word. The ſtory tells us, that the fathers were more moved at this inſtance of modeſty and ingenuity, than they could have been by the moſt pathetic oration; and, in ſhort, pardoned the guilty father for this early promiſe of virtue in the ſon.

I take aſſurance to be the faculty of poſſeſſing a man's ſelf, or of ſaying and doing indifferent things without any uneaſineſs or emotion in the mind. That which generally gives a man aſſurance is a moderate knowledge of the world, but above all a mind fixed and determined in itſelf to do nothing againſt the rules of honour and decency. An open and aſſured behaviour is the natural conſequence of ſuch a reſolution. A man thus armed, if his words or actions are at any time miſinterpreted, retires within himſelf, and, from a conſciouſneſs of his own integrity, aſſumes force enough to deſpiſe the little cenſures of ignorance and malice.

Leſſon III.—Every one ought to cheriſh and encourage in himſelf the modeſty and aſſurance I have here mentioned.

A man without aſſurance is liable to be made uneaſy by the folly or ill-nature of every one he converſes with. A man without modeſty is loſt to all ſenſe of honour and virtue.

It is more than probable, that the prince above mentioned poſſeſſed both theſe qualifications in a very eminent degree. Without aſſurance he would never have undertaken to ſpeak before the moſt auguſt aſſembly in the world; without modeſty he would have pleaded the cauſe he had taken upon him, though it had appeared ever ſo ſcandalous.

From what has been ſaid, it is plain, that modeſty and aſſurance are both amiable, and may very well meet in the ſame perſon. When they are thus mixed and blended together, they compoſe what we endeavour to expreſs when we ſay a modeſt aſſurance: by which we underſtand the juſt mean between baſhfulneſs and impudence.

I ſhall conclude with obſerving, that as the ſame man may be both modeſt and aſſured, ſo it is alſo poſſible for the ſame perſon to be both impudent and baſhful.

We have frequent inſtances of this odd kind of mixture in people of depraved minds and mean education; who, though they are not able to meet a man's eyes, or pronounce a ſentence without confuſion, can voluntarily commit the greateſt villanies or moſt indecent actions.

Such a perſon ſeems to have made a reſolution to do ill even in ſpite of himſelf, and in defiance of all thoſe checks and reſtraints his temper and complexion ſeem to have laid in his way.

Upon the whole, I would endeavour to eſtabliſh this maxim, that the practice of virtue is the moſt proper method to give a man a becoming aſſurance in his words and actions. Guilt always ſeeks to ſhelter itſelf in one of the extremes, and is ſometimes attended with both.

I have, for the eaſe of the learner, diſtributed all the words of each of the foregoing leſſons, into a regular order; and prefixed to each word, or phraſe, its number, as it is numbered in the plate; by which the characters that compoſe it may be ſeen at one view.

In the firſt leſſon is inſerted every conſonant belonging to each word, with the point for every vowel; which practice, as was before obſerved, the learner muſt purſue, till he be very ready in both writing and reading all his exerciſes. The firſt ten words in this leſſon have their characters formed ſeparately from each other.

In the ſecond leſſon no more letters in a word are inſerted than are abſolutely neceſſary to diſcover the ſound. The ſingle conſonant marks are alſo uſed for the words, prepoſitions, and terminations they repreſent; and the characters for the double and treble letters.

The third leſſon is written upon the moſt finiſhed principles of this art. Above all the advantages which it poſſeſſes, in common with the ſecond leſſon, it has, moreover, all the abbreviations delivered in the foregoing rules. And which rules the writer ſhould have very perfectly in his memory.

LESSON I. *(See the Plate.)*

1 I know
2 no
3 two
4 words
5 that
6 have
7 been
8 more
9 abuſed
10 by
11 the
12 different
13 and
14 wrong
15 interpretations
16 which
17 are
18 put
19 upon
20 them,
21 than
22 theſe
23 two,
24 Modeſty
25 and
26 Aſſurance.
27 To
28 ſay,
29 ſuch
30 a one
31 is
32 a modeſt
33 man,
34 ſometimes
35 indeed
36 paſſes
37 for
38 a good
39 character;
40 but
41 at
42 preſent
43 is
44 very
45 often
46 uſed
47 to
48 ſignify
49 a ſheepiſh,
50 awkward
51 fellow,
52 who
53 has
54 neither
55 good
56 breeding,
57 politeneſs,
58 nor
59 any
60 knowledge
61 of
62 the
63 world.
64 Again,
65 a man
66 of
67 aſſurance,
68 though,
69 at

70 firſt,
71 it only
72 denoted
73 a perſon
74 of
75 a free
76 and
77 open
78 carriage,
79 is
80 now,
81 very
82 uſually,
83 applied
84 to
85 a profligate
86 wretch,
87 who
88 can
89 break
90 through
91 all
92 the
93 rules
94 of
95 decency
96 and
97 morality
98 without
99 a bluſh.
100 I ſhall
101 endeavour,
102 therefore,
103 in
104 this
105 eſſay,
106 to
107 reſtore
108 theſe
109 words
110 to
111 their
112 true
113 meaning;
114 to
115 prevent
116 the
117 idea
118 of
119 modeſty
120 from
121 being
122 confounded
123 with
124 that
125 of
126 ſheepiſhneſs,
127 and
128 to
129 hinder
130 impudence
131 from
132 paſſing
133 for
134 aſſurance.

LESSON II.

1 If
2 I were
3 put
4 to
5 define
6 modeſty,
7 I would
8 call
9 it,
10 the
11 reflection
12 of
13 an ingenuous
14 mind,
15 either
16 when
17 a man
18 has
19 committed
20 an action
21 for
22 which
23 he
24 cenſures
25 himſelf,
26 or
27 fancies
28 that
29 he
30 is
31 expoſed
32 to
33 the cenſure
34 of others.
35 For
36 this
37 reaſon
38 a man
39 truly
40 modeſt
41 is
42 as
43 much
44 ſo
45 when
46 he
47 is
48 alone
49 as
50 in
51 company,
52 and
53 as
54 ſubject
55 to
56 a bluſh
57 in
58 his
59 cloſet,
60 as

61 when
62 the
63 eyes
64 of
65 multitudes
66 are
67 upon
68 him.
69 I do not
70 remember
71 to
72 have
73 met
74 with
75 any
76 inſtance
77 of
78 modeſty
79 with
80 which
81 I am
82 ſo
83 well
84 pleaſed,
85 as
86 that
87 celebrated
88 one
89 of
90 the young
91 prince,
92 whoſe
93 father,
94 being
95 a tributary
96 king
97 to
98 the Romans,
99 had
100 ſeveral
101 complaints
102 laid
103 againſt
104 him
105 before
106 the ſenate,
107 as
108 a tyrant
109 and
110 oppreſſor
111 of
112 his
113 ſubjects.
114 The prince
115 went
116 to
117 Rome
118 to
119 defend
120 his
121 father;
122 but
123 coming
124 into
125 the ſenate,
126 and
127 hearing
128 a multitude
129 of
130 crimes
131 proved
132 upon
133 him,
134 was
135 ſo
136 oppreſſed
137 when
138 it
139 came
140 to
141 his
142 turn
143 to
144 ſpeak,
145 that
146 he
147 was
148 unable
149 to
150 utter
151 a word.
152 The ſtory
153 tells
154 us,
155 that
156 the fathers
157 were
158 more
159 moved
160 at
161 this
162 inſtance
163 of
164 modeſty
165 and
166 ingenuity,
167 than
168 they
169 could
170 have
171 been
172 by
173 the moſt
174 pathetic
175 oration;
176 and,
177 in
178 ſhort,
179 pardoned
180 the guilty
181 father
182 for
183 this
184 early
185 promiſe
186 of
187 virtue
188 in
189 the ſon.
190 I take
191 aſſurance
192 to
193 be
194 the faculty
195 of
196 poſſeſſing
197 a man's
198 ſelf,
199 or
200 of
201 ſaying

202 and
203 doing
204 indifferent
205 things
206 without
207 any
208 uneaſineſs
209 or
210 emotion
211 in
212 the mind.
213 That
214 which
215 generally
216 gives
217 a man
218 aſſurance
219 is
220 a moderate
221 knowledge
222 of
223 the world,
224 but
225 above
226 all,
227 a mind
228 fixed
229 and
230 determined
231 in
232 itſelf
233 to
234 do
235 nothing
236 againſt
237 the rules
238 of
239 honour
240 and
241 decency.
242 An open
243 and
244 aſſured
245 behaviour
246 is
247 the natural
248 conſequence
249 of
250 ſuch
251 a reſolution.
252 A man
253 thus
254 armed,
255 if
256 his
257 words
258 or
259 actions
260 are
261 at
262 any
263 time
264 miſinterpreted,
265 retires
266 within
267 himſelf,
268 and,
269 from
270 a conſciouſneſs
271 of
272 his
273 own
274 integrity,
275 aſſumes
276 force
277 enough
278 to
279 deſpiſe
280 the little
281 cenſures
282 of
283 ignorance
284 and
285 malice.

LESSON III.

1 Every
2 one
3 ought
4 to cheriſh
5 and encourage
6 in himſelf
7 the modeſty and aſſurance
8 I have
9 here
10 mentioned.
11 A man
12 without aſſurance
13 is
14 liable
15 to be
16 made
17 uneaſy
18 by the folly or
19 ill-nature
20 of every
21 one
22 he
23 converſes with.
24 A man without modeſty
25 is loſt
26 to all
27 ſenſe
28 of honour
29 and virtue.

30 It is
31 more
32 than probable,
33 that the prince
34 above mentioned
35 poſſeſſed both
36 theſe qualifications in
37 a very
38 eminent
39 degree.
40 Without aſſurance
41 he
42 would never
43 have undertaken
44 to ſpeak
45 before the moſt
46 auguſt
47 aſſembly
48 in the world;
49 without modeſty
50 he
51 would have
52 pleaded
53 the cauſe
54 he
55 had taken
56 upon him,
57 though it had
58 appeared
59 ever
60 ſo ſcandalous.
61 From what
62 has been
63 ſaid,
64 it is
65 plain,
66 that modeſty and aſſurance
67 are
68 both
69 amiable,
70 and may
71 very well
72 meet
73 in the ſame
74 perſon.
75 When
76 they are
77 thus
78 mixed
79 and blended
80 together,
81 they
82 compoſe
83 what
84 we endeavour
85 to expreſs
86 when
87 we ſay
88 a modeſt aſſurance;
89 by which
90 we underſtand
91 the juſt mean
92 between
93 baſhfulneſs & impudence.
94 I ſhall conclude
95 with obſerving,
96 that
97 as the ſame
98 man
99 may be
100 both modeſt and aſſured,
101 ſo it is alſo
102 poſſible
103 for the ſame
104 perſon
105 to be
106 both impudent and baſhful.
107 We have
108 frequent
109 inſtances
110 of this
111 odd
112 kind
113 of mixture
114 in people of
115 depraved
116 minds
117 and mean
118 education;
119 who,
120 though they
121 are not able
122 to meet

123 a man's
124 eyes,
125 or pronounce
126 a ſentence
127 without confuſion,
128 can
129 voluntarily
130 commit
131 the greateſt
132 villanies,
133 or moſt
134 indecent
135 actions.
136 Such a perſon
137 ſeems
138 to have made
139 a reſolution
140 to do ill,
141 even
142 in ſpite
143 of himſelf,
144 and in defiance
145 of all
146 thoſe checks
147 and reſtraints
148 his temper
149 and complexion
150 ſeem
151 to have laid
152 in his way.
153 Upon the whole,
154 I would endeavour
155 to eſtabliſh
156 this maxim,
157 that the practice
158 of virtue
159 is the moſt
160 proper
161 method
162 to give
163 a man
164 a becoming aſſurance
165 in his words
166 and actions.
167 Guilt
168 always
169 ſeems
170 to ſhelter
171 itſelf
172 in one
173 of the extremes,
174 and is
175 ſometimes
176 attended
177 with both.

CHAP. III.

OF VULGAR ARITHMETIC.

SECT. I.

NOTATION OF NUMBERS.

ARITHMETIC is the moſt neceſſary of all the ſciences. From hence we may account for the perfection to which this part of literature is brought, above any other branch of mathematical ſcience.

Notation, ſtrictly ſpeaking, is that part of arithmetic which teaches how to write any number by its proper characters, or figures, and conſequently in their due places; alſo to read, or diſcover, the true value of ſuch number, when written.

All numbers, and the various combinations of them, are noted by theſe ten characters, 1 one, 2 two, 3 three, 4 four, 5 five, 6 ſix, 7 ſeven, 8 eight, 9 nine, 0 cypher or nothing.

Each of theſe characters, when uſed alone, ſtands for no more than its own intrinſic value: thus, 1 ſtands only for one in number, 2 for only 2, &c. But when any of them are joined to other figures, they together ſtand for more than their own real ſeparate value: thus, 1 and 2, joined together thus, 12, ſtands for twelve; 6 and 5, joined together, ſtands for 65, ſixty-five, &c.

In order to diſcover the value of any compounded number it muſt be obſerved, that a number placed in the firſt place towards the right hand, ſtands for no more than its real intrinſic value, but increaſes in value in a tenfold proportion by every remove towards the left hand: thus, in the number 1799, the firſt figure 9 ſtands for 9 only; the ſecond figure being in the ſecond place towards the left hand, its value is increaſed tenfold; thus it repreſents ninety, or ten times nine; which, with the foregoing 9, ſtands for ninety-nine. Again, the figure 7, which ſtands in the third place towards the left hand, is increaſed to ten times as much as it would be if it ſtood in the next inferior place, viz. where the laſt-mentioned 9 ſtands: thus it repreſents ſeven hundred; which, with the two fore-mentioned figures, ſtand for ſeven hundred and ninety-nine. The figure 1, which ſtands in the fourth place, towards the left hand, is alſo increaſed ten times in value to what it would be if it ſtood in the next inferior place, where the 7 is placed; in which caſe it would repreſent one hundred; whereas, in the preſent inſtance, it ſtands for one thouſand; and with the other figures repreſents one thouſand, ſeven hundred, and ninety-nine.

This deſcription of the four foregoing figures may ſerve to give the uninformed an idea of the value of figures, according to the different places they occupy in a compounded number. For every remove of a figure towards the left hand increaſes its value to ten times as much as before; as will more fully appear by the following table, called the Numeration Table:—

The Numeration Table.

Mill. of millions			*Thous. of millions*			*Millions*			*Thous.*					
Hundreds of millions of millions.	Tens of millions of millions.	Millions of Millions.	Hundreds of thousands of millions.	Tens of thousands of millions.	Thousands of millions.	Hundreds of millions.	Tens of millions.	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.
1	2	3,	4	5	6,	7	8	9,	0	9	8,	7	6	5
	4	3,	2	1	0,	1	2	3,	4	5	6,	7	8	9
		9,	8	7	6,	5	4	3,	2	1	0,	1	2	3
			4	5	6,	7	8	9,	0	9	8,	7	6	5
				4	3,	2	1	0,	1	2	3,	4	5	6
					7,	8	9	0,	9	8	7,	6	5	4
						3	2	1,	0	1	2,	3	4	5
							6	7,	8	9	0,	9	8	7
								6,	5	4	3,	2	1	0
									1	2	3,	4	5	6
										7	8,	9	0	9
											8,	7	6	5
												4	3	2
													1	0
														1

In order to read any number with facility and eaſe, it is neceſſary that the learner have all the names of the numbers at the head of the table perfectly in his memory, that he may apply them to any other number he may have occaſion for; calling the firſt figure, on the right hand, units; the ſecond, tens; the third, hundreds; the fourth, thouſands; &c. as in the table.

Thus, the bottom figure in the table, ſtanding under the place of units, is itſelf an unit, or a ſingle *one*. The next higher number in the table conſiſts of two figures, 1 and 0; the firſt whereof, ſtanding in the place of tens, ſtands for ten, being only a figure one: the other figure, being a cypher, and in the place of units, ſtands for nothing, or no *units*: theſe two figures, therefore, expreſs only ten. The next number conſiſts of the figures 432; the four being in the place of hundreds, ſignifies ſo many hundreds; the three, as many tens; and the two, as many units; and is thus expreſſed: four hundred and thirty-two. The fourth number in the table conſiſts of four figures, the firſt whereof ſtands in the place of thouſands; this number, therefore, is thus expreſſed: eight thouſand, ſeven hundred, and ſixty-five. The fifth number has its higheſt figure in the place of tens of thouſands, and is thus expreſſed: ſeventy-eight thouſand, nine hundred, and nine; it having a cypher in the place of tens, which ſtands for nothing. The ſixth number conſiſts of hundreds of thouſands, and is thus expreſſed: one hundred and twenty-three thouſand, four hundred, and fifty-ſix. The higheſt place of the ſeventh number is that of millions: it is expreſſed thus: ſix million, five hundred and forty-three thouſand, two hundred, and ten. The eighth number conſiſts of tens of millions, and is thus expreſſed: ſixty-ſeven million, eight hundred and ninety thouſand, nine hundred, and eighty-ſeven. The ninth number has its higheſt figure in hundreds of millions; it is expreſſed thus: three hundred and twenty-one million, twelve thouſand, three hundred, and forty-five.

The ſix other numbers are expreſſed as follows:—The tenth number: ſeven thouſand, eight hundred and ninety millions, nine hundred and eighty-ſeven thouſand, ſix hundred, and fifty-four. The eleventh number: forty-three thouſand, two hundred and ten million, one hundred and twenty-three thouſand, four hundred, and fifty-ſix. The twelfth number: four hundred and fifty-ſix thouſand, ſeven hundred and eighty-nine million, ninety-eight thouſand, ſeven hundred, and ſixty-five. The thirteenth number:

nine million of millions, eight hundred and ſeventy-ſix thouſand, five hundred and forty-three million, two hundred and ten thouſand, one hundred, and twenty-three. The fourteenth number: forty-three million of millions, two hundred and ten thouſand, one hundred and twenty-three millions, four hundred and fifty-ſix thouſand, ſeven hundred, and eighty-nine. The fifteenth number: one hundred and twenty-three million of millions, four hundred and fifty-ſix thouſand, ſeven hundred and eighty-nine millions, ninety-eight thouſand, ſeven hundred, and ſixty-five.

I have, in the table, diſtinguiſhed every three figures by a point, or comma, beginning at the right hand, as is generally done in public offices, and by men of extenſive buſineſs.

This method alſo affords an eaſier way of enumerating numbers, than by the foregoing table, as every three figures may have a common ſurname appropriated to them (inſerted in italics at the head of the table), beſides their names of units, tens, and hundreds: thus, when the learner can enumerate the firſt three figures in a number, and knows the proper ſurname to apply to each three figures, he may enumerate any number, however large. The firſt three figures on the right hand have no ſurname, as they ſtand ſimply for units, tens, and hundreds; but the next three figures have the ſurname of *thouſands*; the next three have the ſurname of *millions*; the next three, *thouſands of millions*; and the other three figures have the ſurname of *millions of millions*. Thus, to repeat the higheſt number in the table, beginning at the left hand, I ſay, one hundred and twenty-three (to which I add its ſurname of) million of millions; four hundred and fifty-ſix (with its ſurname) thouſands of millions; ſeven hundred and eighty-nine (with its ſurname) millions; ninety-eight (ſurname) thouſands; ſeven hundred, and ſixty-five.

I have been more particular in the deſcription of the Numeration Table, as it is generally found the moſt difficult of all the tables in Arithmetic to a learner; and ſeveral perſons who have arrived to a tolerable proficiency in this ſcience, are, nevertheleſs, very imperfectly acquainted with this moſt eſſential part.

Beſides the foregoing ten characters uſed to expreſs numbers, there are alſo letters employed for the ſame purpoſe, called Numerical Letters. This was the ancient method of expreſſing numbers, and is ſtill made uſe of frequently, in the title-pages of books, and in funeral monuments in Roman hiſtory, to expreſs the date of the year.

I, ſtands for one.
V, five.
X, ten.
L, fifty.
C, an hundred.
D, or IƆ, five hundred.
M, or CIƆ, a thouſand.
IƆƆ, five thouſand.
CCIƆƆ, ten thouſand.
IƆƆƆ, fifty thouſand.
CCCCIƆƆƆƆ, a hundred thouſand.
IƆƆƆƆ, five hundred thouſand.
CCCCCIƆƆƆƆƆ, ten hundred thouſand, or a million.

The letters MDCCCVIII expreſs the number 1808, the date of the preſent year—M ſtanding for one thouſand, D for five hundred, CCC three hundred more, which is eight hundred, and VIII eight; together one thouſand, eight hundred, and eight.

If a letter or letters of inferior value follow one of ſuperior value, they are to be added thereto: thus, VI ſignify ſix, VII ſeven, VIII eight, and DCC ſeven hundred. But when a letter of inferior value is placed before one of ſuperior value, it is then to be deducted therefrom: thus, IV ſignify four, IX nine, XL forty, CD four hundred, &c.

SECT. II.

OF ADDITION.

ADDITION is that part of arithmetic which teaches how to add two or more numbers or ſums together, in order to diſcover the total, or value of the whole.

Addition of whole numbers is principally divided into two parts: namely, Addition of numbers of one denomination; and Addition of numbers of divers denominations.

Addition of numbers of one denomination conſiſts in adding together ſimple numbers or figures; in which it muſt be ſtrictly obſerved, that the units are to be ſet directly under each other, in the ſame column; the tens, in like manner, under each other; the hundreds alſo under each other; the thouſands alſo, tens of thouſands, and thoſe of every degree, are all reſpectively to be placed in their reſpective places, from the right hand, to which their rank entitles them; as in the following examples:—

Yds.	£.	Gallons.
21	171	124731
12	239	642943
33	713	726574
17	604	812973
49	830	861413
54	135	963672
22	100	347365
208	2792	4479671

The ſeveral numbers to be added together being ſet down in a regular order, as ſeen above, they are to be added together; beginning at the bottom figure on the right hand, and proceeding upwards, till you have added all the figures in one column together; then place the firſt figure on the right hand, or unit figure, of the ſum ſo found, under the ſame column, carrying the remaining figure or figures, if any, to be added to the next column: having diſcovered the amount of the ſecond column, place the unit figure alſo under the ſame column, adding the other figure or figures to the next column, proceeding in this manner till the whole be finiſhed, and ſetting down the total amount of the laſt column under the ſame.

Thus, in the firſt example, I ſay 2 and 4 is 6, and 9 is 15, and 7 is 22, and 3 is 25, and 2 is 27, and 1 is 28; this being the amount of the firſt column, I ſet down 8 (which is the figure in the place of units) under the ſame column, and carry the remaining figure 2 to be added to the next column, ſaying, 2 and 2 is 4, and 5 is 9, and 4 is 13, and 1 is 14, and 3 is 17, and 1 is 18, and 2 is 20, the whole amount of the laſt column; wherefore I ſet it down under the column, and the total is thus found to be 208.

Proceeding in the ſame manner, in the ſecond example, I ſay 5 and 4 is 9 (for the 0 ſtands for nothing), and 3 is 12, and 9 is 21, and 1 is 22; wherefore, I ſet down the 2 under the column, and carry the remaining 2 to be added to the next column; ſaying, 2 and 3 is 5, and 3 is 8, and 1 is 9, and 3 is 12, and 7 is 19, the amount of the ſecond column; wherefore I ſet down the 9 under the column, and carry the remaining figure 1 to be added to the next column, ſaying, 1 and 1 is 2, and 1 is 3, and 8 is 11, and 6 is 17, and 7 is 24, and 2 is 26, and 1 is 27, the amount of the laſt column, and to be ſet under the ſame; wherefore the total is 2792 (two thouſand ſeven hundred and ninety-two).

In the ſame manner the third example is wrought; as alſo the three following; in which the operation is purpoſely omitted, for the practice of the learner.

	Gallons.		*Yards.*		£.
	17964017		12794368		16729420
	43706123		64368901		12090736
	94218740		20107036		67342617
	29991815		45736298		74298064
	58997252		12469927		29020721
	55122753		72970617		41739402
Total	300000700	Total	228447147	Total	241220960
	282036683		215652779		224491540
Proof	300000700	Proof	228447147	Proof	241220060

Here it muſt be noted, that when the amount of any column in any ſum has a cypher in the place of units, ſuch cypher is to be placed under the column; as in the firſt of the three laſt examples.

The three laſt examples are proved—which is done in this manner: after the total is found according to the foregoing rules, and placed in the firſt of the three bottom lines, the top line of the ſum is to be ſeparated by a line drawn under it, the remaining part of the ſum caſt up, and the amount of it placed under the aforeſaid total; this laſt amount, and the top line of the ſum, are then to be added together; and if the amount of theſe two lines be equal to the total in the firſt bottom line, the ſum is rightly caſt up; otherwiſe not.

All other ſums in addition are to be proved in a ſimilar manner, whether they be of one denomination or of divers denominations.

Addition of divers denominations conſiſts in adding together numbers of different denominations, whether they be money, weights, or meaſures.

Before the learner proceeds to addition of money, it is neceſſary that he have the following tables by heart; called the *Pence Table*, and the *Shilling Table*.

The Pence Table.		*s.*	*d.*	*Shilling Table.*		*£.*	*s.*
20	pence is	1	8	20	ſhillings is	1	0
30	—	2	6	30	—	1	10
40	—	3	4	40	—	2	0
50	—	4	2	50	—	2	10
60	—	5	0	60	—	3	10
70	—	5	10	70	—	3	10
80	—	6	8	80	—	4	0
90	—	7	6	90	—	4	10
100	—	8	4	100	—	5	0
110	—	9	2	110	—	5	10
120	—	10	0	120	—	6	0

Th

In addition of divers denominations, thoſe numbers which are of the ſame denomination are always placed in the ſame column, and their name or mark generally at the top: thus, in addition of money, the pounds are placed in one column, with an £ at the top for *libra*, the Latin word for *pounds;* the ſhillings in another column, with an *s* at the top for *ſolidi*, the Latin word for *ſhillings;* the pence alſo by themſelves, in another column, with a *d* at the top for *denarii*, the Latin for *pence;* and ſometimes the farthings are placed in a ſeparate column, with a *qr.* at the top for *quadrans;* but the moſt general rule is, to place them immediately after the pence, as a fraction of a penny: thus, ¼ for one fourth part of a penny, or one farthing; ½ for one half, or a halfpenny: ¾ for three fourths of a penny, or three farthings: as in the following examples:—

£.	s.	d.
5	7	9
12	0	2
1	6	10
8	7	9
20	10	4
3	15	11
2	7	2
6	19	11
52	15	10
47	8	1
52	15	10

£.	s.	d.
27	9	4½
136	4	0
144	11	3
90	7	9¼
166	19	10¼
101	7	9½
24	0	0½
107	15	6
778	15	7½
751	6	3
778	15	7½

£.	s.	d.
172	9	2
225	8	7¼
144	19	11
176	12	10¼
238	17	11
734	14	7½
814	17	10
937	0	0
3445	0	11½
3272	11	9¼
3445	0	11½

In addition of numbers of divers denominations, this is the general rule:—To begin at the right hand column, or the column of the leaſt denomination; adding all the numbers in the ſaid column together, and obſerving how many units there be in it of the next higher denomination, and how much overplus: the overplus is to be ſet down under the line, and the units of the next denomination are to be added to the next column. The next column being caſt up, it muſt alſo be noted how many units there are in it of the next higher denomination, and how much overplus: the overplus is to be placed under its own column, and the units of the next denomination added to the next column, as before; and ſo on to every new column.

Thus, in the firſt of the three foregoing examples, I begin with the unit figures of the loweſt denomination, which is pence, ſaying, 1 and 2 is 3, and 1 is 4, and 4 is 8, and 9 is 17, and 2 is 19, and 9 is 28; to which I add the tens in the ſame column, ſaying, 28 and 10 is 38, and 10 is 48, and 10 is 58; now, by the pence table, I know that 50 pence is 4 ſhillings and 2 pence, therefore 58 pence is 4 ſhillings and 10 pence: thus, I muſt ſet down the 10 pence, which is the overplus, under the ſame column; and the 4 ſhillings, which are 4 units of the next higher denomination, I carry to be added to the next denomination, ſaying, 4 and 9 is 13, and 7 is 20, and 5 is 25, and 7 is 32, and 6 is 38, and 7 is 45; to which I add the tens, ſaying, 45 and 10 is 55, and 10 is 65, and 10 is 75: now, by the ſhilling table, I find that 75 ſhillings is 3 pounds 15 ſhillings; the 15 ſhillings are to be ſet under the column of ſhillings, and the 3 pounds, being units of the next denomination, viz. pounds, are to be added thereto: and the pounds, being the laſt and higheſt denomination, are to be added together as ſimple numbers of one denomination, and the total placed under the ſame: thus the total is found to be 52*l.* 15*s.* 10*d.*

In the next example I begin with the farthings, being of the loweſt denomination, ſaying, 2 farthings and 2 is 4, and 3 is 7, and 1 is 8, and 2 is 10; 10 farthings is 2 pence and 2 farthings; the 2 farthings is to be ſet under the line as overplus, and the 2 pence is to be added to the column of pence, which, added up, amounts to 43 pence, which is 3 ſhillings and 7 pence; the 7 pence is to be ſet under the column of pence, and the 3 ſhillings is to be added to the column of ſhillings; in which column, with the 3 added, there is 75 ſhillings, or 3 pounds 15 ſhillings; the 15 ſhillings muſt be placed under the ſhillings, and the 3 pounds added to the column of pounds, and the pounds added together as before, and the total placed underneath: thus the amount of this ſum is 778*l.* 15*s.* 7½*d.*

In the ſame manner the third, and every other ſum in this part of addition, is to be wrought. The ſums are all ready proved, for the eaſe of the learner. The line under the total ſhews the amount of the ſum without the top line: the bottom line ſhews the amount of the line juſt over it added to the top line of the ſum, and proves the work right, by being equal to the firſt total.

More Examples for the Learner's Practice.

£.	s.	d.
472	7	4½
19	2	0
145	19	10¼
7	0	1
0	10	6½
27	8	2
30	7	9¼
145	0	4
301	10	10
29	7	2½
6	6	0
1185	0	2
712	12	9½
1185	0	2

£.	s.	d.
1762	9	6½
1000	3	2
10	10	6¼
12	9	3
17	4	0¾
123	18	0½
129	0	0
71	6	9½
41	2	11
7	4	2¼
8	8	0
3183	16	5¾
1421	6	11¼
3183	16	5¾

£.	s.	d.
179	9	9½
184	19	0¼
200	7	9
301	8	10½
401	6	4½
21	8	7¾
33	0	0¼
12	12	0½
19	4	4
8	8	0¼
6	12	10½
1368	17	9
1189	7	11½
1368	17	9

Addition of Avoirdupois Weight.

By this weight are weighed all kinds of goods whatever, except bread, gold, ſilver, and precious ſtones.

The

The Table of Avoirdupois Weight.

4 quarters	make	1 dram,	—	marked *dr.*
16 drams	—	1 ounce,	—	*oz.*
16 ounces	—	1 pound,	—	*lb.*
28 pounds,	—	1 quarter of a hundred weight,		*qr.*
4 quarters	—	1 hundred weight,	—	*C.*
20 hundred weight		1 ton,	— —	*T.*

Avoirdupois weight is divided into two parts, called the greater and the leſs. Avoirdupois the greater conſiſts of hundreds, quarters, and pounds: avoirdupois the leſs is divided into pounds, ounces, and drams.

	4	28
C.	*qrs.*	*lb.*
7	3	12
12	0	9
13	2	7
37	1	0
6	2	7
40	0	20
3	3	0
2	0	7
123	1	6

	4	28
C.	*qrs.*	*lb.*
24	2	4
10	0	9
6	1	20
24	2	0
36	3	21
40	2	0
50	1	22
40	0	10
227	2	11

	16	16
lb.	*oz.*	*dr.*
9	9	10
20	13	14
37	12	0
49	4	7
62	0	1
10	7	0
11	6	7
14	0	12
215	6	3

Theſe examples are wrought in the ſame manner as the former; but having a regard to the avoirdupois table, inſtead of the pence table. And the number of units in each column, which compoſe an unit in the next higher column, is placed at the head of the column: thus, in the firſt example, as 28 is the number of pounds which form one quarter of an hundred, it is placed over the column of pounds; and 4, which is the number of quarters contained in an hundred, is placed over the column of quarters. And in the laſt example, 16 being the number of drams contained in an ounce, it is placed over the column of drams; and, as 16 ounces form a pound, 16 is placed over the column of ounces; and the like is obſerved in addition of other weights and meaſures.

In the firſt of theſe examples the number of pounds in the firſt column is 62, which is 2 quarters, and 6 pounds over; the 6 is to be ſet under the pounds, and the 2 quarters added to the column of quarters, which, with the amount of the whole column, is 13 quarters, which is 3 hundreds and 1 quarter; the 1 quarter is to be ſet under the quarters; and the 3 hundreds added to the next column, and the whole amount is found to be 123*c.* 1*qr.* 6*lb.*

Addition of Troy Weight.

By this weight are weighed bread, gold, ſilver, and precious ſtones. The uſual denominations are pounds, ounces, pennyweights, and grains.

The Troy Weight Table.

24 grains	make	1 pennyweight.
20 pennyweights		1 ounce.
12 ounces		1 pound troy.*

Examples of Troy Weight.

12	20	24	
lb.	*oz.*	*pw.*	*gr.*
4	9	1	17
2	7	19	7
7	0	17	8
1	2	12	9
0	10	14	20
4	11	10	11
7	4	3	2
28	10	19	2

12	20	24	
lb.	*oz.*	*pw.*	*gr.*
12	7	11	21
14	8	12	19
19	2	19	8
20	10	7	7
7	11	6	12
14	10	0	15
12	0	10	12
102	3	8	22

12	20	24	
lb.	*oz.*	*pw.*	*gr.*
32	9	2	20
154	8	1	0
47	0	19	21
39	2	12	12
249	4	10	10
347	7	0	9
558	9	19	1
1429	6	6	1

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* The goldſmiths divide the pound troy into 12 marks, inſtead of ounces; and each mark into 24 carats; and each carat into 4 grains.

Theſe examples are wrought according to the general rule given in the former part of this ſection, and in the ſame manner as addition of avoirdupois; but paying a ſtrict regard to the table, or the figures at the head of each column.

Note. That the pound avoirdupois is heavier than the pound troy; but the troy ounce is heavier than the avoirdupois ounce, for 1*lb.* 2*oz.* 12*pw.* troy is equal to a pound avoirdupois; and a pound troy is about 13*oz.* 2½*dr.* avoirdupois.

It is unneceſſary to give examples of the other parts of addition, concerning apothecaries' weight, liquid, dry, and long meaſures, &c. as the ſame method ſerves for all of them; having reſpect to the table belonging to each.

The Table of Apothecaries' Weight.

20 grains	make	1 ſcruple,	marked ℈.
3 ſcruples	—	1 dram,	ʒ.
8 drams	—	1 ounce,	℥.
12 ounces	—	1 pound,	℔.

By this weight apothecaries mix their drugs; but they are bought and ſold by avoirdupois weight.

The Table of Wool Weight.

7 pounds	make	1 clove.
2 cloves, or 14lb.		1 ſtone.
2 ſtone, or 28lb.		1 tod.
6 tod and a half		1 wey, or 182lb.
2 weys, or 364lb.		1 ſack.
12 ſacks	—	1 laſt, or 4368lb.
240 lb.	—	1 pack of wool.

Note. That a ſtone is generally 14lb. in moſt parts of England; but among the London butchers it is only 8lb.

The Table of Cloth Meaſure.

4 nails, or 9 inches	make	1 quarter of a yard.
4 quarters, or 36 inches		1 yard.
5 quarters, or 45 inches		1 ell Engliſh.
6 quarters, or 54 inches		1 ell French.
3 quarters, or 27 inches		1 ell Flemiſh.

The Table of Wine Meaſure.

8 pints	make	1 gallon.
42 gallons	—	1 tierce.
63 gallons	—	1 hogſhead.
84 gallons	—	1 puncheon.
2 hogſheads	—	1 pipe, or butt.
2 pipes, or butts		1 tun, or 252 gallons.

Note. That ſweet oil hath only 236 gallons to the tun. All liquids are meaſured by wine meaſure, except beer and ale.

Ale or Beer Meaſure.

8 pints	make	1 gallon.
8 gallons	—	1 firkin.
2 firkins	—	1 kilderkin.
2 kilderkins	—	1 barrel.
1½ barrel	—	1 hogſhead.

The ale and beer gallons are the ſame; but the beer firkin contains 9 gallons; conſequently, the kilderkin 18, the barrel 36, &c.

In a Tun of Wine there are,	*In a Pipe, or Butt, there are,*
2 pipes, or butts.	2 hogſheads.
6 tierces.	3 tierces.
252 gallons.	126 gallons.
504 pottles.	252 pottles.
1008 quarts.	504 quarts.
2016 pints.	1008 pints.

In a Puncheon there are,	*In a Hogſhead there are,*
84 gallons.	63 gallons.
168 pottles.	126 pottles.
336 quarts.	252 quarts.
672 pints.	504 pints.

In a Barrel of Beer there are,	*In a Barrel of Ale there are,*
2 kilderkins.	2 kilderkins.
4 firkins.	4 firkins.
36 gallons.	32 gallons.
72 pottles.	64 pottles,
144 quarts.	128 quarts.
288 pints.	256 pints.

The Table of Dry Meaſure.

2 pints	make	1 quart.
2 quarts	—	1 pottle.
2 pottles	—	1 gallon.
2 gallons	—	1 peck.
4 pecks	—	1 buſhel, land meaſure.
5 pecks	—	1 buſhel, water meaſure.
3 buſhels	—	1 ſack.
4 buſhels	—	1 comb, or half quarter.
2 combs	—	1 quarter.
4 quarters	—	1 chaldron.
5 quarters	—	1 wey.
2 weys	—	1 laſt, or 10 quarters.
4 fats or vats, or 36 buſhels		1 chaldron.

Note. That in wheat flour there are always five buſhels to the ſack: and ſalt and ſea-coal are heaped, or elſe there are five pecks to the buſhel.

In a Laſt there are,	*In a Wey there are,*
2 weys.	5 quarters.
10 quarters.	40 buſhels.
80 buſhels.	160 pecks.
320 pecks.	320 gallons.
1280 pottles.	640 pottles.
2560 quarts.	1280 quarts.
5120 pints.	2560 pints.

Long Meaſure.

3 barley-corns	make	1 inch.
12 inches	—	1 foot.
3 feet	—	1 yard.
3 feet 9 inches	—	1 ell Engliſh.
5 feet	—	a geometrical pace.
6 feet	—	1 fathom.
5½ yards	—	1 pole, perch, or rod.
40 poles, or perches	—	1 furlong.
8 furlongs	—	1 Engliſh mile.
3 miles	—	1 league.

In a Mile there are,

8 furlongs.	5280 feet.
320 poles.	63360 inches.
1760 yards.	190080 barley-corns.

Land Meaſure.

5½ yards	make	1 pole, perch, or rod.
40 ſquare poles	—	1 rood, or quarter of an acre.
160 ſquare poles	—	1 acre.
80 poles in length, and 2 in breadth		1 acre.
40 poles in length, and 4 in breadth		1 acre.
4 poles in length	—	1 chain.
10 chains in length, and 1 in breadth		1 acre

The Table of Time.

60 ſeconds	make	1 minute.
60 minutes	—	1 hour.
24 hours	—	1 day, natural.
7 days	—	1 week.
4 weeks	—	1 month, lunar.
13 months, 1 day, and near 6 hours		1 ſolar year.

In a Year there are,

13 months, 1 day, 6 hours.	8,766 hours.
52 weeks, 1 day, 6 hours.	525,960 minutes.
365 days, 6 hours.	31,557,600 ſeconds.

Note. The ſolar year is divided into 12 unequal months, called calendar months, according to the ancient verſe, which may ſerve to impreſs the number of days each month contains, on the memory:

Thirty days hath September,
April, June, and November;
February hath twenty-eight alone,
And all the reſt have thirty-one.

SECT. III.

OF SUBTRACTION.

Subtraction, vulgarly called *Subſtraction*, teacheth how to take a leſs number from a greater; and ſheweth the remainder, exceſs, or difference. Thus, if I take 7 from 9, there will remain 2.

Rule. Place the leſs number under the greater; obſerving, that the figures of each denomination in the leſs number ſtand directly under the figures of the ſame denomination in the greater number; that is, the units under units, tens under tens, and pounds, ſhillings, pence, ounces, drams, &c. &c. directly under the ſame, as in addition. Then, beginning at the right hand, or the leaſt denomination, take the value of each figure in the leſs number from that in the greater number, which ſtands directly over it; ſetting down the remainder underneath. Proceed in this manner till the work be finiſhed. But if, as it frequently happens, any ſingle figure in the leſs number be greater than that in the greater number, from which it is to be taken; an unit is to be borrowed from the next figure towards the left hand of the greater number, and added to the uppermoſt figure, that the bottom figure may be taken therefrom; which borrowed unit muſt be paid, or added to the next figure of the leſs number on the left hand.

Examples.

From	6954	1729886	370290370	greater numbers.
Take	5443	1098907	128403492	leſs numbers.
Rem.	1511	630979	241886878	
Proof	6954	1729886	370290370	

To ſhew the uſe of this rule, I ſay—Suppoſe a merchant owed 6954*l.* whereof he has paid 5443*l.*: to know what remains to be paid, the ſums are to be ſet orderly one under the other, according to the foregoing rule, and as ſeen in the examples: then, beginning with the unit figures, in the firſt example, I ſay, take 3 from 4 and there remains 1, which I ſet under the line; next take 4 from 5 and there remains 1, which is alſo ſet under the line; again, take 4 from 9 and there remains 5, which muſt be ſet down as before; and, laſtly, take 5 from 6 and there remains 1: thus there remains due 1511*l.*

Subtraction is proved by adding the remainder to the leſs of the two given numbers, and if the total of theſe two numbers amount to the exact ſum of the greater number, the work is right; otherwiſe not: thus, in this example, I add the remainder 1511*l.* to the leſs number 5443*l.* and the amount is 6954*l.*; the ſame as the greater number.

In the ſecond example, I begin with the units, as before: ſaying, take 7 from 6 I cannot, but by borrowing 1 from the next figure 8, and which added to the 6 makes 16 (the 8 being the next ſuperior number), I ſay, 7 from 16 and there remains 9; then, for the 1 that I borrowed, I carry 1 in return to the next figure of the leſs number, ſaying, 1 that I borrowed and 0 is but 1, therefore, 1 from 8 and there remains 7; again, 9 from 8 I cannot, but borrowing 1 as before from the next figure, I ſay, 9 from 18 and there remains 9; then for 1 that I borrowed, I muſt add 1 to the next figure 8, ſaying, 9 from 9 and there remains 0 (which may always be done when the two figures are the ſame); again, 9 from 2 I cannot, but 9 from 12 (borrowing 1 from 7) and there remains 3; then 1 that I borrowed, added to the 0, and taken from the 7, there remains 6: thus the work is finiſhed. The proof demonſtrates it right.

Proceeding in the ſame manner in the third example, I ſay, 2 from 0 I cannot, but 2 from 10 (borrowing one from the 7), there remains 8; then 1 that I borrowed and 9 is 10, 10 from 7 I cannot, but 10 from 17 and there remains 7; again, 1 that I borrowed and 4 is 5, 5 from 3 I cannot, but 5 from 13 (borrowing 1) and there remains 8; then, 4 from 0 (or nothing) I cannot, but 4 from 10 and there remains 6; again, 1, that I borrowed, from 9 and there remains 8; and 4 from 12 and there remains 8; and 1 that I borrowed and 8 is 9, from 10, and there remains 1; and 1 that I borrowed and 2 is 3 from 7 and there remains 4; laſtly, 1 from 3 and there remains 2.

More Examples.

From	178927	22574390	13942683	greater numb.
Take	147839	2107863	734098	leſs numbers.
Rem.	31088	20466527	13208585	
Proof	178927	22574390	13942683	

The number of years ſince any event happened, may be diſcovered by ſubtracting the date of the year the event happened from that of the preſent year. Thus:—

The preſent year	1808	Spaniſh Invaſion	1808	Gunpowder Treaſon	1808
The fire of London	1666		1588		1605
Years ſince	142		220		203
Proof	1808		1808		1808

Subtraction of divers denominations is performed upon the ſame principle as ſubtraction of numbers of one denomination. Obſerving, that when an unit is borrowed of the next higher denomination, it muſt be conſidered according to its true intrinſic value, and muſt be repaid to the lower figure of the ſame denomination; as will be ſeen in the following examples:—

		20	124		20	124
	£.	s.	d.	£.	s.	d.
Money owing	17	14	9½	123	14	7¾
——— paid	10	10	6¾	111	19	9¼
Remains	7	4	2¾	11	14	10½
Proof	17	14	9½	123	14	7¾

In the firſt of theſe examples, beginning with the farthings, I ſay, 3 farthings from 2 I cannot, but borrowing an unit from the next denomination, or 1 penny from the 9 pence (and which added to the 2 farthings makes 6 farthings), I ſay, 3 farthings from 6 farthings and there remains 3 farthings, which I ſet under the farthings; then for the unit I borrowed of the pence, I add 1 to the 6, ſaying 6 and 1 is 7; now 7 from 9 and there remains 2; then 10 from 14 and there remains 4; laſtly, 10 from 17 and there remains 7; thus the remains is 7*l.* 4*s.* 2¾*d.* which is proved in the example.

In the ſecond example, I ſay, 1 farthing from 3 and there remains 2 farthings, or an halfpenny; which I ſet down in its proper place, viz. under the denomination of farthings; then 9 from 7 I cannot, wherefore borrowing 1 ſhilling from the 14, and adding it to the 7 pence, which makes 19 pence, I ſay, 9 from 19 and there remains 10; then 1 that I borrowed and 19 is 20, 20 from 14 I cannot, wherefore I borrow 1 pound from the pounds, which, added to the 14, makes 34 ſhillings; therefore, I ſay, 20 from 34 and there remains 14; then 1 that I borrowed and 1 is 2, 2 from 3 and there remains 1; and 1 from 2 and there remains 1: thus, the anſwer to this ſum is 11*l.* 14*s.* 10½*d.*; and its proof, under the anſwer, ſhews that it is right.

More Examples.

	20	12 4
	£.	*s.*	*d.*
Due	174	10	9½
Paid	99	17	10
Remains	74	12	11½
Proof	174	10	9½

		20	12 4
	£.	*s.*	*d.*
Due	145	6	10¾
Paid	110	12	11½
Remains	34	13	11¼
Proof	145	6	10¾

		20	12 4
	£.	*s.*	*d.*
Due	1294	19	10¾
Paid	1001	19	10¼
Remains	293	0	0½
Proof	1294	9	10¾

If the money paid be paid at ſeveral times, the ſums ſo paid are to be added together, and the total ſubtracted from the ſum firſt due.

	£.	*s.*	*d.*
Due	145	19	6
Paid at ſeveral times	2	0	0
	10	10	0
	30	9	8
	0	10	6
	3	9	4
	12	0	6
Paid in all	59	0	0
Reſts due	86	19	6
Proof	145	19	6

	£.	*s.*	*d.*
Received	1120	10	0
Paid to ſeveral perſons	2	0	0
	0	10	6
	10	8	8
	12	12	0
	20	19	10
	9	0	0
Paid in all	55	11	0
Reſts due	1064	19	0
Proof	1120	10	0

Examples in Avoirdupois Weight.

		4	28
	C.	*qrs.*	*lb.*
From	12	3	20
Take	10	2	21
Remains	2	0	27
Proof	12	3	20

		4	28
	C.	*qrs.*	*lb.*
From	147	2	9
Take	116	3	12
Remains	30	2	25
Proof	147	2	9

		16	16
	C.	*qrs.*	*lb.*
From	19	9	14
Take	12	8	15
Remains	7	0	15
Proof	19	9	14

Examples in Troy Weight.

		12	20	24
	lb.	*oz.*	*pwt.*	*gr.*
From	336	10	18	20
Take	247	11	19	22
Remains	88	10	18	22
Proof	336	10	18	20

	20	24
oz.	*pwt.*	*gr.*
3124	16	21
320	19	22
2803	16	23
3124	16	21

Theſe examples, and all others of other denominations, are wrought in the ſame manner as ſubtraction of money; having reſpect to the table belonging to the denomination: thus, in the firſt example of avoirdupois weight, I ſay, take 21 pounds from 20 I cannot, wherefore I borrow 1 quarter of an hundred from the 3 quarters, and add it to the 20, which makes 48 pounds; then I ſay, 21 from 48 and there remains 27; next, 1 that I borrowed and 2 is 3, 3 from 3 I cannot, wherefore I ſet down 0, as before hinted; laſtly, 10 from 12, and there remains 2.

The figures at the head of the columns ſhew the number of units which each unit of the next higher denomination contains.

SECT. IV.

OF MULTIPLICATION.

Multiplication is, perhaps, the moſt neceſſary rule in arithmetic for buſineſs, on account of its diſpatch in reſolving ſeveral long queſtions.

Multiplication teaches how, from two given numbers, to find a third, that ſhall contain either of the two given numbers as often as the other contains units: thus, 3 times 4 is 12; here 3 and 4 are the two given numbers, and 12 is the third number, or product, which contains 3 as often as 4 contains units, viz. 4 times; or it contains 4 as often as 3 contains units, 3 times.

There are three ends principally to be anſwered by multiplication.

Firſt. It ſerves to bring the greater denominations of money, weights, or meaſures, into ſmall ones; as, pounds into ſhillings, pence, and farthings; hundred weights into pounds, ounces, or drams; miles into yards, feet, barley-corns, &c.

Secondly. Having the length and breadth of a plain ſurface, we find its contents.

Thirdly. Having the rate or value of any one thing, we know the rate or value of any number of ſuch things, however great.

But before the learner can begin this rule, it is abſolutely neceſſary that he have the following table perfectly by heart:

The Multiplication Table.

2 times	2	is	4
	3	—	6
	4	—	8
	5	—	10
	6	—	12
	7	—	14
	8	—	16
	9	—	18
	10	—	20
	11	—	22
	12	—	24
3 times	3	—	9
	4	—	12
	5	—	15
	6	—	18
	7	—	21
	8	—	24
	9	—	27
	10	—	30
	11	—	33
	12	—	36
4 times	4	is	16
	5	—	20
	6	—	24
	7	—	28
	8	—	32
	9	—	36
	10	—	40
	11	—	44
	12	—	48
5 times	5	—	25
	6	—	30
	7	—	35
	8	—	40
	9	—	45
	10	—	50
	11	—	55
	12	—	60
6 times	6	—	36
	7	—	42
	8	—	48
	9	—	54
	10	—	60
	11	—	66
	12	—	72
7 times	7	is	49
	8	—	56
	9	—	63
	10	—	70
	11	—	77
	12	—	84
8 times	8	—	64
	9	—	72
	10	—	80
	11	—	88
	12	—	96
9 times	9	—	81
	10	—	90
	11	—	99
	12	—	108
10 times	10	—	100
	11	—	110
	12	—	120
11 times	11	—	121
	12	—	132
12 times	12	is	144.

The uſe of this table is, to find the product of any two numbers: thus, to find the product of 6 times 7, I look in the brace which has 6 at the point, and in the line which has 7 at the beginning, oppoſite to which is the product, which is 42.

The table is to be thus read:—

Beginning with the firſt brace, I ſay—2 times 2 is 4, 2 times 3 is 6, 2 times 4 is 8, 2 times 5 is 10, 2 times 6 is 12, 2 times 7 is 14, 2 times 8 is 16, 2 times 9 is 18, 2 times 10 is 20, 2 times 11 is 22, 2 times 12 is 24.

Then, proceeding in the ſame manner, I begin with the ſecond brace, ſaying, 3 times 3 is 9, 3 times 4 is 12, 3 times 5 is 15, 3 times 6 is 18, 3 times 7 is 21, 3 times 8 is 24, 3 times 9 is 27, 3 times 10 is 30, 3 times 11 is 33, 3 times 12 is 36.

Again, I begin with the third brace, ſaying, 4 times 4 is 16, 4 times 5 is 20, 4 times 6 is 24, 4 times 7 is 28, 4 times 8 is 32, 4 times 9 is 36, 4 times 10 is 40, 4 times 11 is 44, 4 times 12 is 48.

Then, I ſay, 5 times 5 is 25, 5 times 6 is 30, 5 times 7 is 35, 5 times 8 is 40, 5 times 9 is 45, 5 times 10 is 50, 5 times 11 is 55, 5 times 12 is 60.

Then, 6 times 6 is 36, 6 times 7 is 42, 6 times 8 is 48, 6 times 9 is 54, 6 times 10 is 60, 6 times 11 is 66, 6 times 12 is 72.

Then, 7 times 7 is 49, 7 times 8 is 56, 7 times 9 is 63, 7 times 10 is 70, 7 times 11 is 77, 7 times 12 is 84.

Then, 8 times 8 is 64, 8 times 9 is 72, 8 times 10 is 80, 8 times 11 is 88, 8 times 12 is 96.

Again, 9 times 9 is 81, 9 times 10 is 90, 9 times 11 is 99, 9 times 12 is 108.

Then, 10 times 10 is 100, 10 times 11 is 110, 10 times 12 is 120.

And 11 times 11 is 121, 11 times 12 is 132.

Laſtly, 12 times 12 is 144.

36, and 2 that I carried is 38, 8 and carry 3; 4 times 0 is nothing, wherefore I have nothing to ſet down but the 3 I carried; again, 4 times 1 is 4: thus I have done with the firſt figure in the multiplier. Beginning with the next, I ſay, 9 times 2, or 2 times 9, is 18, 8 and carry 1; the 8 is to be ſet exactly under the multiplier 9; then 9 times 1 is 9, and 1 I carried is 10, 0 and carry 1; 9 times 0 is 0, wherefore I ſet down the 1 I carried; again, 9 times 7 is 63, 3 and carry 6; 9 times 9 is 81, and 6 is 87, 7 and carry 8; 9 times 0 is 0, but 8 was carried; 9 times 1 is 9: thus I have done with the ſecond multiplier. The next and laſt two figures in the multiplier being 12, I multiply by them as by one figure, placing the unit figure of the firſt product under that of the multiplier, which is the 2; ſaying, 12 times 2, or 2 times 12, is 24, 4 and carry 2; 12 times 1 is 12, and 2 is 14, 4 and carry 1; 12 times 0 is 0, but 1 was carried; 12 times 7 is 84, 4 and carry 8; 12 times 9 is 108, and 8 is 116, 6 and carry 11; 12 times 0 is 0, but 11 was carried, therefore I ſet down 1, and carry 1; 12 times 1 is 12, and 1 is 13; which being the laſt, I ſet it down. The work being finiſhed, all the ſeveral products are added together, and the total is the real product.

The work is proved by dividing the multiplier in half, and multiplying the original multiplicand by one half, and the product thence ariſing by the number 2, as ſeen in the foregoing example. But if the multiplier cannot be divided exactly in half, as is the caſe when it conſiſts of an odd number, then a number that is an unit leſs than the multiplier is to be divided in half; and the multiplicand multiplied by the one half, and that product by 2, as before, and the original multiplicand added to the laſt product, as in the following examples:—

Example.

```
     379643052
          4321
     ---------
     379643052
    759286104
  1138929156
 1518572208
 -------------
 1640437627692
```

Proof.

```
     379643052
          2160
  ------------
   22778583120
    379643052
   759286104
  ------------
  820028992320
             2
 -------------
 1640057984640
 1640437627692
```

In proving this example, I cannot divide the multiplier 4321 exactly in half, becauſe it contains an odd number in the place of units; I therefore take a number that is an unit leſs, viz. 4320, and divide it in half thus: ſaying, the half of four thouſand is two thouſand, and the half of three hundred and twenty is one hundred and ſixty; which, together, is 2160 for the firſt multiplier in the proof; the product of which is again multiplied by 2, and that product added to the firſt multiplicand, which gives a product equal to the product in the example: which proves the work right.

When the multiplier has cyphers intermixed with the other figures, the ſignificant figures only are to be regarded as multipliers, obſerving the directions before given, to place the unit figure of each product under that of the multiplier; as in the following examples:—

Example.

```
    2|7|9|6|3|8|2|4|9
             |2|0|4|0
 ---------------------
 |1|1|1|8|5|5|2|9|9|6|0
5|5|9|2|7|6|4|9|8|0|
 ---------------------
5|7|0|4|6|2|0|2|7|9|6|0
```

Proof.

```
     2|7|9|6|3|8|2|4|9
              |1|0|2|0
 ----------------------
   | |5|5|9|2|7|6|4|9|8|0
  2|7|9|6|3|8|2|4|9|0|
 ----------------------
  2|8|5|2|3|1|0|1|3|9|8|0
                       2
 ----------------------
  5|7|0|4|6|2|0|2|7|9|6|0
```

When the multiplier, or multiplicand, or both, conſiſt of cyphers towards the right hand, and ſignificant figures towards the left, ſuch cyphers are omitted in the operation, and placed on the right hand of the product; as follows:—

Example.

	1	8	7	6	4	9	0	7	2	0			
									5	1	0	0	0
	1	8		6	4	9	0	7	2	0			
9	3	8	-	4	5	3	6	0	0				
9	5	7	0	1	0	2	6	7	2	0	0	0	0

Proof.

	1	8	7	6	4	9	0	7	2	0			
									2	5	0	0	0
	9	3	8	2	4	5	3	6	0	0			
3	7	5	2	9	8	1	4	4	0				
4	6	9	1	2	2	6	8	0	0	0	0	0	0
													2
9	3	8	2	4	5	3	6	0	0	0	0	0	0
9	5	7	0	1	0	2	6	7	2	0	0	0	0

In the proof of the laſt example, the multiplicand is added to the product as before directed, when the original multiplier cannot be divided exactly in half.

Multiplication of divers Denominations.

Multiplication of divers denominations is performed by multiplying each denomination by the multiplier; beginning with the leaſt denomination, and carrying the units of the next denomination to be added thereto, as in addition of money.

Multiplication of divers denominations is either ſingle or compound: ſingle, when the multiplier conſiſts of 12 or leſs, and is performed by one multiplier; compound, when the multiplier conſiſts of more than 12, and requires more than one multiplier.

Example.

£.	s.	d.
9	10	8
		8
76	5	4

Proof.

£.	s.	d.
9	10	8
		4
32	2	8
		2
76	5	4

In this example I begin with the pence, ſaying 8 times 8 is 64, which is 5 ſhillings and 4 pence; the 4 pence I ſet down under the pence, and carry the 5 ſhillings to be added to the product of ſhillings; ſaying, 8 times 10 is 80 and 5 is 85, or 4*l.* 5*s.* the 5 ſhillings I ſet under the ſhillings, and carry the 4 pounds to the next product; then 8 times 9 is 72 and 4 is 76.

The work is proved as the former examples, viz. by dividing the multiplier in half, and multiplying by one half, and that product by 2.

But in compound multiplication of divers denominations, or when the multiplier is more than 12, the multiplier is to be reſolved into its two commenſurable parts, if it be a commenſurable number, and the multiplicand multiplied by one of thoſe parts, and that product by the other part: thus, if the multiplier be 36, I multiply by 6, and alſo the product thence ariſing by 6, which are the two commenſurable parts: for 6 times 6 is 36.

But if the multiplier be not a commenſurable number, or one which cannot be reſolved into parts exactly, then the next lower number which is a commenſurable one, is to be taken, and the multiplicand multiplied by the two parts as before, and the multiplicand multiplied by the overplus, and that product, added to the laſt product; as in the number 46. This number cannot be reſolved exactly into two, or three commenſurable parts, there being no ſuch number in the multiplication table, wherefore I take the next lower commenſurable number 45, becauſe 5 times 9 is 45, and multiply by theſe two numbers, and the multiplicand by 1, which muſt be added to the product.

1 *Query.* What is the amount of 36 pieces of cloth, at 7*l.* 14*s.* 6*d.* each piece?—Here I multiply by 6, and that product by 6.

Example.

£.	s.	d.
7	14	6
		6
46	7	0
		6
278	2	0

Proof.

£.	s.	d.
7	14	6
		3
23	3	6
		6
139	1	0
		2
278	2	0

2 *Qu.* What is the amount of 46 ſcore of ſea-coal, at 38*l.* 18*s.* 0*d.* per ſcore?

Example.

£.	s.	d.
38	18	0
		5
194	10	9
		9
1750	10	0
1789	8	0

Proof.

£.	s.	d.
38	18	0
		3
116	14	0
		7
816	18	0
		2
77	16	0
894	14	0
		2
1789	8	0

In the firſt of theſe examples the work is proved by multiplying by 3, and that product by 6 (which is equal to multiplying by 18, half the original multiplier), and the laſt product by 2, as directed before.

In the ſecond example, I multiply by 5, and that product by 9, which is equal to multiplying by 45; and for the other one I add the multiplicand to the laſt product. In the proof of this example, I multiply by 3, and that product by 7, which being equal to 21, wants 2 of half the multiplier 46; I therefore multiply the original multiplicand by 2, and add the product to the laſt product; the whole of which I laſtly multiply by 2.

Examples in Avoirdupois Weight.

3 *Qu.* What is the weight of 107 cheſts of tea; each cheſt weighing four hundred, three quarters, and twenty-two pounds?

Example.

C.	qrs.	lb.
4	3	22
		10
49	1	24
		10
494	2	16
		7
34	2	14
529	1	2

Proof.

C.	qrs.	lb.
4	3	22
		5
24	2	26
		10
247	1	8
		3
14	3	10
262	0	18
		2
524	1	8
529	1	2

In this example 100 is the neareſt commenſurable number, but the multiplier is 107; I therefore multiply by 10 and 10, and the original multiplicand by 7; which laſt product is added to the other.

In the proof, becauſe 107 is an odd number, I take the half of 106, which is 53; multiplying by 5 and 10, for 50, and the original multiplicand by 3; then I multiply the whole product by 2, which doubles it to 106; and laſtly, add the multiplicand to the product, for the 1 that was wanting.

By theſe examples it may be ſeen, that there is no occaſion to have always a commenſurable number, or to come very near one; for, if it want ever ſo much, it may be worked in this manner; always multiplying the original multiplicand by the overplus, and adding ſuch product to the other.

It muſt be obſerved, that if the multiplier, in this part of multiplication, conſiſt of more than 144, it muſt be reſolved into three multipliers at leaſt; and if it conſiſt of more than 1728, it muſt be reſolved into more than three.

4 *Qu.* What is the ſuperficial content of a piece of ground, whoſe breadth is 10274 feet, and length 24640?

Here I multiply the length by the breadth (which is the general rule in ſuperficial meaſure), and the product is 253151360 ſquare feet; for the anſwer.

24640
10274
98560
172480
49280
246400
253151360

5 *Qu.* How many ſolid feet does a piece of timber contain, that is forty feet in length, four in breadth, and three in depth?

Here the general rule is, to multiply the length by the breadth, and that product by the depth: And the product is 480 ſolid feet, for the anſwer.

length	40
breadth	4
	160
depth	3
	480

Multiplication teaches alſo how to multiply different denominations of meaſure by different denominations, called *croſs multiplication*; of which I ſhall ſpeak in Menſuration.

Multiplication alſo ſerves to bring great denominations of money, weights, or meaſures, into ſmall ones: but this more properly belongs to the rule of Reduction.

SECT. V.

OF DIVISION.

DIVISION teacheth how, from two given numbers, to find a third; that ſhall be contained in the largeſt of the two given numbers, as often as the ſmalleſt contains units: or the third number contains units, as often as the ſmalleſt of the two given numbers is contained in the other. Thus, if 15 were to be divided by 3, the anſwer would be 5; for 3 is contained in 15, 5 times: and 5 is contained in 15, 3 times.

As multiplication teacheth how to bring great denominations into ſmall ones; and having the rate or value of one thing, to know the rate or value of many; and from the length and breadth of a ſuperficies, to know its contents, &c.: ſo, on the contrary, diviſion teacheth how to bring ſmall denominations into great ones; and from the rate or value of many things, and the number of them given, to know the rate or value of one; and from the contents of a ſuperficies, and the length to find the breadth; or from the ſuperficies, and the breadth, to find the length; and from the contents of a ſolid, and one dimenſion thereof, to find the other two.

In every ſum in diviſion, there are three parts which are to be particularly remembered, viz.

The *Dividend*, or number to be divided.

Diviſor, or number by which we divide.

Quotient, or anſwer to the work, which ſhews how often the diviſor is contained in the dividend. Thus, in the before-mentioned inſtance, 15 is the dividend, 3 the diviſor, and 5 the quotient.

Beſides theſe three parts, which are in every ſum, there is ſometimes a remainder, when the work is finiſhed; which will always happen when the dividend does not exactly contain the diviſor a certain number of times: as, if I were to divide 13 by 4; here 4 the diviſor is contained 3 times in the dividend 13, and there is a remainder of 1; and in diviſion of divers denominations, it muſt be noted, that the remainder is always of the ſame denomination with the dividend.

Diviſion is either ſingle, or compound: ſingle diviſion is when the diviſor does not conſiſt of more than 12, and the dividend of not more than 144. Any queſtions of this ſort may be anſwered at once by the multiplication table, without ſetting them down: thus, if it were required to divide 110 by 10, by that table I know that 10 times 11 is 110; thus 10 is contained 11 times in 110, and 11 is the quotient.

Compound diviſion is when the diviſor contains more than 12, or dividend more than 144, or both.

In diviſion the dividend is to have a crooked line placed at each end of it; and before the line on the left hand, the diviſor is to be placed, and behind that on the right hand, the quotient, or the anſwer to the work, is placed.

Rule. Mark off from the dividend, by a point placed under the figure, as many figures as there are in the diviſor; but if the diviſor conſiſt of a greater number than the figures ſo pointed off from the dividend, then another figure is to be pointed off from the dividend; then ſeek how often the diviſor, or (if the diviſor conſiſt of many figures) the firſt, or firſt and ſecond figures of the diviſor, is contained in the ſame number of figures pointed off from the dividend; place the anſwer in the quotient, and multiply the diviſor thereby; then place the product of ſuch multiplication under the pointed-off figures, and ſubtract the ſaid product therefrom; but if the product amounts to more than the figures pointed off, the diviſor is to be multiplied by a number that is an unit leſs than the former anſwer, and this anſwer placed in the quotient, inſtead of the former, and this product ſubtracted from the aforeſaid figures in the dividend (if this product be ſtill too large, the dividend muſt be multiplied by an unit leſs); to the remainder, the next figure in the dividend is to be brought down, and placed on the right hand thereof, which number is to be taken for a new dividend: ſeek how often the diviſor is contained in ſuch new dividend, place the anſwer in the quotient, multiply the diviſor thereby, ſubtract the product from the laſt dividend, and bring down the next figure in the dividend, as before. Proceed in this manner till the work be finiſhed; as in the following examples:—

Let 4876 be divided into 7 parts.

In this example, I place the numbers as before directed: and becauſe 7 the diviſor is more than 4, the firſt figure in the dividend, I make a point under the ſecond figure 8 in the dividend, which makes 48 for the firſt dividend; then I aſk how often 7 the diviſor is contained in 48, the firſt dividend, which I find is 6 times. I therefore

7)4876(696
42
67
63
46
42
4

I therefore place 6 in the quotient, and multiply the diviſor 7 thereby, and the product 42 I ſet under the dividend 48, to be ſubtracted therefrom; and to the remainder 6, I bring down the next figure 7, of the original dividend; thus I have 67 for a new dividend. Then I aſk how often 7 is contained in 67, which I find is 9 times: I therefore put 9 in the quotient, and multiply the diviſor thereby, and the product 63 I place under, and ſubtract from, the laſt dividend; and to the remainder 4 I bring down the next and laſt figure 6 in the original dividend, which makes 46 for a new dividend. Then I aſk how often 7 is contained in 46, which I find is 6 times: I place 6 in the quotient, and multiply the diviſor thereby: the product 42 I ſubtract from the laſt dividend 46, and there being no more figures in the original dividend to bring down, there is a remainder of 4 after the work is finiſhed, and which remainder is always leſs than the diviſor, if the work be right.

The remainder may be ſet over the diviſor as a fraction of an unit; thus in the anſwer to this queſtion: if 4876*l.* were to be divided equally among 7 men, what would each man's ſhare amount to? *Anſ.* 696*l.* and 4-7ths of a pound.

Diviſion is proved by multiplication: thus the foregoing example is proved by multiplying the quotient by the diviſor, adding thereto the remainder, and if the product be equal to the dividend the work is right; otherwiſe not.

```
8)61280(7660              11)18727(1702
  56···     8               11···    11
  ──                        ──
   52  61280 proof.          77  18727 proof.
   48                        77
   ──                        ──
    48                        02
    48                         0
    ──                        ──
    00                        27
    ══                        22
                              ──
                               5
                              ══
```

In the firſt of theſe two examples, I ſay, 8 is contained in 61, 7 times; and 7 times 8 is 56, which I place under the dividend, and ſubtracted from it, there reſt 5; to which I bring down the 2, which makes 52 for a new dividend. 8 in 52, 6 times; and 6 times 8 is 48, which ſubtracted from 52, there reſts 4, which with the 8 brought down is 48. 8 in 48, 6 times; and 6 times 8 is 48, which ſubtracted from 48, there remains 0; to which I bring down the next, and laſt figure 0, 8 in 00 no times, wherefore I ſet 0 in the laſt place in the quotient; and thus the work is finiſhed rightly: as ſhewn by the proof.

The ſecond example is wrought in the ſame manner, ſaying 11 in 18 once; and 11 ſubtracted from 18, leaves 7, which with the 7 brought down, is 77. 11 in 77, 7 times; and 7 times 11 is 77, which ſubtracted from 77, reſts 0; to which I bring down the 2. 11 in 2, no times; therefore I place 0 in the quotient; and 0 times 11 is 0, which ſubtracted from 2 there reſts 2, and 7 brought down is 27. 11 in 27, 2 times, and 2 times 11 is 22, which ſubtracted from 27, there remains 5 after the work is ended.

From theſe examples it may be ſeen, that there muſt be always one figure, or cypher, brought down at a time, and no more than one; and for every one, ſo brought down, a figure or cypher muſt be placed in the quotient.

Though the foregoing method is the regular way of performing diviſion, yet there is a more expeditious method, when the diviſor conſiſts of no more than 12, by placing the quotient under the dividend, and performing the ſubtraction in the mind: as in the following examples:—

8)279083(	11)743002(	12)9473684(
34885	67545	789473
Remains 3	Remains 7	Remains 8

In the firſt of theſe examples, I ſay, 8 is contained in 27, 3 times, and there remains 3; the 3 I place under the 7 in the dividend, and the 3 which remains I imagine to be placed before the 9 in the dividend, which makes 39 for a new dividend. Then 8 in 39 is 4 times, and remains 7; the 4 I place in the quotient, aud the 7 placed before the 0 (the next figure in the dividend) gives 70 for a new dividend. Then 8 in 70, 8 times, and remains 6, which, with the next figure 8, make 68. 8 in 68, 8 times, and remains 4, which, with the laſt figure, gives 43 for the laſt dividend. And 8 in 43, 5 times, and there remains 3 at the laſt.

Proceeding in the ſame manner in the next example, I ſay, 11 in 74, 6 times and there remains 8. Then 11 in 83, 7 times, and remains 7. 11 in 60, 5 times, and remains 5. 11 in 50, 4 times, and remains 6. 11 in 62, 5 times, and remains 7.

This is the moſt expeditious manner of dividing by a figure under 12; and is generally done in public offices, ſhop-books, &c.

But when the diviſor conſiſts of more than 12, the work muſt be performed according to the regular method firſt laid down, as the diviſor is to be multiplied by each anſwer placed in the quotient.

```
375)743201(1981          570)2487021(4363
    375...                   2280...
    3682                      2070
    3375                      1710
     3070                      3602
     3000                      3420
      701                       1821
      375                       1710
      326                        111
```

In the firſt of theſe examples, I mark off by a point the three firſt figures on the left hand, becauſe there are three figures in the diviſor; then I aſk how often the two firſt figures in the diviſor are contained in the two firſt figures of this pointed off dividend, which I find to be twice; but multiplying the diviſor by 2, the product is 750, which is greater than the firſt dividend; I therefore place 1 in the quotient, and place the diviſor under the dividend (as I will not increaſe it by multiplication), which I ſubtract from the dividend, and to the remainder I bring down the next figure 2, which makes 3882 for a new dividend; then repeating the queſtion, I proceed in the ſame manner throughout the whole.

When the diviſor conſiſts of ſeveral figures, the moſt expeditious, and at the ſame time equally ſafe, method is, to make a table of the diviſor, which may contain the different products of the diviſor, multiplied by every number from 1 to 9 incluſive; as in the following example:

```
29874308)243098249372(8137
         238994464...
           41037853
           29874308
          111635457
           89622924
           220125332
           209120156
            11005176
```

Table of the Diviſor.

1	29874308
2	59748616
3	89622924
4	119497232
5	149371540
6	179245848
7	209120156
8	238994464
9	268868772

The foregoing table is formed by multiplying the diviſor by the ſignificant figures from 1 to 9, or it may be formed by adding each laſt product to the firſt: thus, the ſecond number in the table is formed by doubling the firſt; the third number is formed by adding the ſecond to the firſt; the fourth number by adding the third to the firſt; the fifth by adding the fourth to the firſt, &c.: and the table may be proved, by multiplying the firſt number (29874308) by 9, and if the product be equal to the laſt number (268868772), the table is right.

The uſe of this table is, to find how often the diviſor is contained in any particular dividend; thus, in this example, after having pointed off the firſt nine figures in the dividend 243098249 (becauſe the firſt eight figures are leſs than the diviſor), I caſt my eye on the table, and find that the next leſs number is 238994464, and which I find by the table contains the diviſor 8 times; I therefore place 8 in the quotient, and the multiple of 8, taken from the table, I place under the dividend, and ſubtract it therefrom, and to the remainder bring down the next figure 3 for a new dividend: then, by the table, I find the diviſor is contained in this dividend but once, I place 1 in the quotient, and ſubtract the multiple of 1 (or the diviſor) from ſuch dividend as before, and to the remainder I bring down the next figure 7: in this dividend (by the table) I find the diviſor is contained 3 times, which I place in the quotient, and take the multiple of 3 from the table to be ſubtracted from the laſt dividend, to which remainder I bring down the laſt figure 2: and, by the table, I find the diviſor is contained in the laſt dividend 7 times; and the multiple of 7 being ſubtracted from this laſt dividend, there remains 11005176 after the work is ended.

When the diviſor has a cypher or cyphers towards the right hand, the dividend may be divided by the ſignificant figures only, and the cyphers ſeparated from the diviſor by a ſtroke of the pen; in which caſe there muſt be as many figures ſeparated from the right hand of the dividend, as there are ſeparated cyphers, and ſuch ſeparated figures are to be ſet down at the laſt as a remainder: and if there be any other remainder, the ſeparated figures are to be ſet on the right hand thereof. When the diviſor has an unit only on the left hand, with nothing but cyphers on the right hand thereof, the diviſion is performed at once, by cutting off as many figures from the right hand of the dividend as there are cyphers in the diviſor: the remainder of the dividend is the quotient. See the following examples:—

```
12,0)2794,3(       22,00)4760,82(216      1,00)39847,24
      232                44..             Rem.          24
Rem, 103                 36
                         22
                         140
                         132
                    Rem. 882
```

Diviſion of divers Denominations.

Diviſion of divers denominations is performed by dividing each denomination in the dividend by the diviſor, placing the quotient of each denomination under the ſame denomination in the quotient, and carrying the overplus of each denomination to be added to the next, as follows:

Divide 437*l.* 19*s.* 10*d.* among 24 men.

```
      £.     s.    d.  £.  s.  d.
  24)437     19    10(18   4   11¾
     24     119   286
     197     96   264
     192     23    22
       5            4
```

In this example, I firſt divide the 437*l.* by 24, the quotient of which is 18*l.* and there remains 5*l.* which, added to the 19*s.* makes 119*s.* for a new dividend; then the diviſor is contained 4 times in 119, wherefore I ſet down 4*s.* in the quotient, and 23*s.* remain, which added to the 10*d.* make 286*d.* for the next dividend; in which the diviſor is contained 11 times, and there remains 22*d.* or 88 farthings, in which the diviſor is contained 3 times. Thus the quotient is 18*l.* 4*s.* 11¾*d.* and there remains 16 farthings.

Sums of this nature may alſo be more expeditiouſly wrought, if the diviſor be a commenſurable number, and can be reſolved into two parts, as in the following example:—

Qu. 1. If the expenſe of a country feaſt be 263*l.* 10*s.* 6*d.* to be paid by 28 ſtewards, what muſt each ſteward pay?—In this example I divide by 4, and the quotient thence ariſing by 7, which is equal to dividing by 28, and the anſwer is found to be 9*l.* 8*s.* 2*d.* for each ſteward to pay.

Qu. 2. If a gentleman ſpend 347*l.* 15*s.* 9*d.* in the ſpace of one year and eight weeks, it is deſired to know how much it is per week on an average?—Here I divide the whole ſum by 5, and the quotient thence ariſing by 6, and that quotient by 2; which is equal to dividing by 60 (the number of weeks in one year and eight weeks), and the anſwer is 5*l.* 15*s.* 11*d.* per week, as in the example.

Qu. 3. If the capital ſtock of a tontine amount to 4372*l.* 14*s.* 0*d.* and there be in it 160 ſhares, what will be the amount of each ſhare?—In this example I divide by 4, and that quotient by 10, and the quotient thence ariſing by 4, which is equal to dividing by 160; as 4 times 10 is 40, and 4 times 40 is 160, and the anſwer is 27*l.* 6*s.* 7*d.* for each ſhare.

Queſtion 1.	*Queſtion* 2.	*Queſtion* 3.
£. s. d.	£. s. d.	£. s. d.
4)263 10 6	5)347 15 9	4)4372 14 0
7) 65 17 7 2	6) 69 11 1 4	10)1093 3 6
9 8 2 5	2) 11 11 10 1	4) 109 6 4 2
	5 15 11	27 6 7

To find the exact remainder in ſums where there are two or more diviſors, as in the foregoing ones, the rule is to multiply the firſt diviſor by the laſt remainder, adding thereto the firſt remainder, if any, and the product will be the true remainder; as if it had been divided by the long method: thus in the firſt of the foregoing examples, I multiply 4, the firſt diviſor, by 5, the laſt remainder, which produces 20, to which adding 2, the firſt remainder, the true remainder is found to be 22, which may be proved at leiſure.

Qu. 4. There is a piece of land, having 4 ſides, containing 1398 acres, 3 roods, 35 perches, and 240 feet in breadth, it is deſired to know how many feet it is in length?

Qu. 5. There is a piece of timber, the ſolid contents of which is 600 feet, its length is 40 feet, and its depth 3 feet; it is required to know its ſuperficial contents?

Queſtion 4.

	A.	R.	P.	
4)	1398	3	35	
6)	349	2	38	3
10)	58	1	6	2
	5	3	12	6

Queſtion 5.

3)600

200

In the fourth queſtion I divide the contents of the land, firſt by 4, and that quotient by 6, and the next quotient by 10, which is the ſame as dividing at once by 240; and the anſwer is found to be 5 acres, 3 roods, and 12 perches, for the length of the piece of land.

In the laſt queſtion I divide the ſolid contents of the piece of timber by 3, the depth, and the quotient 200 feet, is the ſuperficial contents, which if divided again by 40 feet, the length, would give 5 feet for the breadth.

In the ſame manner as the foregoing examples are wrought diviſion of other denominations may be performed, having reſpect to the table of quantity belonging to the ſame.

But in this ſpecies of diviſion, if the diviſor be not a commenſurable number, or one which cannot be divided into parts exactly, the diviſion muſt then be performed by one diviſor.

Diviſion alſo teacheth how to bring ſmall denominations into great ones; but as this part more properly belongs to Reduction, I have deferred treating of it till I come to that rule.

SECT. VI.

OF REDUCTION.

REDUCTION is only the application of the rules of multiplication and diviſion, and teacheth how to bring numbers of one denomination into another denomination without altering their value.

Reduction is either *deſcending* or *aſcending*. Reduction deſcending is performed by multiplication, and ſerves to bring great denominations into ſmall ones; as pounds into ſhillings, pence, or farthings; hundred-weights into pounds or ounces, &c. Reduction aſcending is performed by diviſion, and brings ſmall denominations into great ones; as farthings into pence, ſhillings, or pounds; drams or ounces into pounds, hundred-weights, &c.

Rule. In reduction deſcending, multiply the number by the number of units of the next lower denomination which make an unit of the next greater, and multiply ſuch product by the number of units of the next lower denomination which make one of the next greater; and proceed in this manner till the number be reduced to the denomination required.

EXAMPLE 1. Reduce 250*l.* 0*s.* 0*d.* into farthings.

In this example, I firſt multiply the 250 by 20, which is the number of units of the next lower denomination which make an unit of the next higher; that is, the number of ſhillings contained in a pound, and the product ſhews the number of ſhillings contained in 250*l.*; which product muſt be again multiplied by 12, the number of units of the next lower denomination which make one of the next greater, or the number of pence contained in one ſhilling, and the product gives the number

	250
Shillings in 1 pound	20
Shillings in 250*l.*	5000
Pence in 1 ſhilling	12
Pence in 250*l.*	60000
Farthings in 1 penny	4
Anſwer	240000

of pence contained in 250*l.*; which product, again multiplied by 4, the number of farthings in one penny, the product gives the number of farthings contained in 250*l.*; or the anſwer.

Rule. In reduction aſcending, divide the given number by the number of units of that denomination which make one of the next greater; and divide that quotient by the number of units of the ſame denomination which make one unit of the next higher, and proceed in this manner till the whole is finiſhed.

Thus, as in the foregoing example I reduced 250*l.* into 240000 farthings; ſo inverſely, here I ſay, in 240000 farthings, how many pounds?

Farthings in 1 penny	4)	240000
Pence in 1 ſhilling	12)	60000
Shillings in 1 pound	20)	5000
		250

In this example, I firſt divide the given number of farthings by 4, the number of units of that denomination which are contained in an unit of the next higher denomination; or the number of farthings contained in one penny, and the quotient gives the number of pence contained in 240000 farthings; which quotient is again divided by 12, the number of pence contained in one ſhilling, and that quotient gives the number of ſhillings contained in the given ſum of farthings; and laſtly, theſe ſhillings are again divided by 20, the number of ſhillings in one pound, and the quotient is 250*l.* for the anſwer.

Thus it may be ſeen, that reduction aſcending and deſcending prove each other. For if the ſum be performed by reduction deſcending, it muſt be proved by reduction aſcending, as in the two foregoing examples; and if it be in reduction aſcending, it muſt be proved by reduction deſcending.

In reduction deſcending, when the ſum conſiſts of ſeveral denominations, the number in each denomination, after the firſt, is to be added to the denomination to which it belongs; as in the following example:

Example 3. In 239*l.* 10*s.* 6*d.* how many pence?—In this example, after having reduced the pounds into ſhillings by multiplying by 20, I add to the product the 10 ſhillings which ſtand in the place of ſhillings; and having reduced the ſhillings into pence by multiplying by 12, I add to that product the 6 pence which ſtand in the place of pence; and the anſwer to the work is thus found to be 57,486 pence.

```
239 10 6
 20
4780
  10
4790
  12
57480
    6
57486
```

Example 4. In 24 tons, 10 hundreds, and 3 quarters, how many pounds?—This is performed as the foregoing, but having reſpect to the table of avoirdupois weight. I therefore multiply the tons by 20 to reduce them to hundred weights, as 20 hundred is 1 ton, and to the product I add the 10 cwt. I then multiply by 4 to bring the hundreds into quarters, to which I add the 3 quarters: and, laſtly, multiply the quarters by 28, the number of pounds in a quarter: the product is 54964 for the anſwer.

```
24 10 3
20
480
 10
490
  4
1960
   3
1963
  28
15704
3926
54964
```

Examples in Reduction Aſcending.

Example 5. In 24,649,721 minutes, how many days, hours, and minutes?

Example 6. In 47,398 grains of troy weight, how many ounces, pennyweights, and grains?

Example 7. In 29,472,986 ſquare perches of land, how many acres, roods, and perches?

Example 5.	*Example* 6.	*Example* 7.
6,0)2464972,1	2)47398	4,0)2947298,6
2) 410828.41	12)23699	4) 736824,26
12) 205414	2,0) 197,4 22	184206
17117,10	98 14	
2		
20		

In the firſt of theſe examples I divide the minutes by 60, to bring them into hours, by cutting off the 0 from the 6, and dividing by 6 only, as taught in Diviſion, and the quotient is 410828 hours, and there remains 4 minutes, which, placed before the 1 cut off from the dividend, makes 41 minutes for a remainder. This quotient I again divide by 2, and that quotient by 12, which is equal to dividing by 24, the number of hours in 1 day, and the quotient is 17117 days, and there remains 10; but to find the true remainder, I multiply the 10, the laſt remainder, by 2, the firſt diviſor of the hours, and the product is 20 for the true remainder: thus the anſwer to the queſtion is, 17117 days, 20 hours, 41 minutes.

In the ſecond example, I divide the grains by 2 and 12, to bring them into pennyweights, as 24 grains make 1 pennyweight; the quotient is 1974 pennyweights, and 22 grains remain. The pennyweights I divide by 20, to bring them into ounces, and the quotient is 98 ounces, and 1 pennyweight remains, to which I bring down the 4 I cut off from the dividend, and the laſt remainder is 14 pennyweights: thus the anſwer is 98 ounces, 14 pennyweights, and 22 grains.

In the laſt example, I divide the perches by 40, as 40 perches make 1 rood, and the quotient is 736824 roods, and 26 perches; the roods I divide by 4, to bring them into acres; and the anſwer is found to be 184,206 acres, and 26 perches.

Reduction, both aſcending and deſcending, may be performed by one diviſor or multiplier: thus, to bring farthings into pounds, the pounds may be divided by 960, the number of farthings in a pound, and the quotient will give the number of pounds, and the remainder (if any) muſt be reſolved into the inferior denominations. And to reduce hundredweights into ſingle pounds, they may be multiplied by 112, the pounds in an hundred-weight, and the product is the anſwer. But the method before laid down is the more regular, and at the ſame time the more expeditious way of performing this rule.

Examples of both Kinds for Practice.

Example 8. If 420 pieces of cloth contain 8420 ells Flemiſh, it is required to know how many ells Engliſh they contain? —*Anſ.* 5052 ells Engliſh.

Example 9. In 220 puncheons of rum how many hogſheads?—*Anſ.* 293 hogſheads, and 21 gallons remain.

Example 10. A ſilverſmith hath 1000 ounces of ſilver to be made into ſpoons, ſalts, and tankards; each ſpoon to weigh 2 oz. 12 pwt. each ſalt 3 oz. and each tankard 30 oz. and to make an equal quantity of each, it is deſired to know how many he can make of each?—*Anſ.* 28, and 64 pwt. remain.

Example 8.	*Example* 9.	*Example* 10.		
		oz.	pw.	
8420	220	2	12	1000
3	84	20		20
5)25260	880	52		712)20000(28
5052	1760	3		1424
	7)18480			5760
	9) 2640 7	60		5696
	293 3	30		64
	21	20		
		600		
		712		

In the firſt of theſe examples the 8420 ells Flemiſh are reduced into quarters of a yard, by multiplying by 3 (as there are 3 quarters in an ell Flemiſh), and then brought into Engliſh ells, by dividing theſe quarters by 5, the number of quarters in an ell Engliſh.

The 9th example is wrought in the ſame manner: viz. by reducing the 220 puncheons into gallons, by multiplying by 84, and bringing theſe gallons into hogſheads, and by dividing them by 7 and 9, which is equal to 63, the gallons in a hogſhead.

This method of reduction always takes place when the leſs denomination is not contained any certain number of times exactly in the greater.

In the 10th example I reduce the weight of 1 ſpoon, 1 ſalt, and 1 tankard, into pennyweights, by multiplying by 20, and then add them together; and by the total 712, I divide the 1000 ounces of ſilver, which is alſo reduced into pwts. and it quotes 28 of each, and 64 pwts. remain.

By reduction we are enabled to reduce the coin of one country into that of another, without having recourſe to the rule of three, or exchange.

Example 11. What is the value, in Engliſh coin, of 350 ducats, at 4*s*. 2*d*. per ducat?

Example 12. In 246*l*. 18*s*. 6*d*. Flemiſh money, how much Engliſh, the courſe of exchange being 30*s*. 6*d*. per pound ſterling?

Example 13. How much money Engliſh is there in 4420 pieces of eight, the courſe of exchange being at 49⅝*d*. ſterling?

Example 11.

```
      350
       50
12)17500
2,0) 145,8 4
Anſ. £72 18 4
```

Example 12.

```
30 6     246 18 6
12        20
366      4938
           12
   366)59262(161
       366..
       2266
       2196
        702
        366
        336
         20
   366)6720(18
       366.
       3060
       2928
        132
         12
   366)1584(4
       1464
        120
          4
   366)480(1
       366
       114
```

Example 13.

```
     4420      4420
        5        49
8) 22100     39780
    2762     17680
            216580
              2762
         12)219340
        2,0) 1827,8 4
        Anſ. £913 18 4
```

Anſ. 161*l*. 18*s*. 4¼.

In the 14th example, I multiply the 350 ducats by 50, to bring them into pence, and then divide by 12, and it quotes 1458 ſhillings and 4 pence: I then divide the ſhillings by 20, and the quotient is 72*l.* 18*s.* 4*d.* for the anſwer.

In the 12th example the 246*l.* 18*s.* 6*d.* Flemiſh money is reduced into pence, and then divided by 366, the courſe of exchange; and the anſwer is 161*l.* 18*s.* 4¼*d.* and 114 farthings remain.

The 13th example is wrought in the ſame manner as the 11th: viz. by reducing the pieces of eight into pence; and for the fraction $\frac{5}{8}$ of a penny, I multiply the given number of pieces of eight by 5, the numerator, or upper figure of the fraction, and divide the product by 8, the denominator, or lower figure of the fraction, and to the quotient I add the pence contained in the pieces of eight, and then reduce the whole into ſhillings and pounds, by dividing by 12 and 20, as before, and the anſwer is 913*l.* 18*s.* 4*d.*

This method of reducing foreign coin into Engliſh may ſerve for thoſe who are unacquainted with the rule of practice; for practice performs this much more expeditiouſly, as will be ſhewn in its proper place.

Thoſe ſums in reduction, in which both diviſion and multiplication are uſed, muſt be proved by multiplication and diviſion; as, for example, that part which is performed by multiplication muſt be proved by diviſion, and that part performed by diviſion muſt be proved by multiplication.

The foregoing examples, perfectly underſtood, will be ſufficient to give the learner a complete knowledge of this rule, and the various uſes to which it may be applied.

It is abſolutely neceſſary that the learner be perfectly acquainted with what has been delivered in the foregoing part of this chapter, as all the following rules in arithmetic are performed by one or more of the foregoing rules; I have therefore been more explicit in the former part; being the baſis of the whole.

SECT.

SECT. VII.

THE GOLDEN RULE; OR, SINGLE RULE OF THREE DIRECT.

This rule, which, for its univerſal uſe in moſt parts of the mathematics, is called the *golden rule*, is alſo called a *rule of proportion*, becauſe the number ſought bears a certain proportion to one of the numbers given.

It is called the *rule of three*, becauſe it conſiſts of three given numbers, from which a fourth number is to be found, which, in the direct rule, bears the ſame proportion to the ſecond number as the third does to the firſt.

Rule. Multiply the ſecond and third numbers together, and divide the product by the firſt number, and the quotient is the anſwer ſought, or the fourth number.

1ſt number. *2d number.* *3d number.*
Example 1. If 3 yds. of muſlin coſt 12*s.* what will 9 yds. coſt at that rate?

9
3)108
Anſwer 36*s.*

Here the fourth number or anſwer, 36, bears the ſame proportion to the ſecond number 12, as 9 the third number, bears to the firſt number 3, that is, it contains it three times; or it bears the ſame proportion to the third number 9 as the ſecond number 12 does to the firſt number 3, viz. contains it four times. This proportion is called *direct* proportion; from whence this rule is called the rule of three *direct*, and is always performed as above.

But when the fourth number bears the ſame proportion to the

the ſecond as the firſt does to the third, it is then called *indirect* proportion. Queſtions of this nature belong to the next rule, called the rule of three *inverſe*, of which hereafter.

In order to know which is the ſecond and third number, it muſt be noted, that of the three numbers which are in every queſtion in this rule, that number which [illegible] the queſtion muſt occupy the third place, and is called the third number; and that number which is of the ſame nature with the fourth number or anſwer, muſt be the ſecond number, and conſequently the other number muſt be the firſt. The ſecond and fourth numbers are therefore always of the ſame nature; as are the firſt and third. Thus, in the foregoing example, the number 9 aſks the queſtion, for the queſtion is, how much will [illegible] yards coſt? 9 is therefore the third number. 12 is of the ſame nature with the fourth number, being money; it muſt therefore poſſeſs the ſecond place: and 3, the other number, muſt be the firſt, which is of the ſame nature with the third, viz. yards.

When either the firſt or third numbers conſiſt of different denominations, they muſt both be reduced to the ſame denomination; and when the ſecond number conſiſts of divers denominations, it muſt alſo be reduced to the loweſt:—this reduction muſt be performed before the work can be wrought; and it muſt be obſerved, that the fourth number or anſwer to the work, is always of the ſame denomination with the ſecond number ſo reduced.

The numbers being ſo reduced, the ſecond and third numbers are to be multiplied together, and the product divided by the firſt, as before directed; and the fourth number or anſwer muſt be brought into the proper denominations required by reduction, and if any thing remain after the products of the ſecond and third numbers are divided by the firſt, ſuch remainder muſt be reduced into the next lower denomination, and then divided by the firſt number, as before; and if any thing ſtill remain, it muſt be reduced into the next lower denomination (if there be any lower), and divided by the firſt number; proceed in this manner till the remainder be brought to the loweſt denomination.

Example 2. If 12 gallons of brandy coſt 4*l.* 10*s.* what will 120 gallons coſt at that rate?

1ſt number. *2d number.* *3d number.*
If 12 gallons coſt 4*l.* 10*s.* what will 120 gallons coſt?
20 90
90 12)10800
2,0) 90,0
Anſ. £45

In this example (the numbers being placed as before directed), the ſecond conſiſting of two denominations, viz. pounds and ſhillings, it muſt be reduced to the loweſt denomination (ſhillings), and the product is 90 ſhillings: the queſtion will then be, if 12 gallons coſt 90*s.* what will 120 coſt? I therefore multiply 120 the third number, by 90 the ſecond number, and divide the product by 12 the firſt number, and the quotient 900 is the fourth number, or anſwer to the queſtion, which, becauſe the ſecond number is reduced to ſhillings, is ſhillings alſo, and is divided by 20 to bring them into pounds, and the quotient is 45 pounds, the true anſwer, or price of 120 gallons at that rate.

The Proof.

There are ſeveral methods of proving queſtions in the rule of three, but the trueſt and moſt improving to the learner is, to back ſtate the queſtion: thus, to prove the laſt example, I ſtate the queſtion backwards, making that number which was the fourth number in the queſtion the firſt number in the proof, and that which was the third number here I make the ſecond, and the ſecond I make the third.

1ſt number. *2d number.* *3d number.*
Proof. If 45*l.* purchaſes 120 gallons, what will 4*l.* 10*s.* purchaſe?
20 90
900 9,00)108,00
12

There

In the proof of this example, I reduce the firſt number 45*l.* into ſhillings, becauſe the third number 4*l.* 10*s.* muſt be reduced into ſhillings, conſiſting of pounds and ſhillings; and then multiplying the ſecond and third numbers together, and dividing by the firſt, the anſwer is 12 gallons, as in the example; it therefore proves the work right.

Example 3. If the income of a perſon be 3 farthings a minute, how much is it per annum?

```
    1ſt number.     2d number.                3d number.
Say, if 1 min. produce 3 farthings, what will 365 days 6 hours
produce?                                       24
                                             1466
                                              730
                                             8766 hours
                                               60
                                           525960 minutes
                                                3
                                        4)1577880 farthings
                                        12) 394470 pence
                                      2,0)  3287,2—6
                                             1643
      Anſwer 1643l. 12s. 6d.                   12
```

Here the 365 days 6 hours are reduced into minutes by multiplying firſt by 24 and then by 60, the product is then multiplied by 3, the ſecond number, and the laſt product is the anſwer in farthings, which is brought into pounds and ſhillings by diviſion, and the anſwer is 1643*l.* 12*s.* 6*d.*

In the foregoing example the firſt number is an unit; when this is the caſe, the work is performed by multiplication, and when the third number is an unit, the work is wrought by diviſion; for 1 neither multiplies nor divides; queſtions of this ſort, therefore, properly belong to reduction.

Example 4. If the effects of a bankrupt amount to 2796*l.* 10*s.* and his debts be 9990*l.* 12*s.* it is required to know how much he can pay in the pound?

Say, if 9990*l.* 12*s.* pay 20*s.* in the pound, what will 2796*l.* 10*s.* pay?

```
    20                       20
  ------                  -------
  199812                   55930
                              20
                         ---------
               199812)1118600(5
                        999060
                        -------
                        119540
                            12
                        -------
               199812)1434480(7
                       1398684
                       -------
                         35796
                             4
                       -------
                        143184
                       =======
```

In ſtating this queſtion, I ſay, if 9990*l.* 12*s.* the whole amount of the bankrupt's debts, will pay 20*s.* in the pound, as it certainly will, what will 2796*l.* 10*s.* the net value of the bankrupt's property, pay? —And the *anſwer* is 5*s.* 7*d.* in the pound.

In performing this example, the firſt and third numbers are reduced into ſhillings, and multiplying the ſecond and third numbers together, and dividing by the firſt, the quotient is 5, which is of the ſame denomination with the ſecond number, viz. ſhillings; and there is a remainder of 119540 ſhillings, which is reduced into pence, and then divided by the firſt number as before, and it quotes 7 pence, and there yet remains 35796 pence, which, reduced into farthings, does not contain the diviſor once; theſe pence, therefore, remain over and above the 5*s.* and 7*d.* in the pound which the bankrupt pays.

The foregoing examples will be found ſufficient to inſtruct the learner in the nature and method of working this rule; I ſhall therefore give a few examples for practice, leaving the operation to be performed by the learner.

Example 5. If 56*lb.* of indigo coſt 11*l.* 4*s.* what will 1008*lb.* coſt at that rate?

Say, if 56*lb.* coſt 224*s.* what will 1008*lb.* coſt?—*Anſwer* 4032*s.* or 201*l.* 12*s.*

Example 6. If a debtor owes his creditors 593*l.* 12*s.* and compounds at 7*s.* 6*d.* in the pound, what will pay his creditors at that rate?

Say, if 20s. can be paid by 90d. what will pay 11872s.? —*Anſwer* 222l. 12s.

Example 7. If 100*l.* gain 6*l.* intereſt in 12 months, how much will 340*l.* gain in the ſame time at that rate of intereſt?

If 100*l.* gain 6*l.* what will 340*l.* gain?—*Anſwer* 20*l.* 8*s.*

Example 8. A draper bought 6 packs of cloth, each pack containing 12 pieces, for which he paid 1080*l.* being 8*s.* 4*d.* per ell Flemiſh: how many yards were there in each piece?

If 100*d.* purchaſes 3 *qrs.* what will 259200*d.* purchaſe?—*Anſwer* 7776 *qrs.* which divided by 72, the number of pieces in the whole, it quotes 108 *qrs.* or 27 yards in each piece.

The general uſe of this rule is, from having the rate, value, proportion, produce, intereſt, gain or loſs of one or any other number of things, to find the rate, value, proportion, produce, intereſt, gain or loſs of one or any other number of the ſame things in a direct proportion.

SECT. VIII.

OF THE SINGLE RULE OF THREE INVERSE.

The rule of three inverſe is that which teaches how from three given numbers to find a fourth, which ſhall bear the ſame rate or proportion to the ſecond number as the firſt does to the third; or the third number bears the ſame proportion to the ſecond as the firſt does to the fourth.

Rule. Multiply the firſt and ſecond numbers together, and divide the product by the third.

The method of placing the numbers in this rule is the ſame as that of the rule of three direct, and therefore need not be repeated.

In order to diſcover whether a queſtion belong to the direct or inverſe rule, it muſt be remembered that the firſt and third numbers are called the extremes; if, therefore, the fourth number be greater than the ſecond, the leſs extreme muſt be the diviſor; but if it be leſs, the greater extreme muſt be the diviſor.

And when the firſt number is the diviſor, the work belongs to the rule of three direct; but when the third number is the diviſor, it belongs to the rule of three inverſe; as in the following examples.

Example 1. If 8 men perform any certain piece of work in 6 days, how many men will perform the ſame work in 3 days?

If 6 days require 8 men, what will 3 days require?
6
3)48(16 *Anſwer* 16 men.

In this example I conſider, that if to do the work in 6 days it requires the labour of 8 men, then, to do the ſame work in 3 days, it will require more men, conſequently the fourth number will be greater than the ſecond, therefore I divide the product of the firſt and ſecond numbers multiplied together by the third number (the leaſt extreme), and the quotient is 16 (men) for the anſwer.

The proof of queſtions in this rule, as well as the foregoing rule, is performed by back ſtating the queſtion; but it muſt be obſerved, in all queſtions in the direct rule, the proof is wrought by the rule of three direct; and in queſtions of the inverſe rule the proof is wrought by the rule of three inverſe.

Thus, to prove the foregoing example, I ſay, if 16 men require 3 days to perform the work, how many days will 8 men require to perform it?

If 16 men require 3 days, what will 8 men require?
3
8)48(6 *Anſwer* 6 days.

Example 2. If a loaf at a certain price weighs 4½lb. when wheat is 6*s.* per buſhel, what ſhould it weigh when wheat is 4*s.* 6*d.* per buſhel?

If 72*d.* per buſhel give 9 half lbs. what will 54*d.* give?

9
54)648(12
54
108
108 *Anſ.* 12 half lbs. or 6lb.

In this example, I multiply 72 pence, the price of the buſhel of wheat, by 9, the number of the half pounds in the loaf, and dividing by 54 pence, the price of the buſhel of wheat at the other price, the anſwer is 12 half pounds or 6 pounds for the weight of the loaf at that price of the wheat.

In London the price of the loaf is varied, and the weight continues the ſame. Queſtions which concern the price are wrought by making the price the ſecond number.

Example 3. If a board be 8 inches broad, how much in length will make a ſquare foot?

If 12 in. in breadth require 12 in. in length, what will 8 in. in breadth require?
12
8)144(18 *Anſ.* 18.

Example 4. How many yards of ſhalloon at 3qrs. wide are ſufficient to line throughout the garments made with 1000 yards of cloth at 7qrs. wide?

If 7qrs. wide require 1000yds. what will 3qrs wide require?
7
3)7000(2333⅓ *Anſ.* 2333⅓ yards.

Qu. 5. If 150*l.* be lent for nine months, how long ſhould 90*l.* be lent to gain the ſame intereſt at the ſame rate?—*Anſwer* 15 months.

Qu. 6. If a colonel be beſieged in a town with 1000 men, having proviſions for only 2 months, how many muſt he diſmiſs, that the proviſion may ſerve the remainder 5 months?—*Anſwer* 600, and retain 400.

Qu. 7. If a perſon perform a journey (travelling at an uniform rate) in 24 days, by travelling 12 hours per day, how long will it take to perform the ſame journey by travelling 16 hours per day at the ſame rate?—*Anſ.* 18 days.

Qu. 8. If a carrier carry 12 cwt. 72 miles for 5*l.* how many miles will he carry 18 cwt. for the ſame money?—*Anſwer* 48 miles.

This rule ſerves to find a fourth number to the given three, in an inverted proportion. And it particularly anſwers ſeven ſorts of queſtions: viz. 1. Having the value of two different ſorts of coin, it ſhews how many pieces of the one are equal in value to a given number of the other. 2. From two different values of one commodity, and the value of an article made from the ſame commodity at one value thereof, to find the weight, meaſure, &c. of the ſame article; or, on the contrary, from the values of the commodity and the weight, meaſure, &c. of the article, to find the value thereof (the 2d example is of this nature). 3. From the breadths of two equal rectangular figures, and the length of one of them, to find the length of the other; or, from the two lengths and one breadth, to find the other breadth (of this nature are the third and fourth examples). 4. From the given weight, expenſe of carriage, and number of miles carriage of any goods, to find the number of miles any other weight could be carried for the ſame price: from a given weight and price, and two diſtances, to find the weight anſwerable to the other diſtance (of this nature is the eighth queſtion). 5. From two ſums of money lent, and the time for which one of them is lent, to find the time for which the other ſhould be lent; or, from the two different times, and the ſum which is lent for one of them, to find the ſum which ſhould be lent for the other time (of this nature is the fifth example). 6. From the quantity of work which a given number of men can perform in a given time, to find the number of men that can perform it in any other given time; or, from the number of men, to find the time any other given number would require (of this nature is the firſt example). 7. From the quantity of proviſions, or money, and the number of men, or other creatures, it would ſerve a certain time, to find the number of men, or other creatures, it would ſerve any other time; or, from the quantity of proviſion and the number of conſumers, to find the time it would ſerve any other number of conſumers (of this nature is the ſixth example).

SECT. IX.

OF THE DOUBLE RULE OF THREE DIRECT[*].

In this rule there are five given numbers to find a ſixth, which ſhall bear the ſame proportion to the product of the fourth and fifth numbers, as the third number bears to the product of the firſt and ſecond.

Queſtions in this rule are reſolved either by two operations in the ſingle rule of three direct, or the *rule of three compoſed of five given numbers.*

Each queſtion in this rule conſiſts of two parts, the *ſuppoſition* and the *demand.*

Rule 1. *By two operations.* Place that number which is of the ſame nature with the ſixth number, or anſwer, in the ſecond place in the firſt operation; and the two other numbers in the ſuppoſition in the firſt place, the one over the other; and the two numbers in the demand in the third place, one over the other, in like manner as the two in the firſt place; obſerving that the bottom numbers in the firſt and third places be of like nature, as will alſo the top ones:

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[*] Some modern writers compound the two double rules of three into one; I have, however, given them diſtinctly, being more conſonant to the true theory of ſcience. In the next Section is, nevertheleſs, ſhewn an infallible method of working both by one rule.

The work is then performed by two operations in the ſingle rule of three direct; the anſwer to the firſt operation forming the ſecond number in the ſecond operation.

2. *By one operation.* Multiply the two numbers which ſtand one over the other in the ſuppoſition, together, for the firſt number; the two numbers in the demand for the third number; and the ſecond number in the firſt queſtion will be alſo the ſecond number in the work. Then the anſwer is found by one operation in the ſingle rule of three direct, as in the following example:

Example 1. If 100*l.* principal gain 5*l.* intereſt in 1 year, what will 140*l.* gain in 9 months?

The Queſtion ſtated.

If 100*l.* gain 5*l.*	What will 140*l.*
in 12 months,	gain in 9 months?

In this example, the numbers 100, 5, and 12, belong to the ſuppoſition, and 140 and 9 is the demand; for the meaning of the work is, *ſuppoſe* 100*l.* gain 5*l.* intereſt in 12 months, (then follows the demand) I *demand* to know how much 140*l.* will gain in 9 months at the ſame rate of intereſt?

Thus the queſtions are ſtated according to the foregoing directions: the 5*l.* being the intereſt of the money (and of the ſame nature with the ſixth number or anſwer) muſt be the ſecond number; and the two other numbers in the ſuppoſition 100 and 12 are placed one above the other, as are the two numbers 140 and 9 in the demand. It matters not which of theſe two numbers is uppermoſt, provided that the numbers in each, which are of the ſame nature, occupy the correſponding places reſpectively: thus, in the ſuppoſition, the pounds principal is the uppermoſt number, ſo it is in the demand, and the number of months is undermoſt in both.

The queſtion being thus ſtated, the work is wrought by two operations of the ſingle rule of three direct. The three uppermoſt numbers are the numbers for the firſt operation, and the fourth number, or anſwer to theſe forms, the ſecond number for the ſecond operation; the bottom number of the ſuppoſition forms the firſt number, and the bottom number of the demand the third number; then the anſwer, or fourth number of this ſecond operation, is the true anſwer to the queſtion; as in the following example, which is the foregoing one at large.

Firſt Operation.

If 100*l.* gain 5*l.* what will 140*l.*
5
1,00)7,00(7

Second Operation.

If 12 months gain 7*l.* what will 9 months?
7
12)63(5
60
3
20
Anſwer 5*l.* 5*s.* 12)60(5

The anſwer would have been the ſame if the number of months had been the uppermoſt numbers inſtead of the pounds principal; in which caſe the firſt queſtion would be, if 12 months give 5*l.* intereſt, what will 9 months give? and the anſwer is 3*l.* 15*s.*; then the ſecond queſtion would be, if 100*l.* gain 3*l.* 15*s.* what will 140*l.* gain? and the anſwer is as before, 5*l.* 5*s.*

By one Operation.

If 100*l.* gain 5*l.* what will 140*l.* gain?
months 12 9 months
1200 1260
5
12,00)63,00
5
remains 300
20
12,00)60,00
5 ſhillings.

In this example, the work is ſtated as in the former; the two firſt numbers in the ſuppoſition are multiplied together; and the two numbers in the demand are alſo multiplied together, then theſe two products are made, the one the firſt, and the other the third number, and the ſecond number in the firſt queſtion is alſo the ſecond number. This rule is the moſt ſure and practical method of proving the double rule of three direct, when wrought by two ſingle ones.

The foregoing example worked both ways will be ſufficient to inſtruct the learner. I ſhall, therefore, give a few queſtions, with their anſwers, omitting the operation.

Qu. 2. Suppoſe 468 men conſume 175 quarters of wheat in 168 days, I demand how many quarters will ſerve 5612 men 58 days?—*Anſ.* 724 quarters, and $\frac{38024}{78624}$ of a quarter, or a little more than half a quarter.

Qu. 3. Suppoſe 80 acres of graſs be mowed by 8 men in 14 days, I demand how many acres 28 men will mow in 12 days?—*Anſ.* 240 acres.

Qu. 4. Suppoſe the wages of 12 men for 6 days amount to 7*l.* 4*s.* what are the wages of 25 men for 40 days?—*Anſ.* 100*l.*

Qu. 5. If 150*l.* principal put out to intereſt for 9 months be increaſed, principal and intereſt, to 156*l.* 15*s.* I demand how much is that per cent. per annum?—*Anſ.* 9*l.*

SECT. X.

OF THE DOUBLE RULE OF THREE INVERSE.

THE double rule of three inverſe is when there are five given numbers to find a ſixth, in an inverted proportion.

Rule. Place the numbers as directed in the laſt ſection. Multiply the lower number of the firſt place by the upper one of the third, and make the product the firſt number; next multiply the upper term of the firſt place by the lower one of the third, for the third number: then if the inverſe proportion be found in the three upper numbers, the anſwer is given by one operation in the rule of three direct; but if the inverſe proportion be found in the lower numbers, the work is performed by the inverſe rule (for of every ſum in this rule one queſtion is direct and the other inverſe).*

Example 1. If 100*l.* gain 5*l.* intereſt in 12 months, what principal will gain 5*l.* 5*s.* in 9 months?

5*l.* 100*l.* 5*l.* 5*s.*
months 12 months 9

The pounds intereſt being reduced to ſhillings, and multiplied by the number of months, the queſtion will ſtand, and operation be performed as follows:

1260 100 900
100
9,00)1260,00
140*l.* for the *anſwer.*

If the number of months had been made the upper terms, the upper proportion would then have been direct, and would have been required to have been worked by the direct method. It would in that caſe ſtand thus:

12 100 9
5 5 5*

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* The rule laid down in this ſection will be found quite general, and ſufficient for working all queſtions in the double rule of three both direct and inverſe; and is ſo obvious, as to require no demonſtration. Nevertheleſs, in conſequence of receiving advice from ſome teachers (not the moſt competent) of the mathematics, that I had not given the *improved method* of working this rule, I ſhall ſtate this much-approved rule, verbatim, from a well-known treatiſe, and ſhew its fallacy.

Qu. 2. If 48*l.* ſerve for the maintenance of 12 men 8 days, how long will 288*l.* ſerve for 4 men?—*Anſ.* 144 days.

Qu. 3. If, when a buſhel of wheat coſts 6*s.* 8*d.* a penny loaf weighs 6 ounces, how much will a loaf weigh that coſts 10½*d.* when the wheat is 10*s.* the buſhel?—*Anſ.* 42 ounces.

"*Rule* 1. Let the principal cauſe of loſs, or gain, intereſt or decreaſe, action or paſſion, be put in the firſt place.

"2. Let that which betokeneth time, diſtance, or place, and the like, be in the ſecond place; and the remaining one in the third.

"3. Place the other two terms under their like in the ſuppoſition.

"4. If the blank falls under the third term, multiply the firſt and ſecond terms for a diviſor, and the other three for a dividend; but,

"5. If the blank falls under the firſt or ſecond term, multiply the third and fourth terms for a diviſor, and the other three for the dividend; and the quotient will be the anſwer.

"*Proof.* By two ſingle rules of three.

"*Example* 1. If 14 horſes eat 56 buſhels of oats in 16 days, how many buſhels will be ſufficient for 20 horſes for 24 days?

By two ſingle rules:

hor. bu. hor. bu.
1. As 14 : 56 :: 20 : 80
da. bu. da. bu.
2. As 16 : 80 :: 24 : 120

Or, in one ſtating, worked thus:

hor. da. bu.
14 : 16 : 56
20 : 24 : —

$$\frac{56\times20\times24}{14\times16}=120."$$

WALKINGAME.

That this rule is not founded on mathematical principles is evident from inſpection; for by a different ſtatement of the queſtion (though exactly agreeable to the rules) a different anſwer will ariſe. Thus, how eaſily might the learner, required to work this queſtion by one ſtatement, order the numbers as follows:

hor. bu. da.
14 : 56 : 16
20 : — : 24

$$\frac{56\times24\times14}{16\times20}=58\tfrac{4}{5}$$

According to this ſtatement the anſwer will be 58⅘ buſhels; whereas, the true anſwer is 120 buſhels. The queſtion is, nevertheleſs, ſtated, in this laſt caſe agreeable to the rules.

Qu. 4. If 96 pioneers in 24 days caſt a trench 96 yards long, how many pioneers will caſt a trench 336 yards long in 8 days?—*Anſ.* 1008 pioneers.

Qu. 5. If 10 men mow 40 acres in 8 days, how many days will it require 3 men to mow 150 acres?—*Anſ.* 100 days.

This rule may be proved by two ſingle rules.

The following rules in this chapter, except vulgar fractions and the extractions of roots, are wrought either by ſome of the rules of proportion, delivered in the four preceding ſections, or by the rule of practice, hereafter to be ſpoken of.

SECT. XI.

OF FELLOWSHIP.

FELLOWSHIP is that rule whereby merchants and others, trading in company, and employing a joint capital ſtock, are enabled to aſcertain each partner's particular loſs or gain, according to his ſhare in the ſame joint ſtock.

This rule alſo ſerves to divide a bankrupt's eſtate among his creditors; to pay legacies, when there is a deficiency of the teſtator's effects: and, in fine, to divide the loſs or profit of any joint concern among the loſers or proprietors.

The rule of fellowſhip is either ſingle or double, that is, without regard to time, or with time.

Fellowſhip without time, or ſingle fellowſhip, is when different perſons employ their reſpective ſtocks for the ſame time.

Rule. As the whole ſtock of all the partners is to the whole gain or loſs, ſo is each man's particular ſtock to his particular ſhare of the gain or loſs.

Proof. By adding each perſon's gain or loſs together.

Therefore, the whole ſtock of all the partners is to be made the firſt number in the rule of three, the whole gain or loſs the ſecond number, and the particular ſtock of any one partner the third number; then the fourth number, or anſwer, is that partner's loſs or gain, whoſe ſtock was the third number. This operation in the rule of three muſt be repeated if there be more than two partners, and performed as often as there are partners concerned.

Example 1. Three perſons enter into trade together: A put into the trade 100*l.* B 170*l.* and C 300*l.*; at the making up their accompts, they find they have gained 550*l.* profit: what is each perſon's ſhare?

```
 A's ſtock £.100
 B's ſtock   170
 C's ſtock   300
            ----
Whole ſtock £.570
            ====

Whole Stock.      Whole Gain.        A's Stock.
 570l.             550l.              100l.
                   100  £.  s.  d.
                  -----
         57,0)5500,0(96  9  9¾ 330/570
              513
              ----
               370
               342
               ---
                280
                 20
               -----
         57,0)560,0(9s.
              513
              ---
               470
                12
              -----
         57,0)564,0(9d.
              513
              ---
               510
                 4
             -----
         57,0)204,0(3 far.
              171
              ---
               30
              ===
```

B's Stock.

570*l.* 550*l.* 170*l.*
170
38500
550 £. *s.* *d.*
57,0)9350,0(164 0 8$\frac{1}{4}$ $\frac{390}{570}$
57
365
342
230
228
20
20
57,0) 400(0*s.*
12
57,0)480,0(8*d.*
456
240
4
57,0)96,0(1 *far.*
57
390

C's Stock.

570*l.* 550*l.* 300*l.*
300 £. *s.* *d.*
57,0)16500,0(289 9 5$\frac{1}{2}$ $\frac{420}{570}$
114
510
456
540
513
270
20
57,0)540,0(9*s.*
513
270
12
57,0)324,0(5*d.*
285
390
4
57,0)156,0(2 *far.*
114
420

The Proof.

	£.	s.	d.
A's ſhare	96	9	$9\frac{3}{4}\ \frac{330}{570}$
B's ſhare	164	0	$8\frac{1}{4}\ \frac{390}{570}$
C's ſhare	289	9	$5\frac{1}{2}\ \frac{420}{570}$
Whole ſtock £.	550	0	0

If there be any remainders, ſuch remainders in the proof muſt be added together (if they be fractions of the ſame denomination), and divided by the common diviſor of each queſtion (*i. e.* the total ſtock); and the quotient, which is units of the ſame denomination, added to the particular ſhares: thus, in the foregoing example, I added the three remainders, 330, 390, and 420, together, and dividing the total by 570, the common diviſor, the quotient is 2, which are farthings, as they are fractions of a farthing.

Qu. 2. Four perſons place their money in the public funds: A put in 360*l.* B 480*l.* C 700*l.* and D 860*l.* When the capital came to be ſold out, the principal and intereſt amounted to 4200*l.* what is each man's ſhare of the net profit?—*Anſwer*, A 270*l.* B 360*l.* C 525*l.* D 645*l.*

Qu. 3. A bankrupt is indebted to four creditors in the following ſums: to A 551*l.* 8*s.* to B 608*l.* 14*s.* to C 304*l.* and to D 208*l.* 12*s.* The bankrupt's eſtate is worth only 337*l.* 17*s.* how much will each creditor receive?—*Anſ.* A 111*l.* 7*s.* 5*d.* B 122*l.* 18*s.* 10½*d.* C 61*l.* 8*s.* 0¼*d.* and D 42*l.* 2*s.* 7¾*d.*

Qu. 4. A ſhip being loſt value 1730*l.*; of which A paid 346*l.* B 519*l.* C 692*l.* and D 173*l.* towards building her; ſhe was inſured to the amount of 1360*l.* what was each perſon's loſs?—*Anſwer*, A 74*l.* B 111*l.* C 148*l.* D 37*l.*

In the ſecond example, the whole ſtock firſt placed in the funds is to be found by adding each perſon's ſtock together, and that made the firſt number in the rule of three; the whole gain is to be the ſecond number, the particular ſtock of any one perſon the third number, and the fourth number is his ſhare.

In the third example, the whole amount of the bankrupt's debt is to be made the firſt number; the value of his effects the ſecond number; and any creditor's real due the third number, and the fourth number is that creditor's ſhare.

Fellowſhip with time, or double fellowſhip, is when the different ſtocks are employed for different times.

Rule. Multiply each man's particular ſtock by the time it is employed for; then ſay, by the rule of three, as the total of all theſe products is to the whole gain or loſs, ſo is each man's particular product to his particular ſhare of the gain or loſs.

Thus the rule of three muſt be repeated as often as there are products or perſons concerned, as in the ſingle rule.

Example 1. A put 200*l.* in a certain trade; B five months after put 300*l.*; at the end of one year they find they have gained 350*l.* profit, what is each perſon's ſhare?

Total Product.	Whole Gain.	A's Product.
£.	£.	£.
4500	350	2400

200
12 months.
2400 A's product

300
7
2100 B's product.
4500 Total of the products.

350
2400
140000
70 A's *Share.*
45,00)8400,00(186*l.* 13*s.* 4*d.*
45
390
360
300
270
3000
20
45,00)600,00(13*s.*
45
150
135
1500
12
45,00)180,00(4*d.*
180
0

B's Product.

£	£	£
4500	350	2100

2100
35000
700
B's Share.
45,00)7,350,00(163*l.* 6*s.* 8*d.*
45
285
270
150
135
1500
20
45,00)300,00(6*s.*
270
3000
12
45,00)360,00(8*d.*
360
0

The Proof.

A's ſhare	£.186	13	4
B's ſhare	163	6	8
	£.350	0	0

The method of proof in this rule is the ſame as in the ſingle rule, as may be ſeen above.

Qu. 2. A, B, and C rent a paſture in common, at 40*l.* per annum; A puts in it 3 oxen for 8 months in the year, B puts in 7 for 4 months, and C puts in 2 for the whole year; what muſt each perſon pay towards the rent? *Anſwer*, A 12*l.* 12*s.* 7½*d.* $\frac{24}{76}$; B 14*l.* 14*s.* 8½*d.* $\frac{28}{76}$; C 12*l.* 12*s.* 7½*d.* $\frac{24}{76}$.

Example 3. A, B, and C make a ſtock for 12 months, A put in at firſt 728*l.* at the end of 2 months he put in 20*l.* more; B put in at firſt 200*l.* and at the end of 5 months he took out 150*l.*; C put in at firſt 40*l.* and at the end of 6 months 50*l.* more, and at the end of 10 months 10*l.* more; at the year's end they find they have gained 500*l.*; what is each perſon's profit?

In theſe examples, and all others of the like nature, where the different parts of the ſtock of each perſon are employed for a different time, the different parts of each perſon's ſtock are to be multiplied by their own ſeparate times, and theſe products added together to make up the product of each perſon; and then each perſon's product added together to make the total product.

	£.
Thus, in the laſt example, I firſt multiply the 728*l.* put in by A, by 2, the number of months it is employed alone, and the product is —	1456
Then A put in 20*l.* more, which, added to the 728*l.* is 748*l.* and which multiplied by 10 months, the product is — — —	7480
And which two added together gives A's product	8936
B put in 200*l.* for 5 months, which product is —	1000
At the end of 5 months he took out 150*l.* and then there remained only 50*l.* which multiplied by 7 months, the product is — — —	350
The two products of B added together are his true one	1350
C put in at firſt 40*l.* which continued alone 6 months, that product therefore is — —	240
At the end of 6 months he put in 50*l.* more, which, added to the former 40*l.* is 90*l.* which multiplied by 4 months, gives — — —	360
The laſt 2 months he had 100*l.* in the ſtock, which, multiplied by 2 months, gives — —	200
Therefore the product of C is —	800
The total ſum of all the products	11086

Then the work is wrought as before; ſaying, as 11086 (the ſum of the products) is to 500*l.* (the total gain), ſo is 8936 (the product of A) to 403*l.* 0*s.* 7¼*d.* $\frac{6826}{11086}$, the ſhare of A of the profits; and repeat the operation twice more, to find the ſhares of B and C.

Anſwer.

	£.	s.	d.	
The ſhare of A	403	0	7¼	6826/11086
B	60	17	9	1128/11086
C	36	1	7½	3132/11086
	500	0	0	

SECT. XII.

OF ALLIGATION, MEDIAL AND ALTERNATE.

THE rule of alligation teacheth how to mix different ſubſtances together, and to diſcover the value of any part of ſuch mixture; or to make a mixture from known ſubſtances of any value.

Alligation is either *medial* or *alternate*.

Alligation medial is that which teacheth how to find the rate or price of any mixture or compound, from having the rates or prices, and the quantities of the ſeveral ſubſtances given.

Rule. Multiply each quantity by its rate or price, and add the products together for a dividend; add the ſums of the ſeveral quantities together for a diviſor; and divide the ſum of the products by the ſum of the quantities, and the quotient will be the rate or price of the compound.

Example 1. A mealman mixes 20 buſhels of flour, worth 5*s.* per buſhel, with 12 buſhels worth 3*s.* 4*d.* per buſhel; what is 1 buſhel of this mixture worth?

Price

Price of a buſhel 60*d.*	Price of a buſhel 40*d.*	Quantities.
No. of buſhels 20	No. of buſhels 12	12
1200	480	20
	1200	32

32)1680(52½*d.* or 4*s.* 4½*d.* per buſhel,
for the *Anſwer.*
160
80
64
16
4 *far.*
32)64(2
64
0

In this example the price of the buſhel is reduced into pence (as it moſtly ſhould). The anſwer is the price of the quantity, which is of the ſame denomination with the diviſor, which here is buſhels.

Proof. Find the value of the whole mixture from the value of any part, and if it be equal to the value of the original ſimples, the work is right.

Thus, to prove the foregoing example, I multiply the price of one buſhel of the mixture by 4 and 8, or 32, the number of buſhels in the whole, and the product, I find, is 7*l.*; then I find the value of the ſeveral ſimples, by multiplying the number of buſhels in each by the number of pence in a buſhel (which is already done in the example), and the product 1680 brought into pounds, gives 7*l.* as follows:

The Proof.

The Number of Buſhels multiplied by the Pence in a Buſhel.

12)1680 pence
2,0)14,0 ſhillings
7 pounds

The Price of a Buſhel of the Mixture multiplied by the Number of Buſhels.

s.	*d.*
4	4½
	4
17	6
	8
£7 0	0

Qu. 2. A grocer mixed the following teas together, viz. 15lbs. at 8*s.* per lb. 20lbs at 7*s.* 4*d.* per lb. 10lbs. at 6*s.* 8*d.* per lb. and 24lbs. at 4*s.* per lb. what is one pound of this mixture worth?—*Anſwer* 6*s.* 2½*d.* $\frac{46}{69}$.

Qu. 3. A vintner mixes 5 gallons of wine at 7*s.* per gallon, with 9 gallons at 8*s.* 6*d.* per gallon, and 14½ gallons at 5*s.* 10*d.* per gallon; what is one gallon of this mixture worth?—*Anſwer* 6*s.* 10¾*d.* $\frac{14}{57}$.

Qu. 4. A goldſmith melts 101½ ounces of gold bullion, of 14 carats * fine, with 152½ ounces of 18 carats fine, how many carats fine is this mixture?—*Anſwer* 16$\frac{204}{508}$ carats fine.

Alligation alternate is the method of finding what quantity of ſimples, whoſe rates or prices are given, will form a mixture of a certain given rate or price.

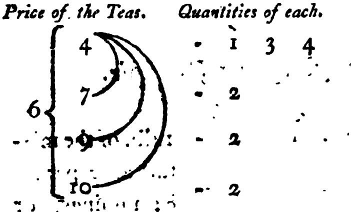
Rule. 1. Write the rates or prices of the ſeveral ſimples under each other. 2. Connect with a curve line the rate or price of each ſimple that is leſs than the rate or price of the mixture, with one or more of theſe rates or prices that are greater than that of the mixture; and each greater rate with one, or more that are leſs: and place the rate or price of the mixture on the left hand of the rates or prices. 3. Write the difference between the rate of the mixture and the rate of each ſimple oppoſite the rate with which ſuch ſimple is connected or linked.

Then theſe differences which ſtand oppoſite any rate is the quantity which that rate requires to form a mixture of the given rate; but if there is more than one difference oppoſite any ſimple, their ſum is the true difference.

---

* Gold is generally mixed with copper or ſome other baſe metal, which is called the allay; and the gold is ſaid to be ſo many carats fine as it contains pure gold: thus, if an article weighs 24 carats, and contains 22 carats of gold, and 2 of allay, it is ſaid to be 22 carats fine. What a carat is may be ſeen, page 135.

Example 1. A grocer would mix teas at 4*s.* per lb. 7*s.* per lb. 9*s.* per lb. and 10*s.* per lb. in such proportion that the mixture may be worth 6*s.* per lb.; what quantity of each must be taken?



The Anſwer.

8lb. at 4*s.* per lb.
2lb. at 7*s.* per lb.
2lb. at 9*s.* per lb.
2lb. at 10*s.* per lb.
14lb. at 6*s.* per lb.

In this example, I firſt ſtate the work as before directed, placing the prices of the teas in a column over each other, with 6, the given price of the mixture, on the left hand.

Secondly, I connect the prices with each other by curve lines; 4 the top figure, being leſs than the rate of the mixture, I connect with 7, 9, and 10, becauſe they are all greater than 6, the rate of the mixture.

Thirdly, I find the difference between 6, the price of the mixture, and 4, that of the firſt ſimple, which is 2; I therefore place 2 oppoſite the 7, 9, and 10, as the 4 is linked to all of them. Then I find the difference between the 6 and the next figure 7, which is 1, I therefore place 1 oppoſite the 4, being the figure to which the 7 is linked. Then the difference between the 6 and 9 is 3, which I place alſo oppoſite the 4 (the 9 being linked thereto), and the difference between 6 and 10, which is 4, I alſo place oppoſite the 4 (as the 10 is alſo linked to it). Theſe differences, ſo placed, contain the true proportion of each ſort of tea at the price oppoſite to each, that ſhould be taken to form a mixture at the deſired rate.

But oppoſite the 4 there are three differences, viz. 1, 3, and 4, which are to be added together, as ſeen in the laſt column. Thus, there muſt be 8lb. of tea at 4*s.* per lb. 2lb. at 7*s.* per lb. 2lb. at 9*s.* per lb. and 2lb. at 10*s.* per lb.; and the whole quantity of the mixture is 14lb. at 6*s.* per lb.

Theſe queſtions are proved in the ſame manner as thoſe in alligation medial, viz. by finding the total value of all the ſimples in their ſeparate ſtate, and the total value of the mixture; and if theſe two values be equal, the work is right.

The Proof of the foregoing Example.

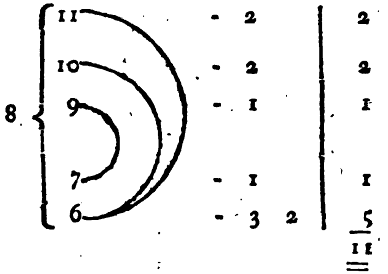
	£.	s.	d.	
8lb. of tea, at 4s. per lb. is	1	12	0	14lb. of the mixture at 6s. per lb.
4				
32 Shillings				84 ſhillings, or 4l. 4s.
2lb. at 7s. per lb. is	0	14	0	
7				
14 Shillings				
2lb. at 9s. per lb. is	0	18	0	
9				
18 Shillings				
2lb. at 10s. per lb. is	1	0	0	
10 Total	4	4	0	
20 Shillings				

Qu. 2. A farmer mixed wheat at 4s. the buſhel, with rye at 3s. the buſhel, and barley at 18d. the buſhel, how much muſt he mix of each to ſell the whole mixture at 22d. the buſhel?—*Anſwer*, 40 buſhels at 18d. per buſhel, 4 buſhels at 3s. per buſhel, and 4 buſhels at 4s. per buſhel.

Qu. 3. A goldſmith has gold to melt of 24 carats fine, 21 carats fine, 19 carats fine, and 16 carats fine; how much of each muſt he take to form an article of gold that ſhall make 17 carats fine?—*Anſwer*, 1 of 24, 1 of 21, 1 of 19, and 13 of 16 carats fine.

When the whole mixture is limited to a certain quantity, after finding the quantity of each of the ſimples as before, ſay (by the rule of three), as the ſum of the quantities is to the given quantity, ſo is the quantity of each ſimple to the required quantity of each.

Example 4. A vintner is deſirous to mix 5 ſorts of wine together: viz. at 11*s.* per gallon, 10*s.* per gallon, 9*s.* per gallon, 7*s.* per gallon, and 6*s.* per gallon, in ſuch proportion as to make 40 gallons of wine, worth 8*s.* per gallon: how much of each ſort muſt he take?



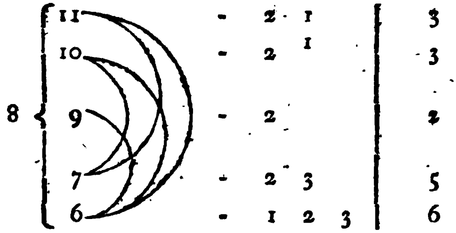
In this example, after linking the prices together, as before directed, I have the quantity of each wine to form a mixture of 8*s.* per gallon; but the whole quantity of the mixture thus found is only 11 gallons, whereas it ſhould be 40 gallons, therefore I ſay,

	Gall.		*Gall.*		*Gall.*		*Gall.*	*Pints.*
As	11	is to	2	ſo is	40	the quantity required to	7	$2\frac{2}{11}$
	11		2		40		7	$2\frac{2}{11}$
	11		1		40		3	$5\frac{1}{11}$
	11		1		40		3	$5\frac{1}{11}$
	11		5		40		18	$1\frac{5}{11}$
						The quantity required	40	0

Queſtion 5. A grocer has ſugar at 10*d.* 8*d.* 6*d.* and 4*d.* per lb. of which he would make a mixture to conſiſt of 60lb. and worth 5*d.* per lb.; how much of each ſort muſt he take?—*Anſwer*, 5lb. at 10*d.* 5lb. at 8*d.* 5lb. at 6*d.* and 45lb. at 4*d.* per lb.

Sometimes it is required to take a certain quantity of any one ſimple to mix with the others, and which generally alters the quantities of the other ſimples. To find what proportion of the others is requiſite, I ſay (by the rule of three), as the quantity of that ſimple whoſe particular quantity is given is to the given quantity, ſo is the found quantity of any other ſimple to the quantity required.

Example 6. A grocer would mix raiſins at 11*d.* per lb. 10*d.* per lb. 9*d.* per lb. and 6*d.* per lb. with 120lb. at 7*d.* per lb.; how much of each ſort muſt he take, that the whole may be worth 8*d.* per lb.?



Hence the ſeveral quantities requiſite are as placed in the example; but to find the quantities which ſhould be taken of each ſort, I ſay (by the rule of three), as 5lb. (the quantity there found) is to 120lb. (the quantity required to be taken), ſo is 3lb. (the quantity at 11*d.* per lb.) to 72lb. (the quantity that ſhould be taken).

	lb.		*lb.*		*lb.*		*lb.*
As	5	is to	120	ſo is	3	to the quantity required	72
	5		120		3		72
	5		120		2		48
	5		120		6		144
						Which, with the	120
						at 7*d.* per lb. is	456

Queſtion 7. A vintner mixes wine at 12*s.* 10*s.* and 6*s.* per gallon, with 20 gallons at 4*s.* per gallon; how much of each ſort muſt he take to make the mixture worth 8*s.* per gallon?—*Anſwer*, 20 gallons at 12*s.* 10 gallons at 10*s.* and 10 gallons at 6*s.*

From the foregoing examples it is evident that there are ſeveral ways of working this rule, according to the method of ſtating the queſtion; as may be partly ſeen in the fourth and ſixth examples, which, though quite different queſtions from each other, yet conſiſt of the very ſame figures, and may be ſtated in the ſame manner; but are here varied for the information of the learner, and admit of ſtill greater variety, as the learner may prove at his leiſure.

All the caution that is neceſſary in linking the numbers together, is, that of every two numbers that are linked together, one muſt be greater and the other leſs than the rate or price of the mixture. Therefore, the firſt example in this rule, having but one number leſs than the rate of the mixture, admits of no other method of ſtating than that deſcribed.

SECT. XIII.

OF VULGAR FRACTIONS.

Reduction, Subtraction, Multiplication, Diviſion, and the Rule of Three.

A FRACTION is any part or parts of an integer or unit, and (in vulgar fractions) is repreſented by two numbers placed one above the other, with a line drawn between them.

The number below the line is called the *denominator*, and ſhews how many parts the integer is divided into; the number above the line is called the *numerator*, and ſhews how many of thoſe parts the fraction repreſents.

Thus the fraction to repreſent *three farthings* is thus written $\frac{3}{4}\begin{smallmatrix}\text{Numerator}\\\text{Denominator}\end{smallmatrix}$ and is properly called three-fourths of a penny;—a penny being the integer, or unit, of which the fraction is a part.

Vulgar fractions are either proper or improper, ſingle or compound.

A *proper* fraction has its numerator leſs than its denominator: as, $\frac{2}{6}$ two ſixths, $\frac{3}{4}$ three fourths, $\frac{3}{9}$ three ninths; and always ſtands for leſs than the integer it is taken from.

An *improper* fraction has the numerator equal to or greater than the denominator, as $\frac{5}{5}$, $\frac{8}{4}$, $\frac{21}{12}$, and always repreſents as much or more than the integer.

A *ſingle* fraction is only a ſingle expreſſion of any aſſigned parts of an integer.

A *compound* fraction conſiſts of more than one fraction, and is a fraction of a fraction, and they have the particle *of* placed between them, as $\frac{1}{2}$ of $\frac{3}{4}$, $\frac{2}{7}$ of $\frac{3}{7}$, &c.

There are alſo mixed numbers, which are whole numbers united with fractions, as $8\frac{3}{4}$, $12\frac{17}{30}$, &c.

The *common meaſure* of two or more numbers, is that number which will divide each of them without a remainder; thus 4 is the common meaſure of 12 and 16.

A number that can be exactly meaſured by two or more numbers is called their *common multiple*; and if it be the leaſt number that can be meaſured, it is called their *leaſt* common multiple: thus, 45 and 60 are the common multiples of both 3 and 5, but their leaſt common multiple is 15.

Before the learner can proceed to reduction of fractions, it is neceſſary that he be able to ſolve the two following Problems.

PROBLEM I.

To find the greateſt common Meaſure of two or more Numbers.

Rule. If there be only two numbers, divide the greater by the leſs: and if there be no remainder, the diviſor is the greateſt common meaſure; but if there be a remainder, the diviſor is to be divided by ſuch remainder, and if there ſtill be a remainder, the laſt diviſor is ſtill to be divided by the laſt remainder till there be no remainder; then the laſt diviſor is the greateſt common meaſure.

But if there be more than two numbers after having found the greateſt common meaſure of two of them, the common meaſure of that common meaſure and one of the other numbers is to be found in the ſame manner; and proceed in this manner through all the numbers; then the laſt common meaſure will be the common meaſure of each of them.

Example 1. What is the greateſt common meaſure of 624, 3126, and 336?

```
624)3126(5              6)336(56
    3120                  30
    ─────                 ──
       6)624(104           36
         6                 36
         ─                 ══
         24
         24
         ══
```

Thus 6 is the greater common meaſure of 624, 3126, and 336.

2. What is the greateſt common meaſure of 81 and 63? —*Anſwer* 9.

3. What is the greateſt common meaſure of 720, 336, and 1736?—*Anſwer* 8.

PROBLEM II.

To find the leaſt common Multiple of two or more Numbers.

Rule. Divide the numbers by any number that will divide two or more of them without a remainder, and ſet the quotient of each number under the dividend to which it belongs; bring down the undivided numbers, or thoſe which cannot be divided, into the ſame line with the quotient. Then divide the ſecond line, in the ſame manner, by any number that will divide two or more of them, and bring down the quotients and undivided numbers, as before. Proceed in this manner as long as there can be found any two numbers that can be exactly divided by one number.

When the numbers are ſo far divided that there are no two numbers that can be divided by one, then multiply all the diviſors and the quotients continually together; and the product is the leaſt common multiple.

Example 1. What is the leaſt common multiple of 7, 8, 4, and 2?

2)7 ,, 8 ,, 4 ,, 2
2)7 ,, 4 ,, 2 ,, 1
7 ,, 2 ,, 1 ,, 1

$$\begin{array}{r} 2 \\ 2 \\ \hline 4 \\ 7 \\ \hline 28 \\ 2 \\ \hline 56 \end{array}$$

Here I divide the numbers firſt by 2, as that number exactly meaſures the 8, 4, and 2; the quotients I place under their dividends; I then divide the next line by 2, as that meaſures 4 and 2; the quotient I place below, and the 7 I bring down.

Then, as there are no two others which admit of a common meaſure, I multiply the diviſors and quotients continually together, and the product 56 is the leaſt common multiple of them all.

Before fractions can be wrought by any of the rules in arithmetic, it is neceſſary to reduce them from compound to ſimple ones, and to bring them into ſeveral other forms; for which purpoſe there are eight ways of altering the form of fractions, without changing their value, as follow:

Caſe 1. To reduce a mixed number to an improper fraction.

Rule. Multiply the integer by the denominator of the fraction, and add thereto the numerator, and the product will form a numerator to a fraction, whoſe denominator is the denominator of the former fraction.

Example 1. Reduce 10$\frac{3}{4}$ gallons to an improper fraction.

$$\begin{array}{r} 10 \\ \underline{4} \\ 40 \\ \underline{3} \\ \underline{\underline{43}} \end{array} \qquad \frac{43}{4} \text{ the fractions required.}$$

2. Reduce 56$\frac{13}{22}$ to an improper fraction.—*Anſwer* $\frac{1245}{22}$.

Caſe 2. To reduce an improper fraction to a whole or mixed number.

This is the reverſe of the former caſe.

Rule. Divide the numerator by the denominator, and the quotient will be the whole number, and the remainder (if any) will be the numerator to a fraction, whoſe denominator is the diviſor.

Thus, to reverſe the firſt example in the former caſe,

Example 1. Reduce 10$\frac{3}{4}$ to its equivalent, whole, or mixed number.

$$\begin{array}{l} 4)43(10 \\ \;\;\underline{4} \\ \;\;\;\underline{\underline{3}} \end{array} \qquad 10\tfrac{3}{4} \text{ the mixed number.}$$

2. Reduce the ſecond example in the former caſe $\frac{1245}{22}$ to its equivalent, whole, or mixed number.—*Anſwer* 56$\frac{13}{22}$.

Caſe 3. To reduce a fraction to its loweſt terms.

Rule. Divide the two terms of a fraction by any number that will exactly divide them without a remainder, and then divide the quotients by a number that will exactly divide them, and ſo on till there can be found no number that will exactly divide the laſt quotients; then theſe quotients will be the fraction reduced to its loweſt terms.

Or, 2dly, find the greateſt common meaſure of the numerator and denominator, as taught in the firſt problem in this ſection, and divide them thereby; then the quotients will be the fraction reduced to its leaſt terms.

Example 1. Reduce $\frac{488}{864}$ to its leaſt terms.

4) $\frac{488}{864}$ | $\frac{122}{216}$ | $\frac{61}{108}$ the *Anſwer*.

Or thus,

```
488)864(1
    488
    ---
    376)488(1
        376
        ---
        112)376(3
            336
            ---
             40)112(2
                 80
                 --
                 32)40(1
                    32
                    --
                     8)32(4
                       32
                       --
                        0
                       ==
```

Therefore

```
8)488(61  | 8)864(108
  48      |   8
  --      |   --
    8     |    64
    8     |    64
    -     |    --
    0     |     0
    =     |    ==
```

Thus, 8 is the greateſt common meaſure of 488 and 864, and theſe, divided by 8, quote 61 and 108 as before *.

2. Reduce $\frac{192}{576}$ to its leaſt terms.—*Anſwer* $\frac{1}{3}$.

3. Reduce $\frac{5184}{6912}$ to its leaſt terms.—*Anſwer* $\frac{144}{167}$.

Caſe 4. To reduce a whole number to an improper fraction.

Rule. Multiply the whole number by the intended denominator, and the product placed over the denominator will form the fraction required.

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* There are ſeveral brief methods of reducing a fraction to its leaſt terms; a few of which I ſhall mention.—1. Any number that ends with 5 or 0 is diviſible by 5: as 25, 55, 105, 40, 100, &c.—2. A number that ends with an even number, or a cypher, is diviſible by 2: as 28, 50, 64, 100, &c.—3. If the two right-hand figures of a number be diviſible by 4, the whole is diviſible by 4: as 312, 4728, &c.—4. If the three right-hand figures be diviſible by 8, the whole is diviſible by 8: as 21104, &c.—5. If the terms of a fraction have a cypher or cyphers on the right-hand, as many cyphers may be cut off from the one as from the other.

Example 1. Reduce 12 to a fraction, whoſe denominator ſhall be 7.

12
7
84 $\frac{84}{7}$ the fraction required.

2. Reduce 135 to an improper fraction, whoſe denominator ſhall be 16.—*Anſwer* $\frac{2160}{16}$.

3. Reduce 1206 to a fraction, whoſe denominator ſhall be 20.—*Anſ.* $\frac{24120}{20}$.

Caſe 5. To find the value of a fraction in the known parts of money, weights, or meaſures.

Rule. Multiply the numerator by the parts of the next inferior denomination, and divide the product by the denominator. If then any thing remain, multiply it as before by the parts of the next inferior denomination, and divide by the denominator; and ſo on till it be brought to the loweſt denomination, and if any thing then remain, it muſt be ſet over the denominator for a fraction of the loweſt denomination.

Example 1. What is the value of $\frac{7}{11}$ of a pound ſterling?

7
multiply by 20 ſhillings
11)140(12 ſhillings
11
30
22
8 remains
12 pence
11)96(8
88
8 remains
multiplied by 4 farthings
11)32(2 farthings
22
10 of a farthing
11 remains.

2. What is the value of $\frac{4}{9}$ of a guinea?—*Anſ.* 9*s.* 4*d.*

3. What is the value of $\frac{3}{1}$ of half a crown?—*Anſ.* 9*d.*

4. What is the value of $\frac{36}{131}$ of a lb. troy weight?—*Anſ.* 3oz. 5pwt. 22$\frac{118}{131}$gr.

5. What is the value of $\frac{5}{8}$ of an acre?—*Anſ.* 2 roods 20 poles.

Caſe 6. To reduce a compound fraction to a ſimple one of the ſame value.

Rule. Multiply all the numerators together for a new numerator, and all the denominators together for a new denominator; and that will be the fraction required.

Example 1. Reduce $\frac{2}{7}$ of $\frac{5}{8}$ of $\frac{3}{4}$ of a pound ſterling to a ſimple fraction.

$$\begin{array}{rr} & 7 \quad 2 \\ & \underline{8} \quad \underline{5} \\ & 56 \quad 10 \\ & \underline{4} \quad \underline{3} \\ \text{denom.} & \underline{\underline{224}} \quad \underline{\underline{30}} \text{ numerator} \end{array}$$

Anſwer $\frac{30}{224}$ or $\frac{15}{112}$.

Thus $\frac{30}{224}$ is the ſimple fraction, equal in value to the compound one in the example; as the learner may prove by finding the value of each part of the compound fraction, by the foregoing caſe, and the value of this ſimple fraction; and the two values will be found to be equal.

2. Reduce $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{8}$ to a ſimple fraction.—*Anſwer* $\frac{30}{120}$, or $\frac{1}{4}$.

3. Reduce $\frac{2}{9}$ of $\frac{2}{5}$ of $\frac{7}{10}$ of $\frac{6}{4}$ to a ſimple fraction.—*Anſ.* $\frac{168}{1800}$ or $\frac{21}{225}$.

If any part of the compound fraction be a whole or mixed number, it muſt be reduced to a fraction by one of the foregoing caſes, before the compound fraction can be reduced to a ſimple one.

Caſe 7. To reduce fractions having different denominators to fractions of the ſame value having a common denominator.

Rule. Multiply each numerator into all the denominators, except its own; and the product is the numerator of that fraction; and multiply all the denominators continually together, and the product is the common denominator.

Example 1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{4}{7}$ to fractions of the ſame value, and having the ſame denominators.

$$\begin{array}{llll}
 & & & 4 \\
 & & 2 & \underline{3} \\
 & 1 & \underline{2} & 12 \\
2 & \underline{3} & 4 & \underline{2} \\
\underline{3} & 3 & \underline{7} & 24 \text{ the numerator for } \frac{4}{7}. \\
6 & \underline{7} & 28 \text{ the numerator for } \frac{2}{3}. & \\
\underline{7} & 21 \text{ the numerator for } \frac{1}{2}. & & \\
42 \text{ common denominator.} & & &
\end{array}$$

Therefore the new fractions equal to the former ones are, $\frac{21}{42}$ equal to $\frac{1}{2}$, $\frac{28}{42}$ equal to $\frac{2}{3}$, $\frac{24}{42}$ equal to $\frac{4}{7}$.

2. Reduce $\frac{1}{30}$, $\frac{3}{4}$ of $\frac{1}{5}$, $5\frac{1}{2}$, and $\frac{8}{19}$ to a common denominator. *Anſ.* $\frac{760}{2280}$, $\frac{342}{2280}$, $\frac{12540}{2280}$, $\frac{960}{2280}$.

In this caſe, as well as the former, if any of the parts be a whole or mixed number, or a compound fraction, they muſt be reduced to ſimple fractions, as in the laſt example.

Caſe 8. To reduce a fraction of one denomination to that of another, without changing its value.

This is either aſcending or deſcending; aſcending is when a fraction of a leſs denomination is brought into a fraction of a greater; and deſcending, when a fraction is brought from a greater to a leſs denomination.

Rule. When a fraction is to be brought from a leſs to a greater denomination, bring the fraction from a ſimple to a compound one, by comparing it with the intermediate denominations, which are between the given fraction and the denomination required; and then, by the rule in Caſe 6, bring the compound fraction to a ſimple one.

Example 1. Bring $\frac{2}{5}$ of a penny to the fraction of a pound.

A penny is $\frac{1}{12}$ of a ſhilling, and a ſhilling is $\frac{1}{20}$ of a pound, therefore $\frac{2}{5}$ of a penny is $\frac{2}{5}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound, which, reduced to a ſimple fraction, is $\frac{1}{600}$ of a pound, as follows:

Numerators.	Denominators.	
2	20	
1	12	
2	240	
1	5	
2	1200	$\frac{2}{1200}$ or $\frac{1}{600}$ of a pound.

Rule. When a fraction is to be brought from a greater to a leſs denomination, multiply the numerator of the fraction by the number of parts of the next inferior denomination, and that product by the number of parts of the next denomination; and ſo on till it be brought to the denomination required; and the product is the numerator of the fraction required.

Example 2. Reduce $\frac{2}{19}$ of a pound ſterling to the fraction of a penny.

Here I multiply the numerator by 20, as that is the number of ſhillings contained in a pound, and that product by 12, as there are ſo many pence in a ſhilling; and the product gives a numerator to the fraction required.

$$\begin{array}{r} 2 \\ \underline{20} \\ 40 \\ \underline{12} \\ 480 \end{array}$$

Thus, if in this caſe the reduction be aſcending, the denominator only of the fraction need be multiplied; and, if deſcending, the numerator only.

3. Reduce $\frac{5}{13}$ of a pound troy to the fraction of a pwt. —*Anſ.* $\frac{1200}{13}$.

4. Reduce $\frac{4}{9}$ of a day to the fraction of a minute. —*Anſ.* $\frac{5760}{9}$ or $\frac{640}{1}$.

5. Reduce $\frac{3}{4}$ of a guinea to the fraction of a penny. —*Anſ.* $\frac{756}{4}$.

Addition of Vulgar Fractions.

Before vulgar fractions can be added together, it is neceſſary that they be brought into a proper form, for which,

Rule. Reduce the compound fractions to ſimple ones, and whole and mixed numbers to improper fractions, and fractions of different denominations to fractions of the ſame denomination. Then reduce all the fractions to fractions of the ſame common denominator. Add all the numerators together, and place the ſum over the common denominator; and this fraction will be equal to the ſum of all the given fractions.

The total thus found, if it be an improper fraction, may be reduced to a whole or mixed number.

Example 1. Add $\frac{1}{9}$, $\frac{2}{7}$, and $\frac{2}{13}$ together.

13	9	7	9
7	2	2	7
91	18	14	63
	13	9	13
	54	126	819
	18		
	234		

The given fractions reduced to a common denominator are, $\frac{91}{819}$ equal to $\frac{1}{9}$, $\frac{234}{819}$ equal to $\frac{2}{7}$, $\frac{126}{819}$ equal to $\frac{2}{13}$.

91
234
126
451

$\frac{451}{819}$, the *anſwer*, or ſum of the given fractions.

2. Add $3\frac{2}{5}$, $\frac{3}{7}$, and $\frac{2}{5}$ of $\frac{3}{10}$ together.

Then $3\frac{2}{5}$ is equal to $\frac{17}{5}$, and $\frac{2}{5}$ of $\frac{3}{10}$ is equal to $\frac{6}{50}$ or $\frac{3}{25}$.

Then the given fractions will be $\frac{17}{5}$, $\frac{3}{7}$, $\frac{3}{25}$.

17	3	3	5
7	5	7	7
119	15	21	35
25	25	5	25
595	75	105	175
238	30		70
2975	375		875

The fractions reduced to a common denominator are, $\frac{2975}{875}$ equal to $\frac{17}{5}$, $\frac{375}{875}$ equal to $\frac{3}{7}$, $\frac{105}{875}$ equal to $\frac{3}{25}$.

105
375
2975
3455

$\frac{3455}{875}$, or $\frac{691}{175}$ or $3\frac{166}{175}$ for the *anſwer*.

In this example, as in the former, I firſt (after having reduced the mixed number $3\frac{2}{5}$ and the compound fraction $\frac{2}{5}$ of $\frac{3}{10}$ to ſimple fractions) reduce all the fractions to a common denominator, by multiplying the numerator of each into all the denominators except its own, for a numerator to each fraction, and all the denominators into each other for a common denominator, as taught in reduction of fractions; the numerators of theſe newly-diſcovered fractions are then added together, and their ſum 3455 placed over the common denominator 875 is the anſwer or fraction equal to the ſum of all the given fractions, which, divided by 5, gives $\frac{691}{175}$, and reduced to a mixed number is $3\frac{166}{175}$.

The proof of addition of fractions is wrought by finding the value of each of the given fractions, and adding them together, and then finding the value of the whole fraction or ſum of the fractions, and if theſe two values be equal the work is right.

Qu. 3. What is the ſum of $\frac{1}{12}$ of a pound and $\frac{3}{7}$ of a ſhilling?—*Anſ.* $\frac{736}{1680}$ of a pound.

In this example the fraction $\frac{3}{7}$ of a ſhilling muſt be reduced to the fraction of a pound before the work can be wrought, as before directed.

Qu. 4. What is the ſum of $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ of $\frac{1}{6}$?—*Anſwer* $2\frac{5}{18}$.

Qu. 5. What is the ſum of $\frac{1}{3}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{2}$ an hour?—*Anſ.* 2 days $14\frac{1}{2}$ hours.

Subtraction of Vulgar Fractions.

Rule. Reduce the fractions in the ſame manner as for addition; then the leſs numerator is to be ſubtracted from the greater, and the difference placed over the common denominator will form the fraction required.

Example 1. Subtract $\frac{3}{9}$ from $\frac{5}{7}$.

$$\begin{array}{ccc} 7 & 9 & 9 \\ \underline{3} & \underline{5} & \underline{7} \\ 21 & 45 & 63 \end{array}$$

$\frac{21}{63}$ equal to $\frac{3}{9}$.

$\frac{45}{63}$ equal to $\frac{5}{7}$.

From 45
Take 21
24 $\frac{24}{63}$ or $\frac{8}{21}$ the *Anſwer.*

Proof. Add the leſs number and the remainder together, as in whole numbers.

Example 2. Subtract $\frac{2}{9}$ from $\frac{3}{8}$ of $\frac{7}{9}$.

$\frac{3}{8}$ of $\frac{7}{9}$ is $\frac{21}{72}$ or $\frac{7}{24}$, therefore I can take $\frac{2}{9}$ from $\frac{7}{24}$.

24 9 24
2 7 9 $\frac{48}{216}$ equal to $\frac{2}{9}$.
48 63 216 $\frac{63}{216}$ equal to $\frac{7}{24}$.

From 63
Take 48
Reſts 15 $\frac{15}{216}$ or $\frac{5}{72}$ the *Anſwer.*

Qu. 3. What is the difference between $\frac{3}{10}$ and 28?—*Anſ.* $27\frac{7}{10}$.

Qu. 4. What is the difference between $26\frac{3}{7}$ and $54\frac{5}{6}$?—*Anſ.* $28\frac{17}{42}$.

Qu. 5. What is the difference between $14\frac{1}{4}$ and $\frac{2}{3}$ of 19? *Anſ.* $1\frac{7}{12}$.

Multiplication of Vulgar Fractions.

Rule. Reduce whole and mixed numbers, compound fractions, and fractions of different denominations, to ſimple fractions of the ſame denomination; then multiply all the numerators together for a new numerator, and all the denominators together for a new denominator, and ſuch fraction will be the true product required.

Example 1. Multiply $\frac{3}{4}$ by $\frac{5}{7}$.

3 4
5 7
15 numerator. 28 denominator. $\frac{15}{28}$ *Anſwer.*

Example 2. What is the product of $2\frac{5}{7}$ multiplied by $\frac{5}{6}$, by $\frac{3}{9}$, and $\frac{4}{7}$ of $\frac{2}{6}$?

$$\begin{array}{rcrcl} 19 & \quad & 7 & & \\ \underline{5} & & \underline{6} & & \\ 95 & & 42 & & \\ \underline{3} & & \underline{9} & & \\ 285 & & 378 & \quad & \textit{Anſ.}\ \frac{1140}{7938} \text{ or } \frac{190}{1323}. \\ \underline{4} & & \underline{21} & & \\ \underline{\underline{1140}} & & 378 & & \\ & & \underline{756} & & \\ & & \underline{\underline{7938}} & & \end{array}$$

In this example $2\frac{5}{7}$ is to be reduced to its equivalent fraction $\frac{19}{7}$, and $\frac{4}{7}$ of $\frac{2}{6}$ muſt be reduced to its ſimple fraction, $\frac{8}{42}$ or $\frac{4}{21}$, then the queſtion will be, what is the product of $\frac{19}{7}$, $\frac{5}{6}$, $\frac{3}{9}$ and $\frac{4}{21}$ multiplied together?

Qu. 3. What is the product of $\frac{3}{4}$ of $\frac{2}{3}$, by $\frac{3}{5}$ of $\frac{1}{2}$?—*Anſ.* $\frac{18}{120}$ or $\frac{3}{20}$.

Qu. 4. What is the product of 24 by $\frac{2}{3}$?—*Anſ.* $\frac{48}{3}$ or 16.

Qu. 5. What is the product of $\frac{1}{2}$ of 7 by $\frac{1}{2}$?—*Anſ.* $1\frac{3}{4}$.

Qu. 6. What is the product of 36 by $\frac{9}{11}$?—*Anſ.* $\frac{324}{11}$ or $29\frac{5}{11}$.

Thus it may be ſeen, that multiplication of vulgar fractions is performed in the ſame manner as reducing a compound fraction to a ſimple one *.

Diviſion of Vulgar Fractions.

Rule. Prepare all the fractions for diviſion in the ſame manner as for addition, ſubtraction, and multiplication; and invert the two terms of the diviſor, placing the denominator at the top and the numerator at the bottom; then proceed exactly the ſame as in multiplication, viz. multiply the numerator of the dividend and that figure that is uppermoſt in the diviſor together for a numerator to the quotient: and multiply the denominator of the dividend and the lower figure of the diviſor for the denominator of the quotient.

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* If any number be multiplied by a fraction, the product will be leſs than the multiplicand, and in proportion as the multiplier is leſs than an unit.

Proof. Multiply the quotient by the diviſor, as in whole numbers.

Example 1. Divide $\frac{3}{4}$ by $\frac{2}{7}$.—$\frac{7}{2}$)$\frac{3}{4}$($\frac{21}{8}$ or $2\frac{5}{8}$ *Anſwer.*

2. Divide $\frac{2}{3}$ of 7 by $\frac{3}{8}$ of $\frac{4}{9}$.

In this example, $\frac{2}{3}$ of 7 is equal to $\frac{14}{3}$, and $\frac{3}{8}$ of $\frac{4}{9}$ is equal to $\frac{12}{72}$ or $\frac{1}{6}$; therefore I ſay, divide $\frac{14}{3}$ by $\frac{1}{6}$.—*Anſ.* $\frac{84}{3}$ or 28.

3. Divide $\frac{7}{10}$ by 3.—*Anſ.* $\frac{7}{30}$.

4. Divide 10 by $\frac{7}{4}$.—*Anſ.* $\frac{40}{7}$ or $5\frac{5}{7}$.

5. Divide $7\frac{1}{2}$ by $\frac{3}{9}$ of $\frac{4}{11}$.—*Anſ.* $\frac{495}{8}$ or $61\frac{7}{8}$ *.

The Rule of Three direct in Vulgar Fractions.

Rule. Prepare the fractions as for the four foregoing rules; and then having placed the three numbers in their proper order according to their proportion, invert the firſt term or fraction, tranſpoſing the numerator and denominator, as in the foregoing rule; then multiply all the three numerators together for a numerator, and all the denominators together for a denominator, and this fraction will be the anſwer required.

Proof. The ſame as in whole numbers.

Example 1. If $\frac{7}{5}$ of an ell Engliſh coſt $\frac{3}{8}$ of a pound, what will $\frac{3}{5}$ of an ell Flemiſh coſt?

In this example I ſay, if $\frac{7}{5}$ of $\frac{5}{4}$ of a yard (which is an ell Engliſh) coſt $\frac{3}{8}$ of a pound, what will $\frac{3}{5}$ of $\frac{3}{4}$ of a yard (which is an ell Flemiſh) coſt?

I then reduce the compound fraction $\frac{7}{5}$ of $\frac{5}{4}$ to a ſimple one, and it is $\frac{35}{20}$ or $\frac{7}{4}$, and alſo the fraction $\frac{3}{5}$ of $\frac{3}{4}$ is $\frac{9}{20}$ of a yard, which is of the ſame denomination with the firſt number, viz. a fraction of a yard; and inverting the firſt term the queſtion will ſtand thus,

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* If a whole number be divided by a *proper* fraction, the quotient will be greater than the dividend; but if any fraction be divided by a whole number, the quotient will be leſs than the dividend.

$\frac{4}{7}$ coſt $\frac{3}{8}$, $\frac{9}{20}$.—The *anſwer* $\frac{108}{1120}$ or 1*s.* 11$\frac{1}{7}$*d.*

Then multiplying the three uppermoſt figures of the fractions together for a numerator, and the three undermoſt figures for a denominator, the *anſwer* is $\frac{108}{1120}$ of a pound, which, reduced to its real value, is 1*s.* 11*d.* $\frac{1}{7}$ of a penny.

Here it muſt be obſerved, that the firſt and third fractions muſt be reduced to the ſame denomination as in whole numbers, as ſeen in the foregoing example, where they are both fractions of a yard; and the fourth fraction is of the ſame denomination with the ſecond.

Qu. 2. If $\frac{3}{5}$ of a gallon of brandy coſt $\frac{1}{3}$ of a pound, what will 12$\frac{2}{3}$ coſt at that rate?—*Anſ.* $\frac{190}{27}$, or 7*l.* 0*s.* 8$\frac{3}{4}$*d.* $\frac{5}{9}$.

Qu. 3. If $\frac{9}{10}$ of a bale of linen coſt 14*l.* 14*s.* what will 7$\frac{1}{2}$ bales coſt at that rate?—*Anſwer.* 122*l.* 10*s.*

Qu. 4. If 12$\frac{1}{2}$lb. of ſugar coſt 15*s.* 9*d.* what is the price of 48$\frac{1}{4}$lb.?—*Anſwer.* 3*l.* 0*s.* 9$\frac{1}{2}$*d.* $\frac{16}{100}$.

Rule of Three inverſe in Vulgar Fractions.

Rule. Prepare all the fractions as for the foregoing rule, then conſider (as taught in the rule of three in whole numbers) whether the queſtion belongs to the inverſe or direct rule, and if it belong to the rule of three inverſe, the third fraction is to be inverted, by tranſpoſing the numerator and denominator; and the work is wrought exactly in the ſame manner as in the direct rule, by multiplying the three uppermoſt terms of the fractions together for a numerator, and the undermoſt terms of the three fractions for a denominator; and the fraction thus formed will be the anſwer.

Proof. As before in whole numbers.

Example 1. If A lent B $\frac{1}{3}$ of 1000*l.* for $\frac{3}{7}$ of a year, how much muſt B lend A for $\frac{9}{10}$ of a year in return?

After diſpoſing of the fractions as before directed, I conſider that $\frac{9}{10}$ of a year being a longer time than $\frac{3}{7}$, it will not require ſo much principal lent; therefore the greater of the firſt and third numbers muſt be the diviſor (as in whole numbers); the third fraction therefore muſt be inverted, and the queſtion will ſtand thus:

If $\frac{3}{7}$ of a year $\frac{1}{3}$ of 1000*l.* $\frac{10}{9}$ of a year?
Anſwer $\frac{30000}{189}$, or $158\frac{138}{189}$, equal to 158*l.* 14*s.* $7\frac{5}{21}$*d.*

Queſtions both in this rule and the former are proved by back-ſtating the queſtion, as in whole numbers; thus the foregoing example may be proved by ſaying, if $\frac{30}{189}$ of 1000*l.* principal require $\frac{9}{10}$ of a year, what will $\frac{1}{3}$ of 1000*l.* require? and the *anſwer* is $\frac{270}{630}$, or $\frac{3}{7}$ of a year.

Qu. 2. How much ſhalloon will it require at $\frac{3}{4}$ of a yard wide to line the garments made with $10\frac{1}{2}$ yards of cloth at $1\frac{3}{4}$ yard wide?—*Anſ.* $\frac{588}{24}$, or $\frac{49}{2}$, equal to $24\frac{1}{2}$ yards.

Qu. 3. If 12 men can mow $24\frac{1}{2}$ acres in $10\frac{2}{3}$ days, how many days will 6 men require to do the ſame?—*Anſwer* $21\frac{1}{3}$ days.

Qu. 4. If a board be $\frac{3}{4}$ of a foot in breadth, how many inches in length will make a ſquare foot?—*Anſ.* 16 inches.

From what has been delivered in this ſection concerning vulgar fractions, it is plain that every other rule in arithmetic may be wrought by vulgar fractions as well as by whole numbers, as the operation in both caſes depends upon the ſame principle; thus, in the rule of three direct in vulgar fractions, inverting the firſt fraction, and multiplying it by the ſecond and third, is the ſame as multiplying the ſecond and third fractions together, and dividing by the firſt; and in the inverſe rule, inverting the third fraction and multiplying it by the firſt and ſecond, is equal to dividing the product of the firſt and ſecond fractions by the third, as the learner may prove at his leiſure.

SECT. XIV.

OF PRACTICE.

Practice is the moſt expeditious rule in arithmetic, and is of general uſe among men of buſineſs, as it readily diſcovers the value of any number of integers from having the value of one.

By this rule are anſwered all queſtions in the rule of three direct that have an unit for their firſt number.

Rule. Divide the given number of integers by one or more aliquot parts of a penny, ſhilling, or pound, or any two or three of them; and the quotient will be the anſwer, and of the ſame denomination of which the diviſor is a part.

An aliquot part of a number is ſuch a part, that being taken any number of times, will exactly meaſure that number without a remainder: thus 2 is an aliquot part of 6, for it is contained exactly 3 times in 6; and 5*s.* is an aliquot part of a pound, for it is contained exactly four times in a pound; but 5*s.* 2*d.* is not an aliquot part, for it is not exactly contained any number of times in a pound without leaving a remainder.

Before the learner can perform this rule, he muſt perfectly underſtand the following tables of aliquot parts, and retain them in his memory.

The aliquot Parts.

d.				
6	is	½	half	of a ſhilling.
4		⅓	third	
3		¼	fourth	
2		⅙	ſixth	
1½		⅛	eighth	
1		$\frac{1}{12}$	twelfth	
¾		$\frac{1}{16}$	ſixteenth	

½	half	of a penny.
¼	fourth	

s.	*d.*			
10	0	is	½	of a pound.
6	8		⅓	
5	0		¼	
4	0		⅕	
3	4		⅙	
2	6		⅛	
2	0		$\frac{1}{10}$	
1	8		$\frac{1}{12}$	
1	0		$\frac{1}{20}$	

Of a Hundred Weight.		
2qrs. or 56lb.	is	½ half
1 or 28		¼ quarter
14		⅛ eighth
7		$\frac{1}{16}$ ſixteenth

Of a Qadrter of a Cwt.		
14lb.	is	½ half
7		¼ quarter
4		$\frac{1}{7}$ ſeventh
3½		⅛ eighth

Theſe tables are ſo plain as to need no explanation; their uſe is to diſcover by what number to divide any given number of integers.

Caſe 1. When the price is leſs than a penny, divide the given number by the aliquot parts of a penny equal to the given price, and the quotient gives the anſwer in pence, which reduce into ſhillings and pounds by diviſion; except the given price be 3 farthings, then it is brought into ſhillings, and anſwered at once by dividing by 16.

Example 1. What is the amount of 8047lb. of old iron, at a halfpenny per pound?

Here I divide the given number 8047 by 2, as 2 farthings is the half of a penny, and the quotient 4023 is the price of the iron in pence, and 1 remains, which is 1 halfpenny, for the remainder is always of the ſame name with the diviſor; I then reduce the pence into ſhillings by dividing by 12, and the quotient is 335 ſhillings, and 3 remains, which is pence; and then reducing the ſhillings into pounds, the anſwer is 16*l.* 15*s.* 3½*d.*

```
  2)8047
 12)4023 1
2,0) 33,5 3
     16 15
```

Example 2. What is the value of 5763 yards of trimming, at 3 farthings per yard?

In this example I divide the given number by 16, as before directed, as 3 farthings is the ſixteenth part of a ſhilling, and the quotient is 360 ſhillings, which reduced into pounds is 18*l.* 0*s.* 2¼*d.* for the 3 that remains in the firſt diviſion is 3 ſixteenths of a ſhilling, or 3 times 3 farthings, equal to 2¼*d.*

```
        2,0)
16)5763(36,0
   48    18
    96
    96
     3
```

Qu. 3. What comes 445, at ¼*d.*?—*Anſ.* 9*s.* 3¼*d.*

Qu. 4. What is the value of 3370, at ½*d.*?—*Anſ.* 7*l.* 5*s.*

Caſe 2. When the price is an aliquot part of a ſhilling, divide the given number by ſuch aliquot part, and the quotient is the anſwer in ſhillings, which muſt be reduced into pounds.

Example 5. What is the value of 879lb. of cheeſe, at 4*d.* per lb.?

Here the given number of pounds is divided by 3, as 4*d.* is $\frac{1}{3}$ of a ſhilling, and it quotes 293 ſhillings, which are brought into pounds; and the *Anſwer* is 14*l.* 13*s.*

3)879
2,0)29,3 ſhillings
14
13

Qu. 6. What is the value of 297lb. of tallow at 3*d.* per lb.?—Here the given number muſt be divided by 4, as 3*d.* is $\frac{1}{4}$ of a ſhilling, and the *Anſwer* is 3*l.* 14*s.* 3*d.*

Example 7. What is the value of 3cwt. of ſugar, at 6*d.* per lb.?—*Anſ.* 8*l.* 8*s.*

2)336
2,0)16,8
8

Example 8. What is the value of 2178lb. of alum, at 1$\frac{1}{2}$*d.* per lb.?—*Anſ.* 13*l.* 12*s.* 3*d.*

8)2178
2,0)27,2 2
13
12

Example 9. What is the value of 486lb. at 2*d.* per lb.?—*Anſ.* 4*l.* 1*s.*

6)486
2,0)8,1
4. 1

Caſe 3. When the price of the integer is pence and farthings, and not an aliquot part of a ſhilling, find what aliquot part of a ſhilling is the neareſt to the given price, and leſs than it, and divide the given number by that aliquot part; and for the remainder of the price conſider what part it is of the given price, and divide the quotient by it; and if there be ſtill a remainder of the given price, conſider what aliquot part this is of the laſt, and divide the laſt quotient thereby; then add all the quotients together for the anſwer.

Example 10. What is the value of 5½cwt. of butter, at 8½*d*. per lb.?

6*d*. is ½ of a ſhilling	2)616	equal to 5½cwt.
2*d*. is ⅓ of 6*d*. therefore divide by	3)308	{equal to 4*d*.
½*d*. is ¼ of 2*d*. therefore divide by	4)102	remains 2, or ⅔ of 6*d*.
	25	remains 2, or 2/4 of 2*d*.
	2,0)43,5	equal to 1*d*.
	21	
Anſwer 21*l*. 15*s*. 6*d*.	15	

In this example, I divide the given number firſt by 2, as 6*d*. is the neareſt aliquot part to the price, and the quotient is 308*s*. which is the price at 6*d*. per lb.: I then divide that quotient by 3, for the other 2*d*. in the price, and it quotes 102*s*. which is the price of the article at 2*d*. per lb. and 2 remains; and for the halfpenny I divide the laſt quotient by 4, as one halfpenny is the fourth part of 2*d*. and the quotient is 25*s*. which is the price of the butter at a halfpenny per lb.: the three quotients added together give the anſwer.

Example 11. What is the value of 137 yards of cloth, at 10½*d*. per yard?

For the 6*d*. I divide by	2)137	
For the 3*d*. I divide by	2) 68	remains 1, or 6*d*.
For the 1½*d*. I divide by	2) 34	3
	17	1½
	2,0)11,9	
Anſwer	£5 19	10½

Qu. 12. What is the value of 520lb. of ſoap, at 7½*d*. per lb.?—*Anſ*. 16*l*. 5*s*.

Qu. 13. What is the value of 860 yards of linen, at 11½*d*. per yard?—*Anſ*. 41*l*. 4*s*. 2*d*.

Caſe 4. When the price is any number of ſhillings under 20.

Firſt. If it be an even number of ſhillings, multiply the given number by half the price, and double the firſt figure on the right hand, which will be ſhillings, and all the other figures are pounds.

Secondly. If the price be an odd number of ſhillings, multiply the quantity by half the next leſs number, which will be an even number, then for the odd ſhilling add $\frac{1}{20}$ of the given number to the laſt product, and the ſum of theſe two quantities will be the anſwer.

Qu. 14. What is the value of 476 yards of cloth, at 4*s.* per yard?

Here I multiply the given number 476 by 2, half the price, ſaying, 2 times 6 is 12, which doubled is 24; I ſet down the 4 ſhillings, and carry the 1 pound to be added to the next product, ſaying, 2 times 7 is 14, and 1 I carried is 15, 5 and carry 1; then 2 times 4 is 8, and 1 is 9; then I cut off the firſt figure 4 for ſhillings, and the reſt are pounds.

$$\begin{array}{r} 476 \\ 2 \\ \hline 95{,}4 \\ \hline\hline \end{array}$$

Anſwer 95*l.* 4*s.*

Example 15. What is the value of 462 yards of cloth, at 7*s.* per yard?

In this example, 7 being an odd number, I take 3 for the multiplier (being the half of 6 the next even number), and multiply as in the former example, doubling the firſt number for ſhillings, and the product is 138*l.* 12*s.* which is the price of the cloth at 6*s.* per yard; then for the other ſhilling I take a twentieth part of the given number, and the quotient is 23*l.* 2*s.* which is the price of the cloth at 1*s.* per yard, and which added to the former price at 6*s.* gives 161*l.* 14*s.* the anſwer.

$$\begin{array}{lr} & 2{,}0)46{,}2 \\ & 3 \\ \hline & 138{,}12 \\ & 23\quad 2 \\ \hline \textit{Anſwer}\ £ & 161\quad 14 \\ \hline\hline \end{array}$$

Examples for Practiſe.

2,0)431 yards at 13*s.* per yd.
258 12
21 11
£.280 3 *Anſwer.*

2,0)324 ells at 17*s.* per ell.
259 4
16 4
£.275 8 *Anſwer.*

2,0)279 yards at 3*s.* per yd.
27 18
13 19
£.41 17 *Anſwer.*

2,0)2880 yards at 19*s.* per yd.
2592 0
144 0
£.2736 0 *Anſwer.*

Qu. 16. What is the value of 5514 yards, at 1*s.* per yard? —*Anſ.* 275*l.* 14*s.*

Qu. 17. What is the value of 2468 yards, at 5*s.* per yard? —*Anſ.* 617*l.*

Note. When the price is 5*s.* as in the laſt queſtion, the given number may be divided by 4, as 5 is ¼ of a pound, and the quotient is the anſwer in pounds.

And when the price is 2*s.* it is done at ſight, by doubling the firſt figure on the right hand for ſhillings, and the other figures are pounds.

Example 18. What is the value of 896 gallons of cyder at 2*s.* per gallon? 89*l.* 12*s.* *Anſwer.*

By this method of working by 2*s* a variety of examples may be wrought very expeditiouſly, by dividing the given price into parts of 2*s.* each, and finding the value of the overplus, if any.

Example 19. What is the value of 444 gallons of Hollands, at 5*s.* 9*d.* per gallon?

	444 gallons	
For 2*s.* per gallon I take	44*l.*	8*s.*
For the other 2*s.* per gallon	44	8
For the 1*s.* half the 2*s.* line	22	4
For the 6*d.* half the 1*s.* line	11	2
For the 3*d.* half the 6*d.* line	5	11
And then add them together for the *Anſ.*	127	13

In this manner examples in this rule may be wrought, though the price conſiſt of any number of ſhillings and pence, and which would ſuperſede the neceſſity of ſome of the following rules. I ſhall nevertheleſs inſert them, that the work appear not deficient.

Caſe 5. When the price is ſhillings and pence, which make an aliquot part of a pound, divide the given quantity by ſuch aliquot part.

Example 20. What is the value of 2796 dollars, at 3*s.* 4*d.* per dollar?

6)2796

Anſwer 466*l.*

Here I divide the given number by 6, as 3*s.* 4*d.* is the ſixth part of a pound.

Qu. 21. What is the value of 3575 yards, at 1*s.* 8*d.* per yard?—*Anſ.* 297*l.* 18*s.* 4*d.*

Qu. 22. What is the value of 2478 gallons of ſpirits, at 6*s.* 8*d.* per gallon?—*Anſ.* 826*l.*

Qu. 23. What is the value of 2795 yards of linen, at 2*s.* 6*d.* per yard?—*Anſ.* 349*l.* 7*s.* 6*d.*

Caſe 6. When the price conſiſts of ſhillings and pence, or ſhillings, pence, and farthings, which do not make an aliquot part of a pound, divide the given number by the greateſt aliquot part of a pound that the price contains; and for the remainder of the price, if any, conſider what aliquot part it is of the former aliquot part, and divide the quotient thereby; and if there ſtill be a remainder of the price, conſider what aliquot part it is of the laſt aliquot part, and divide the laſt quotient thereby: and proceed in this manner as long as there is any remainder of the price: then the ſum of all the quotients will be the anſwer.

Example 24. What is the value of 3726 yards of cloth, at 7*s.* 4½*d.* per yard?

3)3726
10)1242
16) 124 4 0
7 15 3
Anſwer £.1373 19 3

In this example, I divide the given number by 3, for 6s. 8d. as that is the third part of a pound; the quotient I again divide by 10, for the 8d. remaining, as 8d. is the tenth part of 6s. 8d.; and this quotient I again divide by 16, for the halfpenny, as that is the ſixteenth part of 8d.: then the three quotients added together give the anſwer.

Or the price may be divided into the aliquot parts of a pound, and then the given number muſt be divided by each of them; thus, in the foregoing example, the given number of yards may be divided by 5 for 4s. the fifth part of a pound, and 6 for the other 3s. 4d. the ſixth part of a pound, and for the halfpenny the given number may be divided by 24, the aliquot part of a ſhilling, and this laſt quotient is ſhillings; and the three quotients added together is the anſwer.

Qu. 25. What is the value of 784 gallons, at 6s. 9d. per gallon?—*Anſ.* 264*l.* 12*s.*

Qu. 26. What is the value of 1464 gallons, at 12s. 7d. per gallon?—*Anſ.* 921*l.* 2*s.*

Caſe 7. When the price is pounds, ſhillings, pence, and farthings, multiply the given number by the number of pounds, and the product is pounds; then for the remainder of the price, work according to ſome of the former rules, as the caſe may require; and theſe ſums added together will give the anſwer.

Example 27. What is the value of 416 cwt. of ſugar, at 2*l.* 9*s.* 3¾*d.* per cwt.?

		416*cwt.*		
		2		
For the 2*l.* take		832	0	0
For 8*s.*	—	166	8	0
For 1*s.*	—	20	16	0
For 3*d.*	—	5	4	0
For ¾*d.*	—	1	6	0
Anſwer		£.1025	14	0

For the 2*l.* I multiply the given number by 2, and the product 832 is pounds; then for the 8*s.* I multiply the given number by 4, ſetting the double of the unit figure apart for ſhillings, as directed in the fourth caſe; for the 1*s.* I take the eighth part of the 8*s.* line; for the 3*d.* I take the fourth part of the 1*s.* line; and for the ¾ I take the fourth part of the 3*d.* line: and the ſum of theſe is the anſwer.

Qu. 28. What is the value of 1894 pieces of linen, at 4*l.* 15*s.* 10¾*d.* per piece?—*Anſ.* 9077*l.* 7*s.* 9½*d.*

Caſe 8. When the number of integers whoſe value is required is a whole number with fractions annexed, find the value of the whole number by ſome of the foregoing rules, to which add the value of the parts repreſented by the fractions.

Example 29. What is the value of 278cwt. 84lb. of ſoap, at 1*l.* 19*s.* 4*d.* per cwt.?

	278¾*cwt.*		
For 1*l.* take	278	0	0
For 16*s.*	222	8	0
For 3*s.* 4*d.*	46	6	8
For ½cwt.	0	19	8
For ¼cwt.	0	9	10
Anſwer	£.548	4	2

In this example, after finding the value of the 278 cwt. at the given price, I find that 84lb. is ¾ of an cwt. I therefore take ½ of the given price, viz. 19*s.* 8*d.* for ½ the cwt. and for the other ¼ cwt. I take ¼ of the price, 9*s.* 10*d.*

Qu. 30. What is the value of 568½ pieces of cloth, at 2*l.* 10*s.* 6*d.* per piece?—*Anſ.* 1435*l.* 9*s.* 3*d.*

Caſe 9. When the quantity whoſe value is ſought is of ſeveral denominations, firſt, find the value of the greateſt denomination; and for the other denominations, take an equivalent part or parts of the given price; and add the ſeveral ſums together for the anſwer.

Example 31. What is the value of 10cwt. 2qrs. 21lb. of tobacco, at 4*l.* 5*s.* 6*d.* per cwt.?

	4	5	6	per cwt.
			10	cwt.
For 10cwt. take	42	15	0	
For 2qrs.	2	2	9	
For 14lb.	0	10	8$\frac{1}{4}$	
For 7lb.	0	5	4$\frac{1}{8}$	
Anſwer £.	45	13	9$\frac{3}{8}$	

The cwt. being the greateſt denomiuation, is found firſt, by multiplying the price of an cwt. by 10, and for the value of the 2qrs. I take half the price of an cwt. then for the 21lb. I firſt find the value of 14lb. by taking half the price of a qr. or a fourth part of 2qrs. and half this price for the other 7lb.; theſe ſums added together give the anſwer.

Qu. 32. What is the value of 28yds. 3½qrs. of ſuperfine cloth, at 30s. per yard? — *Anſ.* 43*l.* 6*s.* 3*d.*

SECT. XV.

OF BARTER.

Barter is that rule which inſtructs traders to exchange one commodity for another, ſo that neither party may ſuſtain any loſs.

Rule. Find the value of that commodity whoſe quantity is given, by the rule of three or practice; then by the rule of three, practice, or diviſion, find what quantity of the other commodity ſhould be given in exchange.

Example 1. How many gallons of brandy, at 6*s.* per gallon, muſt be given in barter for 7cwt. 2qrs. 14lb. of ſugar, at 2*l.* 10*s.* per cwt.?

cwt. qrs. lb.
7 2 14
2

£.14 0 0
3 10 0
1 5 0
0 6 3

19 1 3
20

381
12

72)4575(63 gallons.
432

255
216

39
8

72)312($4\frac{24}{72}$ pints.
288 *Anſwer* 63 gallons $4\frac{1}{3}$ pints.

24

In this example, I firſt find the value of the given quantity of ſugar by the rule of practice, which I reduce into pence, and it produces 4575 pence; theſe pence are then divided by 72, the pence in 1 gallon of brandy, and it quotes 63 gallons and $4\frac{1}{3}$ of a pint for the anſwer.

Thus it appears that queſtions that concern only one price of each ſort, of two different kinds of goods, may be wrought by practice and diviſion only, as the foregoing; but thoſe of a more complex nature muſt be reſolved by the rule of three.

Qu. 2. A grocer has 120lb. of tea, which coſt him 6*s.* per lb. but he intends to barter it at the rate of 8*s.* per lb. with a diſtiller, for Hollands that coſt him 4*s.* per gallon. At what price muſt the diſtiller rate his Hollands, that he may have as much profit as the grocer; and how many gallons muſt he give for the 120lb. of tea?—*Anſwer*, he muſt rate his Hollands at 5*s.* 4*d.* per gallon, and give 180 gallons for the tea.

In reſolving this queſtion, firſt find what the Hollands muſt be rated at, by the rule of three, ſaying, if 6*s.* require 8*s.* what will 4*s.* require?—*Anſwer* 5*s.* 4*d.* Then by practice (as in the firſt example) find the value of the tea at 8*s.* per lb. which, divided by the price of 1 gallon of Hollands, as before, quotes the anſwer.

Qu. 3. A vintner barters 196 gallons of wine for 14cwt. of ſugar worth 6*d.* per lb. how much was the wine worth at that rate?—*Anſ.* 4*s.* per gallon.

Qu. 4. A barters 320 gallons of gin, at 4*s.* 6*d.* per gallon, with B for 6lb. of tea at 5*s.* per lb. and for ſugar at 8*d.* per lb.; how much ſugar will A receive?—*Anſwer* 11cwt. 1qr.

Qu. 5. A vintner barters 608 gallons of brandy at 14*s.* per gallon, for ſugar at 3*l.* 10*s.* per cwt. and 14*l.* 12*s.* in caſh: how much ſugar ſhould the vintner receive?—*Anſwer* 85cwt. 2qrs. 24lb.

SECT. XVI.

OF LOSS AND GAIN.

Loſs and gain is that rule which diſcovers the loſs or gain from buying and ſelling goods; and inſtructs traders how to fix their price, in order to gain or loſe any certain ſum.

Rule. By the rule of three direct. Though queſtions in this rule may often be anſwered by practice, or other rules.

Example 1. At how much per lb. muſt a grocer ſell tea which coſt him 4*s.* 10*d.* per lb. ſo as to gain 20 per cent. profit?

As 100*l.* is to 127*l.* ſo is 4*s.* 10*d.* to

```
              58       12
             ----      --
             1016      58
             635
      1,00)73,66(73 pence
           73
           --
             66
              4
           ---
      1,00)2,64(2 farthings
           2
           --
            64     Anſ. 6s. 1½d. 64/100 or 16/25 per lb.
           ===
```

When the gain or loſs is required at any rate per cent, where the intereſt has a 5 or a cypher on the right hand, as is moſt commonly the caſe, the anſwer may be readily found, by adding to, or ſubtracting from the given price ſuch a part as the intereſt is of the principal; thus, if it be required to gain or loſe 5 per cent. (as 5 is the twentieth part of an hundred) the anſwer is found by adding to or ſubtracting from the given price one twentieth part; and if the gain or loſs be 10 per cent. then it is one tenth part; and if 15 per cent. it is $\frac{3}{20}$; and 20 per cent. is $\frac{1}{5}$; and 25 per cent. is $\frac{1}{4}$, &c.

Qu. 2. A grocer bought 8½cwt. of ſugar, which coſt 31*l.* 14*s.* 8*d.*; but, it being damaged, he is willing to loſe 12*l.* 10*s.* per cent. in the ſale of it; at how much per lb. muſt he ſell it?—*Anſwer* 7*d.* per lb.

In this example I ſubtract the loſs per cent. from the principal, and the remainder is the ſecond number in the rule of three, the principal the firſt number, the whole price of the ſugar the third number; and the fourth number will be the whole price at the reduced rate, which divided by the number of pounds, gives 7*d.* the price of 1lb.

Qu. 3. A wholeſale factor in Ireland made linen, which coſt him 12½*d.* per yard, the expenſe of ſending it to London 1½*d.* per yard, it was ſold in London at 1*s.* 9*d.* per yard, and the retail trader was allowed 26 per cent. profit, what profit had the wholeſale factor?—*Anſ.* 24 per cent.

In reſolving this queſtion, I ſay, as 1*s.* 2*d.* the expenſe of making and exporting the linen, is to 1*s.* 9*d.* the retail price, ſo is 100*l.* to 150*l.*; thus there is 50 per cent. profit, which, after deducting 26 per cent. the retail trader's profit, leaves 24 per cent. for the factor.

Qu. 4. A merchant bought 100 gallons of brandy, at 6*s.* per gallon, of which quantity 40 gallons were loſt; at what price per gallon muſt he ſell the remainder, that he may gain 10 per cent. profit upon the money it coſt him?—*Anſ.* 11*s.* per gallon.

SECT. XVII.

OF EQUATION OF PAYMENTS.

EQUATION of payments is that rule whereby is diſcovered the time to pay at one payment ſeveral ſums due at different times, ſo that neither party may ſuſtain any loſs.

Rule. Multiply each debt by the time at which it is due, and add all the products together; divide the ſum of the products by the ſum of all the debts, and the quotient will be the anſwer, or the equated time to pay the whole.

Example 1. A is indebted to B in the ſum of 200*l.* to be paid as follows: 60*l.* in 4 months, 40*l.* in 6 months, and 100*l.* in 10 months; what is the equated time to pay the whole?

Debts.		Months.	Products.
60*l.*	in	4	240
40		6	240
100		10	1000
200			2,00)14,80
Anſwer 7⅖ months.			7
			80

Qu. 2. A owes B 1000*l.* of which $\frac{1}{3}$ is to be paid in 6 months, $\frac{2}{5}$ in 8 months, and the remainder in 10 months; what is the equated time to pay the whole?—*Anſwer* $7\frac{13}{15}$ months.

In this example, $\frac{1}{3}$ of 1000*l.* multiplied by 6 months produces 2, and $\frac{2}{5}$ multiplied by 8, produces $3\frac{1}{5}$, and $\frac{4}{15}$ (the remainder of the 1000*l.*) multiplied by 10, produces $2\frac{10}{15}$; and theſe three products added together gives $7\frac{13}{15}$ months for the anſwer.

And, *note*, when the ſums or times of payment are given in fractions, the ſum of the products is not to be divided by the ſum of the debts, as it is in whole numbers.

Qu. 3. A tradeſman owes his creditor 144*l.*; 44*l.* he pays in ready money; 60*l.* is to be paid at the expiration of 6 months, and the remaining 40*l.* at the expiration of 8 months; but the tradeſman deſiring to have more time for the payment of the laſt 40*l.* pays his creditor the 60*l.* due 6 months after, in ready money; how long may he defer paying this laſt 40*l.* to make him amends for this prompt payment?—*Anſ.* 17 months.

In this example, the 44*l.* to be paid in ready money is neglected; but for the 60*l.* paid 6 months before due, I find by the rule of three what intereſt it would gain at any rate per cent. in that time, and then how long the 40*l.* may be lent for that intereſt at the ſame rate, which I find is 9 months, and which added to 8 months, its time of payment, gives 17 months for the anſwer *.

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* This rule, though greatly uſed by men of buſineſs, is not mathematically exact. The reaſon of the rule given by many writers, is, that for the debtor paying a ſum before the time it is due, an equal ſum ſhould be forborn, for as long a time after it is due; but this is a miſtake, for by the debtor paying money before it is due, he has the diſcount only; but keeping the money after it is due, he gains the intereſt, which is greater than the diſcount.

SECT.

SECT. XVIII.

OF THE RULE OF FALSE, SINGLE AND DOUBLE (GENERALLY CALLED POSITION).

THIS rule teacheth to anſwer ſuch queſtions as cannot be reſolved by any direct rule in vulgar arithmetic, and muſt be performed by falſe or feigned numbers.

Poſition is either ſingle or double.

Single poſition is when the queſtion can be reſolved by one falſe poſition or ſet of feigned numbers, and one operation in the rule of three direct.

Rule. Take any number, and proceed exactly the ſame as if it were the true number through all the proportions mentioned in the queſtion.

Then ſay (by the rule of three) as the reſult of this falſe operation is to any of its parts, ſo is the true reſult in the queſtion to the correſponding part required.

Example 1. A ſon aſking his father his age, the father anſwered, I am double the age of your eldeſt brother John, and he is three times the age of your youngeſt brother Henry, and the ſum of all our ages is 80 years; what is each perſon's age?

Falſe Suppoſition.		*The Truth.*	
Suppoſe Henry's age is	6	Henry's age	8
Then John's age will be	18	John's	24
And the father's age	36	The father's	48
The ſum of the ages	60	The ſum	80

Here I ſuppoſe Henry's age to be 6, at which ſuppoſition John's age muſt be 18, the father's 36, and the ſum of theſe three is 60, whereas it ſhould be 80; then I ſay, as 60 the ſum of the falſe ſuppoſition, is to 80 the ſum of the true one, ſo is 6 the ſuppoſed age of Henry to 8 his true age; therefore John's age is 24, and the father's 48, as in the example.

Qu. 2. A aſked B how much money he had in his pocket; B anſwered, If you give me 4 guineas of the money in your pocket, I ſhall have five times as much as you will then have; but if, inſtead of that, I ſhould give you 2 guineas of the money in my pocket, you will then have twice as much as I ſhall then have: how much money had each?—*Anſ.* 6 guineas.

Qu. 3. A perſon hired a horſe for 9 days, on the following terms: for the firſt 3 days he was to pay ¼ of the hire for the next 3 days, and for each of the laſt 3 days as much as the hire for the firſt 6 days; the whole was 2*l.* 8*s.*; what was it per day?—*Anſ.* 1*s.* per day the firſt 3 days, 3*s.* per day the next 3 days, and 12*s.* per day the 3 laſt days.

Double poſition, or the *double rule of falſe*, is when two falſe poſitions are requiſite to give an anſwer to the queſtion.

Rule 1. Take any two convenient numbers, and work with each of them according to the queſtion, as in ſingle poſition. 2. Find the difference between the reſult of each of theſe falſe poſitions and the reſult of the queſtion; theſe differences are called the errors. 3. Multiply each error by the contrary poſition, that is, the firſt error by the ſecond poſition, and the ſecond error by the firſt poſition; then find the ſum and difference of the products. 4. If the errors are both alike, that is, if the reſult of the two poſitions be both greater or both leſs than the reſult of the queſtion, divide the difference of the products by the difference of the errors, and the quotient will be the anſwer. 5. But if the errors be unlike, that is, if the reſult of one poſition be greater and the other leſs than the truth, then the ſum of the products muſt be divided by the ſum of the errors, and the quotient will be the anſwer.

Example 1. Three perſons, A, B, C, built a houſe, which coſt 500*l.* of which B paid half as much again as A, and C paid as much as A and B together; what did each pay?

Firſt Suppoſition.	*Second Suppoſition.*
Suppoſe A paid £90	Suppoſe A paid £96
Then B muſt have paid 135	Then B muſt have paid 144
And C muſt have paid 225	And C have paid 240
Reſult 450	Reſult 480
Firſt error 50	Second error 20
Second poſition 96	Firſt poſition 90
300	Second product 1800
450	
Firſt product 4800	
Second product 1800	

Difference of errors 3,0)300,0 difference of the products.
£100 for A's ſhare, wherefore
B muſt have paid 150 being half as much again as A,
and D muſt have paid 250 being as much as both A and B,
which added together gives £500 the original ſum.

From this example may be ſeen the method of working this rule, which is always the ſame, except when the errors are unlike; then the ſum of the products is to be the dividend, and the ſum of the errors the diviſor as above directed.

Qu. 2. A ſaleſman bought a number of oxen, ſheep, and lambs, for which he paid 115*l.*; for the oxen he paid 10*l.* each, for the ſheep 20*s.* each, and for the lambs 10*s.* each; how many of each ſort did he buy?—*Anſ.* 10 of each.

Qu. 3. Three perſons, A, B, C, have equal incomes; A ſaves $\frac{1}{10}$ of his income every year; B ſpends 10*l.* per annum more than A, and C ſpends 10*l.* per annum more than B. At the expiration of five years, C finds himſelf in debt 50*l.* what is each perſon's income, and what has each ſaved or ſpent?—*Anſ.* the income of each is 100*l.* per annum; A has ſaved 50*l.*; B has ſaved nothing; and C has ſpent 50*l.* more than his income.

Qu. 4. A labourer was hired for 40 days: for every day he wrought he was to receive 2*s.* and for every day he was idle he was to forfeit 1*s.*; at the end of the time he had to receive 44*s.*; how many days did he work, and how many was he idle?—*Anſ.* he wrought 28 days and was idle 12 days.

SECT.

SECT. XIX.

OF EXCHANGE.

Exchange is that rule which teacheth to find what ſum of the money of one country is equal to a given ſum of the money of another country, the courſe of exchange being known.

The *courſe of exchange* is that ſum of the money of one country which is propoſed to be given for a certain conſtant ſum of that of another country: thus, when we ſay the courſe of exchange between England and Holland is 34*s.* Flemiſh per pound ſterling, it ſignifies that 1 pound ſterling is equal to the value of 34*s.* in Flemiſh money. This courſe of exchange varies on the part of the foreign coins, according to the ſtate of public affairs.

The *par of exchange* is that quantity of the coin of one country which is intrinſically equal to a certain quantity of the coin of another country, according to the value of the metal.

Moſt foreign countries have two ſorts of coins, called *current money* and *banco money;* the firſt is that in general uſe throughout the country; the latter is that kept in the banks of thoſe places, and is finer than the other; the difference between any ſum as it is valued in current money and banco money is called the *agio.*

The money uſed in exchange is generally imaginary, and different from that in which the accompts are kept in moſt places: the money uſed in exchange alſo differs from current money in its value.

Before the learner can reſolve any queſtions in this rule, it is neceſſary that he know how the country with which the exchange is to be made keep their accompts.

Holland, Flanders, and Germany.

In theſe countries accompts are kept in gilders, ſtivers, and pennings, ſimilar to the Engliſh pounds, ſhillings, and pence.

But the different denominations of their money are contained in the following table:

8 pennings	make one	grote or penny
2 grotes		ſtiver
6 ſtivers		ſchilling
20 ſtivers		florin or gilder
2½ florins		rix-dollar
6 florins		pound Flemiſh

Note. The money of Holland and Flanders is called Flemiſh money, and they exchange by the pound ſterling.

The courſe of exchange with theſe countries has moſtly been (except during the troubles on the Continent) from 33*s.* 4*d.* to 36*s.* 6*d.* Flemiſh per pound ſterling; and the agio from 3 to 6 per cent.

As the exchange with all countries is ſuppoſed to be made in banco money, the current money muſt be turned into banco before the exchange can be made.

Rule. By the rule of three, or practice.

Proof. By reverſing the queſtion.

To change current money into banco, and banco money into current.

Say, by the rule of three, as 100*l.* with the agio added to it, is to 100*l.* ſo is the given ſum current money to its value in banco.

And, as 100*l.* is to 100*l.* with the agio added to it, ſo is the given ſum banco to its value in current money.

Example 1. In 110*l.* 10*s.* 6*d.* ſterling, how many florins, ſtivers, and pence current, the courſe of exchange being at 35*s.* 7½*d.* Flemiſh per pound ſterling, agio 4*l.* per cent.?

For 1*l.* take	£110	10	6
For 10*s.* take	55	5	3
For 5*s.*	27	12	7½
For 6*d.*	2	15	3 6/10
For 1¼*d.*	0	13	9¼ 3/20
	£196	17	5¼ ¾

Then to find the value of this ſum in current money, I ſay, as 100*l.* is to 104*l.* ſo is 196*l.* 17*s.* 5¼*d.* to 204*l.* 14*s.* 11*d.*

6

1228 9 6

Anſwer 1228 flor. 9 ſtiv. 6 pence.

By the rule of practice, I firſt reduce 110*l.* 10*s.* 6*d.* ſterling, into Flemiſh money, and it produces 196*l.* 17*s.* 5¼*d.* ¾. Then by the rule of three, as before directed, I bring this money into current money, and it produces 204*l.* 14*s.* 11*d.* omitting the fraction of a farthing; and this multiplied by 6, the number of florins in a pound Flemiſh, produces 1228 florins 9 ſtivers 6 pence, for the anſwer.

Qu. 2. In 912*l.* 16*s.* ſterling, how many rix-dollars current, agio 4¾ per cent. exchange 36*s.* 1½*d.*?—*Anſ.* 4141 rix-dollars.

Qu. 3. In 1876 florins 7 ſtivers 1 grote, current, agio 5¾ per cent. how many pounds ſterling, exchange at 35*s.* 11*d.*? —*Anſ.* 165*l.* 1*s.* ¼*d.*

In Hamburgh

Accompts are kept in marks and ſols lub, but the exchange is by the pound ſterling, as in Holland.

2 deniers gros	make one	ſol lub
6 ſols lub		ſol gros
16 ſols lub		mark
2 marks		drittle, or Hamburgh dollar
3 marks		rix-dollar
7½ marks		livre gros, or pound Flemiſh

The exchange with this place has moſtly been from 32 to 35*s.* Flemiſh per pound ſterling, and agio from 18 to 20 per cent.

Example 4. In 886 marks 12 ſols lub banco, how many pounds ſterling, exchange 36 ſols gros 2 deniers per pound ſterling?

36 ſols gros 2 den.
12
434

886 marks 12 ſols lub
16
5318
887
14188 ſols lub
2
434)28376(65*l.*
2604
2336
2170
166
20
434)3320(7*s.*
3038
282
12
434)3384(7*d.*
3038
346
4
434)1384(3*f.*
1302
82

Anſwer 65*l.* 7*s.* 7¾*d.*

Here the marks and ſols are reduced into deniers, as are alſo thoſe in the courſe of exchange; then the courſe of exchange I make the diviſor, and the foreign coin the dividend, and the quotient is 65*l.* 7*s.* 7¾*d.* for the anſwer.

Qu. 5. In 1072*l.* ſterling, how many marks, the exchange at 36*s.* 4*d.* Flemiſh per pound ſterling?—*Anſ.* 14606 marks.

Qu. 6. In 1686*l.* 2*s.* 5½*d.* ſterling, how many rix-dollars and ſols lub current, exchange at 33 ſols gros 9¼ deniers, agio 18¼ per cent.?—*Anſwer* 8434 rix-dollars 23 ſols lub.

In France

Accompts are kept in livres, ſols, and deniers; and the exchange is by the ecu, or crown tournois.

The exchange has moſtly been from 30*d.* to 32*d.* ſterling per ecu *.

12 deniers	make one	ſol
20 ſols		livre
3 livres		crown tournois, or ecu
10 livres		piſtole
24 livres		Louis d'or, or guinea

Example 7. In 2465 livres 12 ſols 9 deniers, how many pounds ſterling, exchange 31½*d.* per ecu?

Here the livres, ſols, and deniers are brought into crowns tournois, by dividing by 3, and that quotient divided by the aliquot parts of the courſe of exchange gives the anſwer.

		liv.	ſol.	den.
	3)	2465	12	9
30*d.*	8)	821	17	7
1½*d.*	20)	102	14	8¾
		5	2	8¼
Anſwer		£107	17	5

Qu. 8. In 1543*l.* 15*s.* ſterling, how many French piſtoles, exchange 30⅞*d.* per ecu?—*Anſwer* 3600.

In Spain

Accompts are kept in piaſtres, rials, and marvadies; and the courſe of exchange is by the piaſtre, and is generally from 38 to 42*d.* ſterling per piaſtre.

4 marvadies vellon, or 2⅛ marvadies of plate	make one	quarta
2½ quartas, or 34 marvadies vellon		rial vellon
16 quartas, or 34 marvadies of plate		rial of plate, or dollar
8 rials of plate		piſo, piaſtre, or piece of eight
5 piaſtres		Spaniſh piſtole
2 piſtoles		doubloon

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* The courſe of exchange mentioned in every part of this ſection means the exchange as it generally ſtood before the late unhappy affairs on the Continent.

Example 9. In 8746 rials of plate, how many pounds ſterling, exchange at 41½*d.* per piaſtre or piſo?

Here the rials are brought into piaſtres or piſos, by dividing by 8, as the exchange is by that piece, and the remainder of the work is wrought as the 7th example.

		rials.		
	8)	8746	0	0
For 40*d.*	6)	1093	5	0
For 1*d.*	40)	182	4	2
For ½*d.*	2)	4	11	1¼
		2	5	6½
Anſwer	£	189	0	9¾

Qu. 10. In 178*l.* 5*s.* 11*d.* ſterling how many rials of plate, &c. exchange at 40½*d.* per piaſtre?—*Anſwer* 8452 rials plate 8 quartas.

In Portugal

Accompts are kept in reas and milreas; and the exchange is by the milrea, and generally from 60 to 67*d.* per milrea.

400 reas } make one { cruſado
1000 reas, or } make one { milrea
2½ cruſadoes }

Example 11. In 1864 milreas 108 reas, how many pounds ſterling, exchange at 5*s.* 9*d.* per milrea?

For 5*s.*	4)	1864	0	0
For 6*d.*	10)	466	0	0
For 3*d.*	2)	46	12	0
		23	6	0
For 108 reas take		0	0	7¼
Anſwer	£	535	18	7¼

Here the milreas are brought into pounds and ſhillings by practice.

Qu. 12. In 1500*l.* 19*s.* how many cruſadoes, the exchange at 64½*d.* per milrea?—*Anſwer* 14045 cruſadoes.

In Venice and Leghorn

The accompts are kept in dollars, ſoldi, and denari. The exchange is by the ducat and piaſtre, and from 52*d.* to 54*d.* per ducat, and from 45*d.* to 54*d.* per piaſtre, and the agio 20 per cent.

12 denari	make one	ſoldi
20 ſoldi		lira, or piaſtre of Leghorn
5⅙ ſoldi		groſſe
24 groſſe		ducat

Example 13. In 846 piaſtres 10 ſoldi 8 denari, how many pounds ſterling, exchange 48½*d*. per piaſtre?

		pia.	ſol.	den.
For 40*d*.	6)	846	10	8
For 8*d*.	5)	141	1	9⅓
For ½*d*.	16)	28	4	4
		1	15	3¼
Anſwer	£	171	1	4½

Qu. 14. In 831*l*. 14*s*. 8*d*. ſterling, how many ducats, &c. current, exchange at 53*d*. per ducat, agio 20 per cent.?—*Anſ*. 4519 ducats 14 groſſe.

The foregoing examples will be found ſufficient to inſtruct the learner in the method of performing this rule; I ſhall therefore inſert the remainder of the tables of foreign coin, and a few queſtions with their anſwers, omitting the operation for the practice of the learner.

In Ruſſia

Accompts are kept in rubles and copecs; and the exchange is by the ruble; and when made immediately with London, is from 4*s*. to 5*s*. per ruble; but when made by the way of Hamburgh or Amſterdam is from 48 to 50 ſtivers per ruble.

3 copecs	make one	altine
10 copecs		grivena
25 copecs		polpolitin
2 polpolitins		politin
2 politins		ruble
2 rubles		ducat

Qu. 15. In 2337 rubles 73 copecs, how many pounds ſterling; exchange, by way of Amſterdam, at 122 copecs per rix-dollar of 50 ſtivers; and exchange between Holland and England at 34*s*. 7*d*. Flemiſh per pound ſterling?—*Anſ*. 230*l*. 17*s*. 3*d*.

Qu. 16. For 943*l.* 14*s.* 8*d.* paid in London, how many rubles muſt be received at Peterſburgh, exchange by way of Holland at 34*s.* 9*d.* Flemiſh per pound ſterling, and exchange from Holland to Peterſburgh at 50 ſtivers per ruble?—*Anſ.* 7870 rubles 36 copecs.

In Poland and Pruſſia

Accompts are kept in florins, gros, and penins. The exchange is by the way of Holland, and from 240 to 295 gros per pound Flemiſh.

18 penins	make one	gros
18 gros		oort
30 gros		florin, or Poliſh gilder
3 florins		rix-dollar
2 rix-dollars		gold ducat

Qu. 17. In 437*l.* 17*s.* 4*d.* ſterling, how many rix-dollars Poliſh, exchange 290 gros per pound Flemiſh, and 34*s.* 4*d.* Flemiſh per pound ſterling?—*Anſ.* 2422 rix-dollars 4 gros 13 penins.

In Sweden

Accompts are kept in copper dollars, and oorts or ſilver dollars. The exchange is by the copper dollar, and moſtly from 46 to 50 copper dollars per pound ſterling.

8 penins	make one	runſtychen
3 runſtychens		ſtiver
8 ſtivers		marc
10 ſtivers and 2 runſtychens, or 32 runſtychens		copper dollar
3 copper dollars and 32 ſtivers, or 96 runſtychens, or 4 marcs		ſilver dollar
24 marcs		copper rix-dol.

Qu. 18. In 5838 ſilver dollars 9 runſtychens 3½ penins, how many pounds ſterling, exchange at 49 copper dollars per pound ſterling?—*Anſ.* 357*l.* 8*s.* 8¼*d.*

In Ireland, America, and the Weſt Indies,

Accompts are kept in pounds, ſhillings, and pence, as in England, but the courſe of exchange between England and Ireland is from 5 to 12 per cent. in favour of England, and the courſe of exchange of the paper of America and the Weſt Indies is never at any certain ſtandard.

Qu. 19. If 2172*l.* be remitted to Ireland, how much money ſterling may be drawn for it, exchange at 8 per cent.? —*Anſ.* 2345*l.* 15*s.* 2¼*d.*

Qu. 20. A merchant ſells goods in London, and remits to his correſpondent in Boſton, the value amounting to 1151*l.* 19*s.* how much muſt the merchant at Boſton receive in paper currency, exchange at 33⅓ per cent. in favour of England?—*Anſ.* 1535*l.* 18*s.* 8*d.*

Queſtions of the nature of theſe two are reſolved by the rule of three: thus, in the 19th queſtion, I ſay, if 100*l.* require 108*l.* what will 2172*l.* require? and the anſwer is 2345*l.* 15*s.* 2¼*d.*

The *arbitration of exchange* is the method of finding the courſe of exchange between any two places, by having the courſe of exchange between each of theſe two places and a third place.

By comparing the par of exchange thus found with the courſe of exchange, a perſon can tell which way to draw or remit his money to the moſt advantage.

All queſtions in arbitration of exchange are reſolved by the rule of three direct, comparing the courſe of exchange between any two places, with that of a third place to a fourth.

Example 21. If the exchange between London and Paris be 33*d.* per ecu, and the exchange between London and Amſterdam be 34*s.* 6*d.* per pound ſterling, what is the par of arbitration between Paris and Amſterdam?

As 240*d*. London is to 34*s*. 6*d*. Amſterdam, ſo is 33*d*. Paris to

```
                12
               ———
               414
                33
               ———
              1242
             1242
             ————
     24,0)1366,2(56d.
             120
             ———
              166
              144
              ———
              222
                4
              ———
      24,0)88,8(3f.
              72
              ———
              168     Anſwer 56¾d. 7/10 Flemiſh per ecu
              ═══                  for Paris.
```

Here I ſay, as 240*d*. the peace in a pound ſterling of London, is to 34*s*. 6*d*. the Flemiſh exchange, ſo is 33*d*. the exchange for an ecu, or crown tournois of Paris, to 56*d*. 3 $\frac{7}{10}$ farthings the Flemiſh exchange for an ecu, and is the courſe of exchange between Paris and Amſterdam.

This example will be ſufficient, as the rule never varies, though the courſe of exchange between ſeveral countries be given to find a proportional courſe between any two; in which caſe they may be reſolved into as many queſtions as is neceſſary in the rule of three, and all the firſt numbers multiplied together for a diviſor, and the ſecond and third numbers in each queſtion multiplied together for a dividend, then the quotient will be the anſwer.

Qu. 20. A merchant in London is drawn upon by his correſpondent in Ruſſia for money to the amount of 12500 rubles; the exchange between England and Ruſſia being at 50*d*. per ruble; between Ruſſia and Holland 90*d*. per ruble; and between England and Holland 36*s*. 4*d*. per pound ſterling; which is the moſt advantageous method for the London merchant to pay by; directly to Ruſſia or by way of Holland, and what is the advantage?—*Anſwer*, the London merchant will gain 23*l*. 17*s*. 10¼*d*. by making payment by way of Holland.

SECT.

SECT. XX.

OF THE RELATION OF NUMBERS.

Any ſet of numbers that conſtantly increaſe or decreaſe by a common difference are ſaid to bear a relation to each other, which relation is called progreſſion, and is divided into two kinds: arithmetical progreſſion and geometrical progreſſion.

Arithmetical progreſſion is, when any ſet or ſeries of numbers conſtantly increaſe or decreaſe by a given number, called from thence their common difference; ſuch are the natural orders of numbers, 1, 2, 3, 4, 5, 6, 7, 8, &c. each of which increaſes by 1; and the ſame in inverted order 8, 7, 6, 5, 4, &c. decreaſe by 1; 1 is therefore their common difference.

Again, the numbers 2, 5, 8, 11, 14, 17, &c. increaſe by 3; and 27, 23, 19, 15, 11, 7, &c. decreaſe by 4: therefore 3 is the common difference of the former, and 4 that of the latter.

The numbers themſelves that form the ſeries are called the *terms* of the progreſſion.

Of every ſeries of arithmetical progreſſion having any three of the five following terms the other two may be found readily;

1, the firſt term or number }
2, the laſt term or number } called the *extremes*.
3, the number of terms.
4, the common difference.
5, the ſum of all the terms.

PROBLEM I.

The firſt and laſt terms, and number of terms, being given, to find the ſum of all the terms, and the common difference.

Rule. To find the ſum of all the terms, multiply the ſum of the extremes by the number of terms, and half the product will be the anſwer; and to find the common difference of the terms, divide the difference of the extremes by the number of terms made leſs by 1, and the quotient will be the anſwer.

Example 1. What is the ſum of the ſeries, and the common difference of that progreſſion, whoſe firſt term is 3, laſt term 67, and number of terms 33?

laſt term 67 — laſt term 67 — 33
firſt term 3 — firſt term 3 — 1
70 — difference 64 — 32
number of terms 33
210 — 32)64
210 — 2 common difference
2)2310
1155 *Anſwer*, ſum of the ſeries.

In the firſt operation, 70, the ſum of the extremes, is multiplied by 33 the number of terms, and half the product 1155 is the ſum of the ſeries, or ſum of all the numbers, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, added together.

In the ſecond operation the difference of the two extremes 67 and 3, or 64, is divided by 32, which is one leſs than the number of terms, and the quotient 2 is the common difference.

PROBLEM II.

Having the two extremes and the common difference, to find the number of terms.

Rule. Divide the difference of the extremes by the common difference, and 1 added to the quotient will be the number of terms.

Example 2. What is the number of terms of that progreſſion whoſe extremes are 3 and 67, and common difference 2?

Thus 64, the difference of the extremes, divided by 2, the common difference, quotes 32, to which adding 1 there is 33, the number of terms; as may be ſeen annexed.

$$\begin{array}{rl} 67 & \rbrace \text{ extremes} \\ \underline{3} & \\ 2)\underline{64} & \text{difference} \\ 32 & \\ \underline{1} & \\ \underline{\underline{33}} & \text{number of terms.} \end{array}$$

In every arithmetical progreſſion the ſum of any two terms is equal to the ſum of any two other terms, taken at an equal diſtance from the former, and on oppoſite ſides: thus, in the foregoing progreſſion, 15 and 19 is equal to 23 and 11; viz. 34; and 31 and 33 is equal to 23 and 41, viz. 64. And the double of any one term is equal to the ſum of any two terms taken on each ſide of it, and at an equal diſtance from it; thus the double of 25 is equal to 19 and 31, viz. 50.

Qu. 3. How many ſtrokes does an Engliſh clock ſtrike in 8 days?—*Anſ.* 1248 ſtrokes.

Qu. 4. If a traveller go a journey of 10 days, travelling 3 miles the firſt day, and increaſing three miles every day, how many miles will he travel in the 10 days? and how many miles the laſt day?—*Anſ.* 165 miles in the whole, and 30 miles the laſt day.

Geometrical Progreſſion.

When any ſeries of numbers increaſe or decreaſe by a conſtant multiplication or diviſion, they are ſaid to be in *geometrical progreſſion.*

Thus the numbers 4, 8, 16, 32, 64, &c. and 243, 81, 27, 9, 3, 1, are in geometrical progreſſion, the former *increaſing* by multiplying each preceding number by 2, and the latter *decreaſing* by dividing each preceding number by 3.

Thus, in geometrical progreſſion, the numbers are increaſed by multiplication, and decreaſed by diviſion; whereas in arithmetical progreſſion they are increaſed by addition and decreaſed by ſubtraction.

The number in geometrical progreſſion by which the ſeries are multiplied or divided, is called the *ratio;* whereas in arithmetical progreſſion the number by which the ſeries are increaſed or diminiſhed is called the *common difference.*

In geometrical progreſſion, as in arithmetical progreſſion, having any three of the following terms, the other two may be readily found.

1, the firſt term } the extremes.
2, the laſt term }
3, the number of terms.
4, the ratio.
5, the ſum of all the terms.

PROBLEM I.

Having the firſt and laſt term and the ratio, to find the ſum of the terms.

Rule. Multiply the laſt term by the ratio, and from the product ſubtract the firſt term, and the remainder divided by one leſs than the ratio will quote the ſum of the ſeries.

Example 1. What is the ſum of the ſeries of a geometrical progreſſion whoſe extremes are 1 and 65536, and the ratio 4?

laſt term 65536
ratio 4
262144
firſt term 1
one leſs than the ratio 3)262143
the ſum of the ſeries 87381

Here the laſt term is multiplied by the ratio 4, the firſt term 1 ſubtracted from the product, and the remainder divided by 3, which is 1 leſs than the ratio, and the quotient 87381 is the ſum of the ſeries.

Qu. 2. What is the ſum of a ſeries of numbers, in geometrical progreſſion, whoſe extremes are 1024 and 59049, and the ratio 3?—*Anſwer* 88061,

PROBLEM II.

Having the firſt term and the ratio, to find any other term required.

Rule. Write down a few of the leading terms of the ſeries, and place their indices over them, beginning with a cypher *.

Then add any of thoſe indices together, which will make an index an unit leſs than the number which expreſſes the place of the term required.

Multiply the terms of the ſeries together belonging to thoſe indices, and the product will be a dividend, to be divided by the product of the firſt term, multiplied by a number an unit leſs than the number of terms multiplied; and the quotient will be the anſwer.

Example 3. What is the laſt term of that geometrical ſeries whoſe firſt term is 3, ratio 2, and number of terms 10?

Indices	0,	1,	2,	3,	4,	5,	6,
Leading terms	3,	6,	12,	24,	48,	96,	192.

$$\begin{array}{rl} 48 & \qquad 4 \\ \underline{96} & \qquad \underline{5} \\ 288 & \qquad \underline{\underline{9}}\ \text{index to the 10th term.} \\ \underline{432} & \\ 3)\overline{4608} & \\ \underline{\underline{1536}} & \text{the } \textit{Anſwer}\text{, or the 10th term.} \end{array}$$

---

* The indices of the terms in this rule are numbers expreſſing the place of each term according to the natural order of numbers; thus the index of the firſt term is 0, that of the ſecond term 1, the third term 2, the fourth 3, &c.; but when the firſt term of the ſeries is equal to the ratio, the indices muſt begin with an unit, and the product of the terms will be the anſwer, without dividing by the product of the firſt term, as directed above.

In this example, the indices 4 and 5 added together are 9, which is 1 leſs than the number of the term ſought; then the term belonging to theſe indices, 48 and 96, are multiplied together for a dividend, the firſt term 3 multiplied by 1 leſs than the number of terms, which are multiplied together, is the diviſor; and the quotient 1536 is the anſwer.

Note. The number of terms multiplied together in this example are 2; therefore 3 the firſt term multiplied by 1 leſs than 2, or 1 only, produces 3.

Example 4. What is the 15th term of a ſeries whoſe firſt term is 3, and ratio 3?

```
1,  2,  3,  4,  5,   6,  Indices
3,  9, 27, 81, 243, 729, Leading terms
       729              6
       243              4
      ----              5
      2187             --
     2916              15 index to the 15th term
    1458               ==
   -------
    177147
        81
   -------
    177147
   1417176
  --------
  14348907 the 15th term, or Anſwer.
  ========
```

Here, as the firſt term is equal to the ratio, the indices muſt begin with 1, inſtead of 0; and as many indices muſt be taken as will make the entire index to the term ſought, viz. 15, and the product of the terms muſt not be divided, as in the former caſe.

Qu. 5. What is the laſt term of that geometrical ſeries where the firſt term is 1, ratio 2, and number of terms 23? —*Anſ.* 4194304.

Qu. 6. A perſon having an elegant houſe to diſpoſe of, offered it to ſale on the following terms: for the firſt window the purchaſer was to pay 1 farthing, for the ſecond window 2 farthings, for the third window 4 farthings, and ſo on, doubling the price for every ſucceeding window; there were 32 windows in the houſe: what would be the price of the houſe at that rate?—*Anſ.* 4,473,924*l.* 5*s.* 4*d.*

Qu. 7. An Indian of the name of Seſſa having invented the game of cheſs, ſhewed it to his prince Shehram, who was ſo pleaſed with the invention, that he bade Seſſa ſay what he would have as a reward for his ingenuity. Seſſa requeſted 1 grain of wheat for the firſt ſquare on the cheſs-board, 2 grains of wheat for the ſecond ſquare, 4 for the third, 8 for the fourth, and ſo on, doubling the quantity of grains of wheat for every ſucceeding ſquare: now the whole number of ſquares on the cheſs-board is 64. Suppoſing a buſhel to contain 640,000 of theſe grains, how many ſhips would it require to export the whole quantity of wheat, each ſhip being 100 tons burden?—*Anſ.* 7,205,759,403 ſhips, and about $\frac{4}{5}$ of a ſhip.

There are many other queſtions in progreſſion which favour more of curioſity than real utility. This rule was much admired formerly, before the nature of numbers was ſo well underſtood as at preſent, on account of its ſurpriſing increaſing power: it is, however, of little uſe, except in calculating tables, &c.

SECT. XXI.

OF EVOLUTION, OR THE EXTRACTION OF ROOTS, SQUARE AND CUBIC.

BEFORE the learner proceeds to extraction of roots, he ſhould underſtand the nature of involution, or the raiſing of a number to any power required.

A *power* is the product of any number multiplied into itſelf any certain number of times.

If a number be multiplied into itſelf only once, the product is called the ſecond power, or the *ſquare*; thus 4 is the ſecond power, or ſquare of 2, being the product of 2, multiplied by 2; and 9 is the ſecond power of 3. But if a number be multiplied twice into itſelf continually, the product is called the third power, or *cube*; thus 27 is the cube, or third power of 3; and 64 is the third power of 4.

When a number is multiplied 3, 4, 5, or 6 times into itſelf, &c. the product is called the fourth, fifth, ſixth, or ſeventh power reſpectively.

Example 1. What is the fifth power of 6?

6
6
36 ſecond power
6
216 third power
6
1296 fourth power
6
7776 fifth power

Fractions are raiſed to powers in the ſame manner as whole numbers; multiplying them as taught in multiplication of fractions.

A TABLE

OF THE

Firſt nine Powers of each of the nine ſingle Numbers.

1ſt	*2d*	*3d*	*4th*	*5th*	*6th*	*7th*	*8th*	*9th*
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

The uſe of this table is, to find any power leſs than the 10th power of any number under 10; thus, to find the 7th power of 6, I look in that column which has 7 at the top, and in that line which has 6 on the left hand, and in the junction of theſe two lines is 279936.

The *root* of any number is that number which, being multiplied into itſelf a certain number of times, will produce the given number; thus 2 is the ſquare root of 4, becauſe 2 multiplied by 2 produces 4; and 3 is the cube root of 27, for 3 multiplied 3 times into itſelf continually produces 27.

There are many numbers the given roots of which can never be exactly found, though by decimal fractions we may approximate towards it to any degree of exactneſs; but the power of a given number can always be found exactly.

Thoſe roots, which only approximate towards the truth, are called *ſurd* roots, and thoſe which perfectly expreſs the root of the given power are called *rational* roots: thus the ſquare root of 5 is a ſurd root, as it cannot be exactly expreſſed; but the cube-root of 8 is a rational root, being exactly 2.

To extract the Square Root.

Rule 1. Divide the given number into periods of two figures each, by placing a point over every other figure, beginning with that in the place of units.

2. Find the greateſt ſquare root of the firſt period, and place it on the right hand of the given number, like a quotient in diviſion.

3. Subtract the ſquare of this root from the aforeſaid period, and to the remainder bring down the next period, and call it the reſolvend.

4. Double the figures in the quotient for a new diviſor to the reſolvend, and find how often this diviſor is contained in the reſolvend, excluſive of the unit figure, and place the anſwer both in the quotient, and on the right hand of the diviſor.

5. Then

5. Multiply the diviſor thus increaſed by the laſt quotient figure, and ſubtract the product from the reſolvend, and to the remainder bring down the next period for a new reſolvend.

6. Double the figures in the quotient for a new diviſor to this laſt reſolvend, and ſeek how often it is contained therein, excluſive of the unit figure, placing the anſwer both in the quotient and diviſor as before: and proceed in the ſame manner through each new reſolvend, and the figures placed in the quotient will be the anſwer, or ſquare root of the given number.

Example 1. What is the ſquare root of 5934096?

```
            . . . .
            5934096(2436 root
            4
        44 |193
         4 |176
       483 | 1740
         3 | 1449
      4866 | 29196
         6 | 29196
The Anſwer     2436
```

After dividing the given number into periods of two figures each by the points, I find the neareſt ſquare foot of 5 (as that is pointed for the firſt period), which is 2; I therefore place 2 in the quotient, and the ſquare of 2, which is 4, I ſubtract from this firſt period 5, and to the remainder 1 I bring down the next period 93; thus I have 19 for a new reſolvend (for the unit figure 3 is to be neglected), then I take the double of the figure 2 in the quotient for a diviſor, and ſeeking how often it is contained in 19, the reſolvend, the anſwer is 4, which I place in the quotient, and alſo on the right hand of the diviſor 4; then I multiply the diviſor thus increaſed by 4, and the product 176 I place under and ſubtract from the laſt reſolvend, and to the remainder 17 I bring down the next period 40; thus I have 174 for a new reſolvend, and 48 the double of the figures in the quotient for a new diviſor; then I ſeek how often the diviſor 48 is contained in 174, and the anſwer 3 I place both in the quotient and diviſor as before, and multiply the diviſor alſo thereby, and the product 1449 placed under and ſubtracted from the reſolvend, there remains 291, to which bringing down the laſt period 96, I have 2919 for the laſt reſolvend, and the double of 243 or 486 for a diviſor: this laſt quotient is 6, which I place in the quotient and diviſor, and multiply the diviſor thereby, and the product is equal to the laſt reſolvend: thus 2436 is the anſwer.

Qu. 2. What is the ſquare root of 152399025?—*Anſwer* 12345.

Qu. 3. What is the ſquare root of 22071204?—*Anſwer* 4698.

To extract the Cube Root.

Rule 1. Divide the given number into periods of three figures each, by placing a point over every third figure, beginning with the unit figure.

2. Find the greateſt cube in the firſt period, and place its cube root on the right hand of the given number.

3. Subtract the cube from the ſaid period; and to the remainder bring down the next period, which is called the reſolvend.

4. Place three times the root under the reſolvend, and alſo three times the ſquare of the root, the latter being removed one place to the left; theſe two added together form a diviſor.

5. Seek how often this diviſor is contained in the reſolvend, excluſive of the place of units, and place one leſs than the anſwer in the quotient.

6. Under the diviſor place the cube of the laſt quotient figure, the ſquare of it multiplied by three times the reſt of the quotient, and three times the root multiplied by the ſquare of the quotient; each of the two latter numbers being removed one place farther towards the left hand than the

foregoing, and the ſum of theſe three numbers is called the *ſubtrahend*.

7. Then ſubtract this ſubtrahend from the reſolvend, and to the remainder bring down the next period for a new reſolvend, with which proceed as before, and ſo on till the work be finiſhed.

Example 4. What is the cube root of 164566592?

	164566592(548	
	125	cube of the firſt period
765)	39566	firſt reſolvend
	15	three times the root 5
	75	three times the ſquare of 5
	765	firſt diviſor
	64	cube of 4
	240	ſquare of 4 multiplied by 3 times 5
	300	three times 4 multiplied by the ſquare of 5
	32464	ſubtrahend
87642)	7102592	ſecond reſolvend
	162	three times 54
	8748	three times the ſquare 54
	87642	ſecond diviſor
	512	cube of 8
	10368	ſquare of 8 multiplied by 3 times 54
	69984	three times 8 multiplied by the ſquare of 54
	7102592	ſecond ſubtrahend

Anſwer 548 the cube root required.

Qu. 5. What is the cube root of 389017?—*Anſ.* 73.

Qu. 6. What is the cube root of 405 $\frac{28}{125}$?—*Anſ.* $7\frac{2}{5}$.

There are many other methods of extracting the cube root given by different mathematicians; but none of which I have ſeen are ſo ſimple and eaſy to be remembered as the method here laid down.

After what has been ſaid, it need hardly be mentioned, that the extraction of the ſquare root is proved by multiplying the root into itſelf; and the extraction of the cube root, by multiplying the root three times into itſelf.

CHAP. IV.

OF DECIMAL ARITHMETIC.

SECT. I.

REDUCTION OF DECIMAL FRACTIONS.

A DECIMAL fraction is that fraction whoſe denominator has an unit in the firſt place on the left hand, with as many cyphers annexed as neceſſary: thus $\frac{5}{10}$, $\frac{25}{100}$, $\frac{48}{1000}$, $\frac{572}{10000}$, &c. are decimal fractions. But theſe fractions are uſually expreſſed in writing without a denominator, by writing the numerator, and prefixing as many points or cyphers before it on the left hand, as there are more places of figures in the denominator than in the numerator. Thus the foregoing fractions are written .5, .25, .048, .0572, and expreſſed five tenths, twenty-five hundredths, forty-eight thouſandths, &c.

Cyphers placed on the right hand of decimal fractions make no alteration in their value: thus, if a cypher be annexed to the foregoing fraction .5, it will be then .50 fifty hundredths or half an integer, as before; if two cyphers .500 it will be five hundred thouſandths, or $\frac{1}{2}$, and the ſame of the others.

But cyphers placed on the left hand of a decimal fraction decreaſe their value, every cypher decreaſing it in a tenfold proportion: thus .5, .05, .005, are five tenths, five hundredths, five thouſandths parts reſpectively.

The value of every figure in a decimal fraction increaſes in a tenfold proportion from the firſt place on the right hand, as in whole numbers.

The figures in the firſt place of a decimal fraction on the left hand are called *primes*, thoſe in the ſecond place *ſeconds*, thoſe in the third place *thirds*, &c.

DECIMAL TABLES

OF

Coin, Weight, and Meaſure.

TABLE I.

Engliſh Coin.

1*l.* and 1*s.* the Integer.

Sh.	*Dec.*	*Sh.*	*Dec.*
19	,95	9	,45
18	,9	8	,4
17	,85	7	,35
16	,8	6	,3
15	,75	5	,25
14	,7	4	,2
13	,65	3	,15
12	,6	2	,1
11	,55	1	,05
10	,5		

Pence.	*Decimals*
6	,025
5	,020833
4	,016666
3	,0125
2	,008333
1	,004166

Farths.	*Decimals.*
3	,003125
2	,0020833
1	,0010416

TABLE II.

Engliſh Coin 1*s.*
Long Meaſ. 1 foot
Troy Weight 1oz.

Pence & Inches.	*Decimals.*
6	,5
5	,416666
4	,333333
3	,25
2	,166666
1	,083333

Farths.	*Decimals.*
3	,0625
2	,041666
1	,020833

TABLE III.

Troy Weight.

1lb. the Integer.

Penny-weight.	*Decimals.*
10	,041666
9	,0375
8	,033333
7	,029166
6	,025
5	,020833
4	,016666
3	,0125
2	,008333
1	,0041666

Grains.	*Decimals.*
12	,002083
11	,001910
10	,001736
9	,001562
8	,001389
7	,001215
6	,001042
5	,000868
4	,000694
3	,000521
2	,000347
1	,000173

1oz. the Integer.

Pennyweights the ſame as Shillings in the Firſt Table.

Grains.	*Decimals*
12	,025
11	,022916
10	,020833
9	,01875
8	,016666
7	,014583
6	,0125
5	,010416
4	,008333
3	,00625
2	,004166
1	,002083

TABLE IV.

Avoirdup. Wt.

112lb. the Integer

Qrs.	*Decimals.*
3	,75
2	,5
1	,25

Pounds.	*Decimals.*
14	,125
13	,116071
12	,107143
11	,098214
10	,089286
9	,080357
8	,071428
7	,0625
6	,053571
5	,044643
4	,035714
3	,026786
2	,017857
1	,008928

Ounces.	*Decimals.*
8	,004464
7	,003906
6	,003348
5	,002790
4	,002232
3	,001674
2	,001116
1	,000558

¼oz.	*Decimals.*
3	,000418
2	,000279
1	,000139

TABLE V.

Avoirdup. Wt.

1lb. the Integer.

Ounces.	*Decimals.*
8	,5
7	,4375
6	,375
5	,3125
4	,25
3	,1875
2	,125
1	,0625

Drams.	*Decimals.*
8	,03125
7	,027343
6	,023437
5	,019531
4	,015625
3	,011718
2	,007812
1	,003906

TABLE VI.

Liquid Meaſure.

1 Tun the Integer.

Gallons	*Decimals.*
100	,396825
90	,357141
80	,317460
70	,277777
60	,238095
50	,198412
40	,158730
30	,119047
20	,079365
10	,039682
9	,035714
8	,031746
7	,027777
6	,023809
5	,019841
4	,015873
3	,011904
2	,007936
1	,003968

Pints.	*Decimals.*
4	,001984
3	,001488
2	,000992
1	,000496

1 Hogſhead the Integer.

Gallons	*Decimals.*
30	,476190
20	,317460
10	,158730
9	,142857
8	,126984
7	,111111
6	,095238
5	,079365
4	,063492
3	,047619
2	,031746
1	,015873

Pints.	*Decimals.*
3	,005952
2	,003968
1	,001984

TABLE VII.

Meaſure.

Liquid. *Dry.*

1 Gall. 1 Quarter

the Integer.

Pints	*Dec.*	*Buſh.*
4	,5	4
3	,375	3
2	,25	2
1	,125	1

¼ *pts.*	*Dec.*	*Pk*
3	,09375	3
2	,0625	2
1	,03125	1

Decimals.	¼ *pecks*
,0234375	3
,015625	2
,0078125	1

Decimals.	*Pints.*
,005859	3
,003906	2
,001953	1

Caſe 1. To reduce a vulgar fraction to a decimal one of the ſame value.

Rule. Divide the numerator by the denominator, and the quotient will be the decimal fraction required.

If the numerator be too ſmall to be divided by the denominator, add one or more cyphers thereto, in which caſe as many places muſt be ſeparated from the quotient for decimals as the difference of the number of places of decimals in the dividend and the diviſor, and the remaining figures in the quotients (if any) are integers.

But if the number of places in the quotient be leſs than the required number of decimal places, as many cyphers muſt be prefixed on the left hand thereof as are neceſſary.

Example 1. What is the decimal fraction of a pound equal to ¼?

Here two cyphers muſt be added to the numerator, which divided by 4 quotes 75, which I mark for decimals, as there are two decimals added to the dividend, and the quotient is the ſeventy-five hundredth part of a pound, or $\frac{3}{4}$, equal to 15*s.*

4)300
75

Example 2. What is the decimal fraction of a pound equal to 9*d.*?—*Anſ.* .0375.

24,0)90000(.0375

As there are but three places of figures in the quotient, I place a cypher on the left thereof, for there ſhould be four figures pointed off for decimals, being ſo many cyphers annexed to 9; thus the quotient is $\frac{375}{10000}$ of a pound.

Example 3. What is the decimal fraction of a pound for 3 farthings?—*Anſ.* .003125.

96,0)300000,0(.003125

Here the vulgar fraction for 3 farthings is $\frac{3}{960}$; I therefore place as many cyphers as are neceſſary to the numerator 3, which in this caſe is 6 cyphers, and the quotient is 3125, but there muſt be 6 decimals in the quotient, as there are 6 in the dividend and none in the diviſor; I therefore prefix two cyphers to the left hand thereof as before directed.

Example 4. What is the decimal fraction of a year for 73 days?—*Anſwer* .2, equal to $\frac{73}{365}$.

365)730(.2

Caſe 2. To find the value of a decimal fraction in money, weight, or meaſure.

Rule. Multiply the decimal fraction by the number of parts of the next inferior denomination, and from the product cut off as many figures from the right hand ſide as there are decimal places in the fraction.

Then multiply theſe figures ſo ſeparated on the right hand by the number of parts of the next inferior denomination, and from the product cut off as many figures as this laſt multiplicand has decimal places, as before.

Proceed in the ſame manner through all the denominations, then the ſeparated figures on the left hand ſide will be the anſwer.

Example 1. What is the value of .564 of a pound ſterling?

Here I multiply the given decimal by 20, the number of parts of the next leſs denomination, viz. ſhillings, and from the product I ſeparate the three figures on the right hand, 280, as there are three places of decimals in the given number.

,564
20 ſhillings in a pound
11,280
12 pence in a ſhilling
3,300
4 farthings in a penny
1,440

Anſwer 11*s*. 3¼*d*. 440 of a farthing.

I then multiply theſe ſeparated figures 280 by 12, and from the product 3360 I ſeparate the three firſt figures 360, as before, to be multiplied by the parts of the next denomination, viz. 4; then the figures ſtanding on the left hand of theſe ſeparated figures will be the anſwer.—11*s*. 3¼*d*.

The ſame rule ſerves to diſcover the value of any other integers, having regard to the number of parts contained in each inferior denomination.

Thus to find the value of a decimal fraction of a pound troy, I firſt multiply the fraction by 12 to find the ounces, then the ſeparated figures by 20 to find the pennyweights, and laſtly by 24 to diſcover the grains.

Qu. 2. What is the value of .6725 of a cwt.?—*Anſwer* 2qrs. 19lb. 5oz.

Qu. 3. What is the value of .61 of a ton of wine?—*Anſ.* 2hhds. 27galls. 2qts. 1pt.

Caſe 3. To find the value of a decimal of a pound ſterling, by inſpection.

Rule. Double the firſt figure of the decimal on the left hand for ſhillings; and if the next figure be 5, or above 5, add 1 ſhilling more to it: then call the figures in the ſecond and third places farthings (after deducting 5 from the ſecond figure if neceſſary), and if the number of farthings be above 12 abate one, and if above 37 abate 2. The other figures in the fraction, if more, are neglected.

Example 1. Find the value of .564 of a pound by inſpection.

Here I take the double of 5, the firſt figure, for ſhillings, which is 10, alſo 1 ſhilling more for the 5 contained in the 6, and the remaining 14 (viz. 1 in the 6, and 4) I conſider as farthings, abating 1 becauſe they are above 12, which make 3¼*d.*: thus the decimal .564 of a pound is 11*s.* 3¼*d.* as may be ſeen in Caſe 4.

For the 5 take	10*s.*	
For the 5 in the 6 take	1	
For the 14 remaining	0	3¼
	11	3¼

Qu. 2. Find the value of .7880 of a pound by inſpection, —*Anſwer* 15*s.* 9*d.*

Qu. 3. Find the value of .14729 of a pound by inſpection, —*Anſ.* 2*s.* 11¼*d.*

Caſe 4. To find the decimal fraction of a pound equal to any given number of ſhillings and pence.

Rule. Write half the greateſt even number of ſhillings for the firſt figure in the decimal, and the number of farthings in the pence and farthings for the ſecond and third figures, obſerving to add 5 more to the ſecond figure, if the number of ſhillings be odd; alſo add one more to the third figure, if the farthings exceed 12, and add 2 if they exceed 37.

Example 1. Find the decimal fraction of a pound equal to 11*s.* 3¼*d.*

Here for the 11*s.* I take ½ of the next greateſt even number, which is 5, for the firſt figure of the decimal, and for the odd ſhilling, place 5 to be added to the ſecond figure; then for the 3¼*d.* I take 14 to be added to the ſecond and third numbers, as there are 13 farthings in 3¼*d.* and 1 is added, as the farthings are above 12; thus, the decimal is .564 of a pound.

For half 10 take	.5
For the odd ſhilling	.05
For the 3¼	.014
	.564

Qu. 2. Find the decimal equal to 18*s.* 4½*d.*—*Anſ.* .919.

Qu. 3. Find the decimal equal to 17*s.* 6¾*d.*—*Anſ.* .878.

SECT. II.

OF ADDITION OF DECIMALS.

Addition of decimal fractions is performed in the ſame manner as addition of whole numbers, except in the diſpoſition of the figures.

Rule. Place the figures exactly under each other, according to the value of their places, that is, the whole numbers (if any) under each other, as in addition of whole numbers, and the fractions are alſo to be placed according to their values, viz. primes under primes, ſeconds under ſeconds, &c.

Then find their ſum as in whole numbers, and point off as many places for decimals as are equal to the greateſt number of decimal places in any of the given numbers.

Example 1. What is the ſum of 22.5709, 1.03, 1492.001, 2.971, .00726?

Here the numbers are added together like whole numbers, and the number of places pointed off for decimals are 5, equal to the greateſt number of decimal places in any of the given numbers. The figures on the left hand of the decimal point are whole numbers or integers.

22.5709
1.03
1492.001
2.971
.00726
1518.58016

More Examples.

2.219	22.179	224.010
24.002	.025	.120
.125	.3006	2.24
75.002	41.2278	112.01
100.1002	112.002	7.8602
201.4482	175.7344	346.2402

SECT.

SECT. III.

OF SUBTRACTION OF DECIMALS.

DECIMAL fractions are ſubtracted in the ſame manner as whole numbers, but the numbers are placed and the decimals pointed off according to the rule given in the foregoing ſection.

Example 1. What is the difference of 247.0729 and 3746.805732?

Here the uppermoſt number conſiſts of 6 places of decimals, wherefore the remainder ſought muſt have as many decimal places; in every other reſpect it is wrought like ſubtraction of whole numbers.

3746.805732
247.0729
3499.732832

Examples.

20.0192	928.12004	3.7031
14.129732	400.002	.002397
5.889468	528.11804	3.700703

SECT. IV.

OF MULTIPLICATION OF DECIMALS.

RULE. Multiply the given numbers one by the other, as in whole numbers.

Then point off as many figures for decimals as there are decimal places in both the numbers; but if there be not ſo many places in the product, as many cyphers muſt be prefixed on the left hand thereof as will make up the number of decimal places required.

Example 1. Multiply 2·03271 by .0056.

In this example the number of decimal places in both the multiplier and multiplicand is 9; but the product conſiſts of only 8 figures, therefore I muſt prefix one cypher on the left hand thereof, which makes the true product.

2.03271
.0056
1219626
1016355
.011383176

Examples.

27.02943
4.30020
540 58860
810882 900
10811772
116.23195 48860

2.20434
.43970
154 30380
1983 906
6613 02
88173 6
.96924 82980

Thus it appears that in multiplication of decimal fractions there is no neceſſity for placing thoſe figures of the ſame value under each other, as in the two foregoing rules *.

SECT. V.

OF DIVISION OF DECIMALS.

THIS rule is alſo performed as diviſion of whole numbers, but from the quotient point off as many figures for decimals as the decimal places in the dividend exceed thoſe in the diviſor. But if the places in the quotient be not ſo many as ſhould be pointed off for decimals, as many cyphers muſt be placed on the left hand as neceſſary.

---

* There is a contracted method of working both this and the following rule given by many writers on arithmetic. They are, however, not very ſimple, and may very well be ſuperſeded, by omitting ſome of the decimals in both numbers, and working according to the general rule.

If at any time there be a remainder, and if it be required to have an anſwer to a greater degree of exactneſs, as many cyphers may be affixed to the right hand of the dividend as is thought proper.

Example 1. Divide 1836.88305 by 23. 15.

```
23.15)1836.88305(79.347
      1620.5
       216 38
       208 35
         8 033
         6 945
         1 0880
           9260
           16205
           16205
```

In this example there are three more decimal places in the dividend than in the diviſor; there muſt therefore be three decimals in the quotient.

Examples.

```
2
146)27.0024(.1849
    146
    12 40
    11 68
       722
       584
       1384
       1314
         70
```

```
3
1027)472.0000(4595
     4108
      61 20
      51 35
       9850
       9243
        6070
        5135
         935
```

In the ſecond example there are 4 decimal places in the dividend, and none in the diviſor, therefore there muſt be 4 places of decimals in the quotient.

And in the third example as the number of decimal places in the diviſor and dividend are equal, being 4 in each, there muſt be no decimals in the quotient.

Every other rule in arithmetic may be performed by decimal fractions as well as by whole numbers, as the ſame rule ſerves for both; thus in the rule of three direct in decimals, the ſecond and third numbers are multiplied together for a dividend, and the firſt number is the diviſor as in whole numbers; and the like may be obſerved of every other rule.

CHAP.

CHAP. V.

OF STOCK-HOLDING.

SECT. I.

THE NATURE OF THE PUBLIC STOCKS.

THE public ſtocks of Great Britain is an invention in imitation of the ſtate of Florence, which, in the year 1344, owing 60,000*l.* and being unable to pay the principal, converted the debt into a public ſtock, transferable from one individual to another, with intereſt at 5 per cent.: theſe public ſtocks were called a *bank* or *mount*, and the prices varied according to the ſtate of public affairs.

In imitation of this, ſoon after the revolution in Great Britain, the expenſes of ſettling the new eſtabliſhment, carrying on long wars on the Continent for reducing the French monarchy, ſecuring the Dutch barrier, ſettling the Spaniſh ſucceſſion, &c. increaſed to ſuch a degree, that it was thought adviſable to draw upon poſterity, rather than reduce the ſtate by an immediate payment.

It was therefore the policy of miniſters of that day to raiſe a public ſtock with transferable ſhares, thus converting the debt into a new ſpecies of property, for the ſecurity of which Parliament pledged their faith; and this example has been cloſely followed ever ſince.

This ſyſtem laid the foundation of what is called the national debt, to which every ſtock-holder is a creditor.

Thus the property in the kingdom in idea has been greatly increaſed, compared to that of former times; but if we inquire into the nature of this property, we ſhall find it exiſts only in name, in paper, and public faith; but that is ſufficient for the public creditor to rely upon, for the pledge is nothing leſs than the land, the trade, and perſonal induſtry of the ſubject, from which ſources the money muſt ariſe to pay the intereſt; in theſe, therefore, the property of the public creditors does really exiſt, conſequently the value of the land, the trade, and induſtry of the ſubject, are diminiſhed in proportion as the debt increaſes.

To demonſtrate this, if a debtor owes 100*l.* the creditor has a demand upon him to that amount, and though he may not call upon him for the principal, yet ſo much of the debtor's eſtate is always transferable to the creditor; and the ſame principle holds good with regard to the public creditor.

Therefore the property of every ſtockholder conſiſts in a certain portion of the national debt.

To pay the intereſt, and, if required, the principal of this debt, certain taxes are appropriated: theſe taxes were originally ſeparate and diſtinct funds, and each of them was a ſecurity for a certain ſum advanced; they were afterwards united together in three capital funds, which are the principal funds of any account, called the *general fund*, the *aggregate fund*, and *South Sea fund;* and the funds thus united together are become mutual ſecurities for each other, and the whole produce of them liable to pay the intereſt charged upon each.

The ſurpluſſes of theſe three funds, over and above the intereſt charged upon them, are directed to be at the diſpoſal of the Parliament, and are uſually called the *ſinking fund*, becauſe originally deſigned to reduce the national debt: to this have lately been added ſeveral other duties granted in later years.

This ſinking fund, with the other duties added thereto, is the laſt reſort of the nation, on which muſt depend all our hopes of diſcharging our incumbrances. But by a ſtatute paſſed in the 26th year of Geo. III. Parliament veſted one million annually in commiſſioners for the reduction of the debt: in which act ſeveral proviſions are made to prevent this fund from being appropriated to any other purpoſe at any future time. In conſequence of this act 8,200,000*l.* had been redeemed in February 1792.

Since which period impoſts have been applied to the ſame purpoſe, particularly the redemption of the land-tax; and ſeveral new loans have been opened on different conditions.

SECT. II.

SIMPLE AND COMPOUND INTEREST.

SIMPLE intereſt is the allowance made by the borrower of any ſum of money to the lender, according to a certain rate per cent. per annum, which is limited by law to 5 per cent. per annum, that is, 5*l.* for the uſe of 100*l.* for one year, and 10*l.* for the uſe of it for two years, or 10*l.* for the uſe of 200*l.* for one year.

Principal is the ſum of money lent.

Intereſt or *rate* is the ſum agreed upon for the uſe of the principal.

Amount is the principal and intereſt added together *.

---

* In the courts of law intereſt for money lent is computed for years, quarters of years, and ſingle days. But for the intereſt on public bonds belonging to the South Sea and India Companies, and the ſecurities of the Bank of England, the time is generally computed in calendar months and days, and on Exchequer bills in quarters of a year and days.

Simple Intereſt.

Rule. Queſtions that concern intereſt are reſolved by the rule of three; thus—

Multiply the principal by the rate of intereſt, and divide the product by 100, and the quotient will be the intereſt for one year.

Then multiply the intereſt for one year by the number of years given, and the product is the intereſt for that time.

But when parts of a year are given, then find what aliquot part or parts of a year ſuch part is, and take an equivalent part of the intereſt for one year; or find the intereſt for ſuch part by the rule of three direct.

Example 1. What is the intereſt of 245*l.* 10*s.* for 2 years 4 months and 50 days, at 3½ per cent. per annum?

```
  £.  s.              Days.   £.  s.  d.          Days.
 245  10           If 365 gain 8  11  10, what will 50 gain?
       3½ per cent.                    5
 -----------                 -----------
 736  10                      42  19  2
 122  15                              10
 -----------                 -----------
1,00)8,59   5(8l.         365)429  11  8(1l.
     ----                     365
       59                     ---
       20                      64
     ----                      20
1,00)11,85(11s.               ---
     -----                365)1291(3s.
        85                    1095
        12                    ----
     -----                     196
1,00)10,20(10d.                 12
     -----                    ----
        20                365)2360(6d.
         4                    2190
        --                    ----
        80                     170
        ==                       4
                              ----
 £.  s.  d.
 8  11  10  1 year's intereſt  365)680(1f.
         2                         365
 -----------                       ---
17   3   8  2 years intereſt       315
 2  17   3¼ 4 months intereſt      ===
 1   3   6¼ 50 days intereſt
 -----------
21   4   5½
 ===========
```

In this example I firſt multiply the 245*l.* 10*s.* by the rate per cent. 3½, and the product divided by 100 quotes 8*l.* 11*s.* 10*d.* for the intereſt of one year. Then by the rule of three direct I find the intereſt for 50 days, which is 1*l.* 3*s.* 6½*d.*

And by the rule of practice I find the intereſt for 2 years 17*l.* 3*s.* 8*d.* and alſo the intereſt for 4 months, or one third of the year, 2*l.* 17*s.* 3½*d.* which 3 added together, the ſum is 21*l.* 4*s.* 5¾*d.* for the anſwer.

Queſtions concerning intereſt alſo are reſolved by the double rule of three; thus the foregoing example may be wrought, ſaying, if 100*l.* gain 3*l.* 10*s.* per cent. in 1 year, what will 245*l.* 10*s.* gain in 2 years 4 months and 50 days?

Qu. 2. What is the intereſt of 461*l.* for 1 year, at 4 per cent. per annum?—*Anſ.* 18*l.* 8*s.* 9⅗*d.*

Qu. 3. What is the amount of 345*l.* for 3 years, at 4½ per cent. per annum?—*Anſ.* 391*l.* 11*s.* 6*d.*

Qu. 4. What is the intereſt of 411*l.* 10*s.* for 3 months, at 4 per cent. per annum?—*Anſ.* 4*l.* 2*s.* 3½*d.*

Qu. 5. What is the amount of 241*l.* for 2½ years, at 4¾ per cent. per annum?—*Anſwer* 269*l.* 12*s.* 4*d.*

Qu. 6. What is the amount of 400 guineas for 4 years 7 months and 25 days, at 4½ per cent. per annum?—*Anſwer* 507*l.* 18*s.* 9*d.*

Qu. 7. What is the intereſt due upon an Exchequer bill of 900*l.* at 3¾ per cent. per annum, for 2¾ years and 67 days?—*Anſwer* 99*l.* 0*s.* 1½*d.*

A TABLE

SHEWING

The Number of Days from any Day of one Month to the ſame Day of another Month.

	From any day of											
To	Jan.	Feb.	Mar	Apr	May	June	July	Aug	Sep.	Oct.	Nov	Dec
Jan.	365	334	306	275	245	214	184	153	122	92	61	31
Feb.	31	365	337	306	276	245	215	184	153	123	92	62
Mar.	59	28	365	334	304	273	243	212	181	151	120	90
Apr.	90	59	31	365	335	304	274	243	212	182	151	121
May	120	89	61	30	365	334	304	273	242	212	181	151
June	151	120	92	61	31	365	335	304	273	243	212	182
July	181	150	122	91	61	30	365	334	303	273	242	212
Aug.	212	181	153	122	92	61	31	365	334	304	273	243
Sept.	243	212	184	153	123	92	62	31	365	335	304	274
Oct.	273	242	214	183	153	122	92	62	31	365	334	304
Nov.	304	273	245	214	184	153	123	92	61	31	365	335
Dec.	334	303	275	244	214	183	153	122	91	61	30	365

The uſe of this table is, to find how many days are contained from any day of one month to the ſame day of any other month: thus, to find how many days there are from the 10th of March to the 10th of October following, I look in that column which has March at the top, and in that line which has October on the left hand, and in the junction of theſe two lines I find 214 for the anſwer.

Compound Intereſt.

Compound intereſt is that intereſt which ariſes from the principal and a former intereſt taken together as a principal; thus, if 100*l.* be put out to intereſt for any number of years at ſimple and compound intereſt, the intereſt as it ariſes is added to the principal, and their ſum becomes a new principal; and the intereſt due upon this is called compound intereſt.

Rule. Find the amount of the given principal and the firſt payment of intereſt, by the rule of ſimple intereſt. Then this amount will be the principal for the ſecond payment of intereſt, which ſecond payment of intereſt being added to the laſt amount, forms a third principal, and ſo on through all the payments to the laſt, always accounting the laſt amount the principal for the next intereſt.

Example 1. What is the amount of 540*l.* 10*s.* for 5 years at 5 per cent. per annum, compound intereſt?

£.	*s.*	*d.*	
540	10	0	firſt year's principal
27	0	6	firſt year's intereſt
567	10	6	ſecond year's principal
28	7	6¼	ſecond year's intereſt
595	18	0¼	third year's principal
29	15	10¾	third year's intereſt
625	13	11	fourth year's principal
31	5	8¼	fourth year's intereſt
656	19	7¼	fifth year's principal
32	16	11¾	fifth year's intereſt
689	16	7	the *Anſwer.*

After finding the firſt year's intereſt by the foregoing rule of ſimple intereſt, I add it to the principal; thus I have 567*l.* 10*s.* 6*d.* for the ſecond year's principal, the intereſt of which is 28*l.* 7*s.* 6¼*d.* and which added to its principal gives 595*l.* 18*s.* 0¼*d.* for the third year's principal, to which is alſo added its intereſt 29*l.* 15*s.* 10¾*d.* and the ſum 625*l.* 13*s.* 11*d.* forms a principal for the fourth year, which is alſo added to its intereſt 31*l.* 5*s.* 8¼*d.* and the ſum 656*l.* 19*s.* 7¼*d.* is the principal for the fifth year, which added to the laſt intereſt gives 689*l.* 16*s.* 7*d.* for the anſwer required.

Qu. 2. What is the amount of 500*l.* for 4 years, at 5½ per cent. per annum, compound intereſt?—*Anſ.* 619*l.* 8*s.* 2¾*d.*

Qu. 3. What is the compound intereſt of 450*l.* for 5 years, at 4 per cent. per annum?—*Anſwer* 97*l.* 9*s.* 10½*d.*

Intereſt by Decimal Fractions.

Simple intereſt is alſo wrought by decimal fractions, as well as by whole numbers.

Rule. Multiply the principal, the ratio, and the time together, and the product will be the intereſt required.

Ratio is the ſimple intereſt of 1*l.* for 1 year at the rate agreed upon; thus the ratio

	per cent. is	
at	1½	.015
	2	.02
	2½	.025
	3	.03
	3½	.035
	4	.04
	4½	.045
	5	.05
	5½	.055
	6	.06
	6½	.065
	7	.07
	7½	.075
	8	.08
	8½	.085
	9	.09
	9½	.095
	10	.1

The fraction oppoſite each rate per cent. is the decimal of a pound, equal to the intereſt of 1*l.* for a year; it alſo ſhews the decimal parts the intereſt is of the principal.

Thus, .05 of a pound is the intereſt of a pound for one year at 5 per cent. being equal to 1*s.*; it alſo ſhews that the intereſt of any principal for one year, at 5 per cent. is .05 parts of the principal.

Example 1. What is the intereſt of 846*l.* 15*s.* for 3 years, at 4 per cent. per annum, ſimple intereſt?

Here for the 15*s.* in the principal, I take the equivalent fraction .75, as taught in Caſe 4 of reduction of decimal fractions; then multiplying the principal by the ratio, and the product by the number of years, I have 101.6100*l.* for the intereſt.

Principal	846.75
Ratio	.04
	33.8700
Number of years	3
Pounds	101.6100
	20
Shillings	12.2000
	12
Pence	2.4000
	4
Farthings	1.6000

Then I find the value of the decimal .6100 in ſhillings and pence, by the rule in Caſe 2, of reduction of decimals, and it produces 12*s.* 2¼*d.*; thus the anſwer is 101*l.* 12*s.* 2¼*d.*

Qu. 2. What is the ſimple intereſt of 254*l.* 17*s.* 6*d.* for 5 years, at 4 per cent. per annum?—*Anſwer* 50*l.* 19*s.* 6*d.*

Qu. 3. What is the ſimple intereſt of 1760*l.* for 2½ years, at 3½ per cent. per annum?—*Anſ.* 154*l.*

Compound intereſt is alſo wrought by decimal fractions.

Rule for which is, to find the amount of 1*l.* for 1 year, at the given rate per cent. Then raiſe the amount to ſuch a power as is expreſſed by the number of years. Multiply this power by the principal given, and the product will be the amount, from which if the principal is extracted the remainder will be the intereſt.

Example 1. What is the compound intereſt of 520*l.* for 4 years, at 4 per cent. per annum?

1.04	
1.04	
4 16	
1 04	
1.08 16	the ſecond power of 1.04
1.04	
432 64	
1 0816	
1.1248 64	the third power of 1.04
1.04	
4 4994 56	
1 12 4864	
1.16 9858 56	the fourth power of 1.04
5 20	the principal
2 33 9717 120	
58 49 2928 0	
608.3 2645 120	the whole amount
520	
88.3 2645 120	*Anſwer*, or intereſt required.

In this example I raiſe the amount of 1*l.* for 1 year, 1.04, to the fourth power, and multiplying this power by the 520*l.* principal, the product is 608.32645120 for the whole amount of principal and intereſt, and ſubtracting the principal therefrom, the remainder is 88.32645120 or 88*l.* 6*s.* 6½*d.*

Qu. 2. What is the amount of 450*l.* for 5 years, at 4 per cent. per annum?—*Anſ.* 547*l.* 9*s.* 10½*d.*

Moſt writers on compound intereſt give tables to diſcover at once the compound intereſt of any ſum for any certain time, at different rates per cent.; I ſhall, therefore, give one to ſhew the manner in which they are conſtructed.

A TABLE

A TABLE

OF

Compound Intereſt, at 3½ per Cent. per Annum, calculated without the Loſs of a ſingle Fraction, for Twenty Years.

Year	1. £.
	.035
1ſt	1.035
	.036225
2d	1.071225
	.037492875
3d	1.108717875
	.038805125625
4th	1.147523000625
	.040163305021875
5th	1.187686305646875
	.041569020697640625
6th	1.229255326344515625
	.043023936422058046875
7th	1.272279262766573671875
	.044529774196830078515625
8th	1.316809036963403750390625
	.046088316293719131263671875
9th	1.362897353257122881654296875
	.047701407363999300857900390625
10th	1.410598760621122182512197265625
	.049370956621739276387926904296875
11th	1.459969717242861458900124169921875
	.051098940103500151061504345947265625
12th	1.511068657346361609961628515869140625
	.052887403007122656348656998055419921875
13th	1.563956060353484266310285513924560546875
	.054738462112371949320859992987359619140625
14th	1.618694522465856215631145506911920166015625
	.056654308286304967547090092741917205810546875
15th	1.675348830752161183178235599653837371826171875
	.058637209076325641411238245987884308013916015625
16th	1.733986039828486824589473845641721679840087890625
	.060689511393997038860631584597460258794403076171875
17th	1.794675551222483863450105430239181938634490966796875
	.062813644292786935220753690058371367852207183837890625
18th	1.857489195515270798670859120297553306486698150634767625
	.065012121843034477953480069210414365727034435272216796875
19th	1 922501317358305276624339189507967672213732585906982421875
	.067287546107540684681851871632778868527480640506744384765625
20th	1 989788863465845961306191061140746540741213226413726806640625

The foregoing table ſhews the amount of 1*l.* principal, at 3½ per cent. per annum, compound intereſt, for any number of years not exceeding 20: thus, to find the amount of 1*l.* for 15 years, I ſeek in the table in the firſt line of the fifteenth year, and the anſwer is £1.675348, &c. equal to 1*l.* 13*s.* 6*d.*

Though the table be formed only for 1*l.* principal, yet queſtions that concern 10*l.* 100*l.* 1000*l.* or any multiple of 10, may be readily anſwered by it, by only removing the decimal point as many places further towards the right hand as the pounds principal contain cyphers: thus, to find the amount of 10*l.* for 15 years at 3½ per cent. compound intereſt, I place the decimal point one place farther towards the right hand, and the anſwer is £16.753, &c. or 16*l.* 15*s.* 0¾*d.* and for 100*l.* the anſwer is £167.5348, &c. or 167*l.* 10*s.* 8¼*d.* by inſpection, &c.

The Conſtruction of the Table.

The foregoing table, and all others of the ſame nature, are formed by multiplying the principal by the ratio, and the product gives the ſimple intereſt for one year, which added to the principal, the ſum is the amount or principal for the next year; which is to be again multiplied by the ratio, and the product again added to the laſt principal for a new principal, &c.: thus, in the foregoing table, I multiply the principal 1*l.* by .035, the ratio at 3½ per cent. and the product is .035 for the intereſt for one year, which added to its principal 1*l.* is 1.035 for the ſecond year's principal, which is again multiplied by the ratio, and the product is .036225, which is again added to its principal 1.035, and the ſum is .071225 for the principal for the third year, &c.

In forming tables of compound intereſt, it is ſufficient, in general, if there be only three places of decimals, which will give the anſwer to a farthing.

SECT. III.

OF DISCOUNT OR REVERSION.

Discount is that allowance which is made for the payment of any ſum before it is due, according to a certain rate per cent.

Diſcount may be wrought either by whole numbers or decimal fractions.

The preſent worth of any ſum which is due ſome time hence is that ſum which if put to intereſt for that time, at the ſame rate per cent. will amount to the ſum or debt then due, whether the intereſt be ſimple or compound. Diſcount is wrought by the rule of three.

Rule. As the amount of 100*l.* at the given rate and time is to 100*l.* ſo is the ſum given to the preſent worth. Then ſubtract the preſent worth from the given ſum, and the remainder is the diſcount required.

Or, as the amount of 100*l.* at the given rate and time, is to the intereſt of 100*l.* for that time, ſo is the given ſum to the diſcount required.

Example 1. What is the diſcount of 573*l.* 15*s.* due 3 years hence, at 4½ per cent. per annum, ſimple intereſt?

```
                        £. s.
                         4 10
   £.  s.                   3            £.  s.
   113 10               13 10            573 15
    20                  20                20
  ----                 ---              -----
  2270                 270              11475
                                          270
                                       ------
                                       803250
                                       22950     20)
                                 227,0)3098250(1364
                                       227      -----
                                       ---       68 4
                                        828
                                        681
                                        ---
                                        1472
                                        1362
                                        ----
                                         1105
                                          908
                                         ----
                                          197
                                           12
                                         ----
                                     227)2364(10
                                         227
                                         ---
                                           94
                                            4
                                         ----
                                     227)376(1
                                         227
                                         ---
Answer 68l. 4s. 10¼d.                    149
```

In this example, the amount of 100*l.* for 3 years, at 4½ per cent. is made the firſt number in the rule of three direct, the intereſt for the 100*l.* for that time, at that rate per cent. is the ſecond number; and the given ſum, whoſe diſcount is required, is the third number; and the fourth number is the anſwer.—68*l.* 4*s.* 10¼*d.*

Diſcount by Decimals.

Rule. As the amount of 1*l.* for the given time, at the given rate, is to 1*l.* ſo is the intereſt of the debt for the ſame time to the diſcount required.

Example 1. What is the diſcount of 573*l.* 15*s.* due 3 years hence, at 4½ per cent. per annum?

As

As 1.135 is to 1 ſo is 77.45625

```
1.135)77.45625(68.243
      68 10
      -----
       9 356
       9 080
       -----
        2762
        2270
        ----
         4925
         4540
         ----
          3850
          3405
          ----
           445
```

68.243, equal to 68*l.* 4*s.* 10¾*d.* the *Anſwer.*

In this example, I find the amount of 1*l.* for the given time, at the given rate per cent. which is 1.135, and the intereſt of the debt for the given time, at the given rate per cent. is 77.45625. I therefore divide this intereſt of the debt by the amount (as it is the ſecond number of the rule), and the quotient is 68.243, or 68*l.* 4*s.* 10¾*d.*

Qu. 2. What is the diſcount of 1401*l.* 12*s.* for 10 months, at 3⅞ per cent. per annum?—*Anſ.* 43*l.* 16*s.* 11*d.*

Qu. 3. What is the preſent worth of 75*l.* due 15 months hence, at 5 per cent. per annum, ſimple intereſt?—*Anſwer* 70*l.* 11*s.* 9*d.*

Qu. 4. What is the preſent worth of 120*l.*; 50*l.* payable at 3 months hence, 50*l.* at 5 months, and the reſt at 8 months, diſcount at 6 per cent. per annum?—*Anſ.* 117*l.* 5*s.* 5*d.* *

Queſtions

---

* Diſcount is that allowance required for paying a debt before it becomes due, and is founded upon this ſuppoſition, that the debtor paying a ſum of money before it is due, loſes that intereſt which the money would otherwiſe gain him from the time of paying it till it is due, and conſequently the creditor has that advantage; therefore it is plain, that the benefit from diſcharging the debt before it is due, by a preſent payment, ſhould be the ſame as employing the whole ſum at intereſt till it becomes due; for if the diſcount be put out to intereſt for the

Queſtions concerning the buying and ſelling of Stock.

Example 1. What is the purchaſe of 2220*l.* 10*s.* Bank ſtock, at 105½ per cent.?

	£.	*s.*	*d.*
1/20	2220	10	0
1/10	111	0	6
	11	2	0½
	2342	12	6½ *Anſwer.*

---

the given time, its amount will be the intereſt of the whole debt for the ſame time.

In the method of diſcounting bills bankers commit a great error, for they reckon the intereſt of the ſum drawn for from the time the bill is diſcounted till it becomes due, including the days of grace; thus they take the intereſt inſtead of the diſcount, and make it more than it ſhould be.

To make this more plain:—A had a bond of 1000*l.* due 1 year hence, which he offers to B to diſcount for him at 5 per cent.; B, in imitation of the cuſtom of bankers, aſked A 50*l.* to diſcount the bill; but, ſays A, 50*l.* is the intereſt of 1000*l.* for 1 year, at 5 per cent. therefore who is to have the intereſt of the 50*l.* you or I? If you will return me the intereſt for the 50*l.* which is 2*l.* 10*s.* I will agree to your terms. A added farther, If you will give me as much money for my bond as with the intereſt will make up 1000*l.* at the year's end, I will deal with you—otherwiſe not; to which B agreed, and A calculated the intereſt thus: as 1.05 is to 1*l.* ſo is 1000*l.* to 47.619 or 47*l.* 12*s.* 4½*d.* which is the diſcount, and which ſubtracted from 1000*l.* leaves 952*l.* 7*s.* 7½*d.* the preſent worth.—Thus B got all the money he ought to have; for if B lends out the diſcount at 5 per cent. it will bring him juſt 2*l.* 7*s.* 7½*d.* and the amount will be juſt 50*l.* at the year's end, and ſuppoſing A put out the 952*l.* 7*s.* 7½*d.* at 5 per cent. it will juſt bring in 47*l.* 12*s.* 4½*d.* the money paid in hand to B, and he would have his 1000*l.* entire at the year's end.

But for goods bought or ſold, when payment is made in ready money, without any regard to time, one year's intereſt of the ſum is generally eſteemed the diſcount.

Here it muſt be noted, that in compound intereſt, if the intereſt be paid half-yearly, it increaſes the amount to ſomewhat more than when paid annually; and alſo if the diſcount be ſubtracted every half year, it makes the reverſion ſomewhat leſs than when computed annually.

Queſtions of this nature are reſolved by the rule of practice: thus, in the foregoing example, I take the principal ſum of 2220*l.* 10*s.* for the price of the Bank ſtock, at 100*l.* per cent. and for the other 5 per cent. in the price of the ſtocks, I take a twentieth part of the above ſum 111*l.* 0*s.* 6*d.* and for the remaining ½ per cent. I take a tenth part of the 5*l.* line, which is 11*l.* 2*s.* 0⅗*d.*; and theſe three lines added together is the anſwer.

Qu. 2. What is the purchaſe of 1975*l.* 10*s.* India ſtock, at 113⅞ per cent.?—*Anſ.* 2249*l.* 12*s.*

Qu. 3. What is the purchaſe of 1633*l.* 4*s.* Bank annuities, at 89⅜ per cent.?—*Anſ.* 1459*l.* 13*s.* 5¼*d.*

Qu. 4. What is the purchaſe of 4100*l.* 12*s.* in the 3 per cent. conſolidated Bank annuities, at 55¼ per cent.?—*Anſ.* 2265*l.* 8*s.* 3¾*d.*

CHAP. VI.

OF MERCHANTS ACCOUNTS.

SECT. I.

OF BOOK-KEEPING THE ITALIAN WAY; OR, BY DOUBLE ENTRY.

THERE are ſeveral books uſed by mercantile men, of which the principal are the *waſte-book*, the *journal*, and the *ledger*. Theſe books are generally uſed by all men of buſineſs; I ſhall therefore give an example of the uſe of each of them.

Of the Waſte-Book.

This book, by ſome called the *day-book*, is uſed to note down whatever occurs in the way of trade, as buying, ſelling, paying, receiving, delivering, &c. &c. without omitting any one thing.

This book has one marginal line on the left hand, and three lines for pounds, ſhillings, and pence, on the right hand; the day of the month and the date of the year are inſerted in the middle of the page.

Of the Journal.

Every thing which is written in the waſte-book muſt be tranſcribed into the journal in the proper mercantile terms, and in a fairer hand; and every article is to be ſet down according to the order of time, without any intermiſſion, to make the book of more validity in caſe of any controverſy or diſpute.

In this book every article bought or ſold is diſtinguiſhed by the name of debtor or creditor, and to this book recourſe muſt be had for the particulars of any account.

In the journal the day of the month is alſo placed in the middle of the page; it is ruled with three lines, for pounds, ſhillings, and pence, as the waſte-book is, and one, and ſometimes two marginal lines on the left hand, to refer to the ledger.

Of the Ledger.

Every article is poſted into this book from the journal, but in ſhort, in one line; every left-hand page of this book is called the debtor page, and each right-hand page the creditor. Every debtor page muſt have a creditor page of the ſame number; thus, the articles or perſons which are called the debtors (in the mercantile phraſe) muſt be entered on a page of the ſame number with thoſe called the creditors.

The day of the month, in this book, is ſet in a narrow column on the left hand; at the top of each page is the name, the place of reſidence, and the year of our Lord.

I ſhall give a few examples of the uſe of each of theſe books; but it muſt firſt be obſerved, that in the waſte book every article is written down in ordinary plain language, and entering the ſame articles into the journal is nothing more than the ſetting them down in the mercantile phraſe, diſtinguiſhing each perſon or article by the name of debtor or creditor, for which the following rule muſt be ſtrictly obſerved:

All things received, or the receiver, are debtors to the delivered, or the deliverer.

Examples of the Uſe of the Waſte-Book.

April 2d, 1808.		£.	s.	d.
Bought of Henry Smith, of Cheapſide, 100 yards of Iriſh linen, at 2s. 6d. per yard, to be paid in 3 months — — —		12	10	0
Sold to John Jones, 20 yards of broad cloth, at 18s. per yard, to be paid in 6 months —		18	0	0
Bought of William Williams, 1 cwt. of ſugar, at 1l. 18s. per cwt.	1 18			
Bought of John Edwards, 60lb. of tea, at 5s. per lb. —	15 0			
		16	18	0

Journal Entry.

April 2d, 1808.

½	Iriſh linen debtor to Henry Smith, 12l. 10s. for 100 yards, at 2s. 6d. per yard, to be paid in 3 months — — —	12	10	0
	John Jones debtor to broad cloth for 20 yards, at 18s. per yard, to pay in 6 months —	18	0	0

Liſbon

		£	s	d.
4	Liſbon ſugar debtor to William Williams for 1cwt. at 1*l.* 18*s.* per cwt. — —	1	18	0
5	Green tea debtor to John Edwards for 60lb. at 5*s.* per lb. — — —	15	0	0

In the firſt of theſe examples I make Iriſh linen the debtor, as that is the article received, and Henry Smith creditor, as he is the deliverer.

In the ſecond example John Jones is the debtor, as he is the receiver, and broad cloth the creditor. The ſame may be obſerved in all the other examples which follow.

Thus an experienced perſon may keep a journal without a waſte-book, as they both import the ſame thing.

I ſhall, however, give a few more examples of each; and firſt the form of an inventory, as that is always taken down.

Waſte-Book.

London, January 1ſt, 1808.

An inventory of all the money, goods, and debts, belonging to me, A. B. of London, merchant: viz.

In caſh — —	£4250	5	0			
In Canary wine —	1124	9	10			
In French brandy —	526	13	3			
In Florence oil —	224	0	0			
In 1000 ells of Iriſh linen at 2*s.* 4*d.* per ell — —	116	13	4			
Due from John Smith, by note of hand — —	50	0	0			
				6292	1	5

January 20, 1808. Received from Benjamin Hughes, in Jamaica, 50 puncheons of rum, at 18*l.* per puncheon, to pay in 6 months — — —	900	0	0

	£.	s.	d.
January 30, 1808.			
Bought of Henry Jones, 20 yards of broad cloth, at 17s. 6d. per yard £17 10 Bought 20 yards of ſerge, at 3s. 6d. per yard, to pay in 6 months — 3 10	21	0	0
February 3, 1808.			
Sold to Henry Brown 20 gallons of rum, at 8s. per gallon, in 3 months — —	8	0	0
February 4, 1808.			
Received of George Jones 100*l.* 10s. due upon bonds — — —	100	10	0
March 4, 1808.			
Paid to Thomas Williamſon 50*l.* due by note of my hand — — —	50	0	0
March 5, 1808.			
Bought of John Edwards 2 pipes of Madeira, at 30*l.* per pipe, to pay in 3 months —	60	0	0
March 6, 1808.			
Sold to Henry Williams 40 gallons of brandy, at 12s. per gallon, to pay in 6 months —	24	0	0
March 6, 1808.			
Bought of John Thomas 12 puncheons of Hollands, at 15*l.* per puncheon, to be paid in 9 months — — — —	180	0	0
March 6, 1808.			
Bought 2 pieces of muſlin, at 4*l.* per piece, for ready money — — —	8	0	0

Journal.

Journal.

Inventory, &c. London, January 1, 1808.

			£.	s.	d.
5/6	Sundry accounts Dr. to stock	£6292 1 5			
	Cash — —	4230 5 0			
	Canary wine —	1124 9 10			
	French brandy —	526 13 3			
	Florence oil — —	224 0 0			
	1000 ells of Irish linen, at 2*s.* 4*d.* per ell — —	116 13 4			
7/8	Note of hand of John Smith	50 0 0	6292	1	5
	January 20, 1808.				
8/9	Rum Dr. to Benjamin Hughes, in Jamaica, for 50 puncheons, at 18*l.* per puncheon, to pay in 6 months — — —		900	0	0
	January 30, 1808.				
10/11	Sundries Dr. to Henry Jones, viz.				
	Broad cloth, for 20 yards, at 17*s.* 6*d.* per yard — —	£17 10			
	Serge, for 20 yards, at 3*s.* 6*d.* per yard, to pay in 6 months —	3 10	21	0	0
	February 3, 1808.				
11/12	Henry Brown Dr. to rum, for 20 gallons, at 8*s.* per gallon, to pay in 3 months —		8	0	0
	February 4, 1808.				
12/13	Cash Dr. to George Jones, for 100*l.* 10*s.* due upon bonds — — —		100	10	0
	March 4, 1808.				
13/14	Thomas Williamson Dr. to cash, for 50*l.* due by note of hand — — —		50	0	0
	March 5, 1808.				
14/15	Madeira Dr. to John Edwards, for 2 pipes, at 30*l.* per pipe, to pay in 3 months —		60	0	0
	March 6, 1808.				
14/15	Henry Williams Dr. to brandy, for 40 gallons, at 12*s.* per gallon, to pay in 6 months		24	0	0
	March 6, 1808.				
14/15	Hollands Dr. to John Thomas, for 12 puncheons, to pay in 9 months — —		180	0	0
	March 6, 1808.				
14/15	Muslin Dr. to cash, for 2 pieces, at 4*l.* per piece		8	0	0

Ledger.

(1)

1808			Caſh Dr.		£.	s.	d.
Jan.	1		To ſundry accounts — —		86	15	3
Feb.	28	R	To balance the nett produce of my ſtock	3	6920	5	1
					7006	18	4

(1)

1808			Per contra — Cr.		£.	s.	d.
Jan.	17	6	By ſundry accounts — —	4	6292	1	5
Feb.	28	R	By profit and loſs gained — —		714	16	11
					7006	18	4

			Caſh Dr.		£.	s.	d.
Jan.	1	1	To ſtock — —	1	4250	5	0
Feb.	2		To ſundry accounts received this month		201	10	0
					4451	15	0

			Per contra Cr.		£.	s.	d.
Jan.	31	1	By ſundry accounts paid this month		506	7	0
Feb.	28		By ſundry accounts paid this month		101	2	6
		R	By balance remaining on hand	7	3844	5	6
					4451	15	0

			Canary Dr.	*Pip.*	*Mk.*		£.	s.	d.
Jan.	1	1	To ſtock, at 40*l.* 3*s.* 2½*d.* per pipe — — —	28	I B	1	1124	10	0
—	4	2	To caſh, at 50*l.* per pipe —	12	O C	1	600	0	0
			To profit and loſs gained by this account				220	0	0
							1944	10	0

			Per contra Cr.	*Pipes.*	*Mk.*		£.	s.	d.
Jan.	19	2	By Thomas Jones, at 50*l.* per pipe — — —	10	A B	4	500	0	0
—	20	4	By John Hughes, at 50*l.* per pipe — — —	5	O D	2	250	0	0
Feb.	2	2	By Edward Brown, at 45*l.* per pipe — — —	20	A B	5	900	0	0
—	5	2	By Robert Day, at 58*l.* 18*s.* per pipe	5	OC	4	294	10	0
							1944	10	0

(2) (2)

1808							£	s.	d.	1808							£	s.	d.
			French brandy Dr.	*Pun.*	*Mk.*								Per contra Cr.	*Pun.*	*Mk*				
Jan.	1	1	To ſtock, at 40*l.* 10*s.* 3*d.* per puncheon	13	A	1	526	13	3	Jan.	2	1	By John Green, at 44*l.* per puncheon	6	A O	2	264	0	0
			To profit and loſs				37	16	9	—	9	1	By John Thomas, at 43*l.* 10*s.* per puncheon	7	A B	2	304	10	0
							564	10	0								564	10	0
			Florence oil Dr.	*Jar.*	*Mk.*								Per contra Cr.	*Jar.*	*Mk.*				
Jan.	1	1	To ſtock, at 28*l.* per jar	8	I A	1	224	0	0	Jan.	3	2	By John Jones, at 26*l.* per jar	4	I A	3	104	0	0
										—	5	2	By John Hawkins, at 30*l.* per jar	4	I A	3	120	0	0
																	224	0	0
			Iriſh linen Dr.	*Ells.*	*Mk.*								Per contra Cr.	*Ells.*	*Mk.*				
Jan.	1	1	To ſtock, at 2*s.* 4*d.* per ell	1000	BE	1	116	13	4	Jan.	7	4	By John Edwards, at 3*s.* per ell	500	B E	5	75	0	0
			To profit and loſs				20	16	8	—	9	4	By John Thomas, at 2*s.* 6*d.* per ell	500	C D	5	62	10	0
							137	10	0								137	10	0
			John Smith Dr.										Per contra Cr.						
Jan.	1	1	To ſtock, by note of hand			1	50	0	0	Jan.	5	5	By caſh received in full			6	50	0	0

(3)

1808			Pun. / Gall.	Mk.		£.	s.	d.
		Rum Dr.						
Jan. 20	6	To Benjamin Hughes, at 18*l.* per puncheon, to pay in 6 months	50	A B	5	900	0	0
— 26	6	To profit and loſs gained by this account				70	0	0
						970	0	0
		Sundries Dr.						
Jan. 30	7	To Henry Jones, viz.						
		To broad cloth, 20 yds. at 17*s.* 6*d.* per yd.			8	17	10	0
		To ſerge, 20 yards, at 3*s.* 6*d.* per yard, to pay in 6 months —				3	10	0
Feb. 8	6	To profit and loſs gained by this account			7	7	10	0
						28	10	0
		Henry Brown Dr.	*Gall.*					
Feb. 3	9	To rum, at 8*s.* per gallon, to pay in 3 months — — —	20		3	8	0	0
		Caſh Dr.						
Feb. 4	7	To George Jones, due on bond —			8	100	10	0

(3)

1808			Pun. / Yds.	Mk.		£.	s.	d.
		Per contra Cr.						
Jan. 22	2	By W. Jones, at 15*l.* per puncheon	10	C D	4	150	0	0
— 26	4	By John Williams, at 20*l.* per puncheon — —	10	F G	5	200	0	0
— 29	4	By John Thomas, at 22*l.* per puncheon — —	10	A B	6	440	0	0
	R	By balance unſold, at 18*l.* per puncheon — —	10			180	0	0
						970	0	0
		Per contra Cr.	*Yds*					
Feb. 2	3	By caſh, at 1*l.* 2*s.* per yard	10		2	11	0	0
— 4	3	By caſh, at 1*l.* 5*s.* per yard	10		2	12	10	0
— 8	3	By George Green, at 5*s.* per yard	20		3	5	0	0
						28	10	0
		Per contra Cr.						
Feb. 3	2	By Henry Brown, to pay in 3 months			3	8	0	0
		Per contra. Cr.						
Mar. 1	2	By caſh, on bond of George Jones			3	100	10	0

(4)

1808			Thomas Williamſon Dr.						
Mar.	4	7	To caſh, on note of hand — —			2	50	0	0
			Madeira Dr.						
Mar.	5	7	To John Edwards, 2 pipes —			7	60	0	0
—	7	8	To profit and loſs gained by this account			7	20	0	0
							80	0	0
			Henry Williams Dr.	Gall.					
Mar.	6	8	To brandy, at 12s. per gallon	40		8	24	0	0
			Hollands Dr.	Pun.	Mk.				
Mar.	6	7	To John Thomas, at 15l. per puncheon — —	12	A R	7	180	0	0
			John Thomas Dr.						
Mar.	6	7	To caſh — — —			8	20	0	0
			Muſlin Dr.	Pie.	Mk.				
Mar.	6	8	To caſh, at 4l. per piece —	2	O E	8	8	0	0

(4)

1808			Per contra Cr.						
Mar.	7	3	By Thomas Williamſon, on note of hand			2	50	0	0
			Per contra Cr.	Hhd.	Mk.				
Mar.	1	3	By caſh, at 28l. per hhd.	1	A L	4	28	0	0
—	7	4	By John Evan, at 18l. 10s. per hhd. —	2	O P	3	37	0	0
		5	By balance, remaining unſold	1	O H		15	0	0
							80	0	0
			Per contra Cr.	Gall.	Mk.				
Mar.	6	2	By Henry Williams, at 12s. per gallon —	40	O P	7	24	0	0
			Per contra Cr.	Pun.	Mk.				
Mar.	6	2	By John Abraham, at 15l. per puncheon — —	12	A P	2	180	0	0
			Per contra Cr.						
Mar.	6	2	By John Thomas, in part of Holland			2	20	0	0
			Per contra Cr.	Pie.	Mk.				
Mar.	6	2	By muſlin unſold, at 4l. per piece	2	O B	2	8	0	0

Every article in the journal is poſted into the ledger, and thoſe perſons or things to which the other party is ſaid to be debtor are entered on the debtor ſide, or left-hand page, with the word *To*; and thoſe perſons or things belonging to the ſame account, to which the other party is ſaid to be creditor, are entered on the right-hand ſide, or creditor page, with the word *By*, as may be ſeen in the foregoing examples.

The figures in the narrow column in the ledger after the day of the month, both on the debtor and creditor ſide, ſhew what pages in the journal the entries are taken from; and the letter R in theſe columns ſignifies the remainder on the balance of that particular account. Thoſe figures in the narrow column on the debtor ſide, before the money, ſhew in what page of the ledger that entry ſtands on the creditor ſide; and the figures in the like column on the creditor ſide ſhew what page of the ledger the entry ſtands in on the debtor ſide.

Each debtor page of the ledger muſt be numbered the ſame as that part of the ledger where the creditors to the ſame accounts are entered.

The ſmall figures at the beginning of each journal entry ſhew in what pages of the ledger ſuch entry is poſted; the upper figure ſhews the debtor ſide, and the lower figure the creditor.

Theſe three books are or may be uſed by every man of buſineſs, in any line whatever, as well as by the merchant.

There are ſeveral other books uſed by merchants, as follows:

Firſt, the *debt-book*, in which is entered the day whereon all ſums become due, whether to be paid or to be received, by bills of exchange, notes, bonds, &c. that by comparing receipts and payments together, proviſion may be made in time.

Second, the *caſh-book*, which is ruled and paged like the ledger, wherein all receipts of money are entered on the left-hand folio, and all payments on the right, with the day of the month (the year being ſet on the top of the page), for whoſe account the money was received or paid; and the total debit or credit on each ſide is uſually once a month poſted into the ledger to the account of caſh therein, with references in the caſh-book to the ſeveral folios in the ledger.

Third, the *invoice-book*, or book of factories, wherein are copied all invoices of goods ſhipped, and alſo of goods received from abroad, which muſt always be entered on the left-hand ſide, leaving the right-hand ſide blank; and on the advice of the ſale of goods which are ſent abroad, and alſo on the ſale of goods received from abroad; which are to be entered on the blank or right ſide. The uſe of this book is, to ſave the journal from any eraſures, which would be unavoidable in taking invoices of the ſeveral goods received, ſent, or ſold.

Fourth, the *receipt-book*, wherein are given receipts for money paid: the receipt to be given by the receiver; for whoſe account or uſe, or for what it is received; and the day of the month, and date of the year.

Fifth, the *book of charges of merchandiſe*, wherein are entered all the charges and expenſes of any goods bought, ſold, received, or ſhipped; ſuch as porterage, wharfage, warehouſe-room, &c. &c.; the whole account is once a month transferred into the caſh-book, on the creditor ſide, with references to the book of charges; and the ſame in the debtor ſide on the ſame account in the ledger.

Sixth, the *book of houſehold expenſes*, wherein are entered the charges of what is expended in houſekeeping, ſuch as apparel, houſe-rent, ſervants' wages, pocket expenſes, victuals, &c. &c.: this is once a month ſummed up, and carried to the caſh-book.

Seventh, the *letter-book*, which contains copies of all letters ſent or received from abroad, and alſo inland letters.

Laſtly, the *note* or *memorandum book*, wherein are inſerted affairs that occur, for the help of memory, and of great uſe where there is a multiplicity of buſineſs.

General Rules.

The principal rule in book-keeping has already been delivered, in the beginning of this ſection, viz. To make all things received or the receiver debtor to the delivered or the deliverers. But I ſhall add a few more rules, which may ſerve to exemplify the foregoing rule more fully.

Firſt, *ſtock is debtor to profit and loſs* for all money received and nothing given for it; as the profit of every commodity in the trade, legacies left, money received with an apprentice, &c. &c.

Per contra, creditor, by profit and loſs, for all money paid and nothing received for it; as the diſcount of money received before due, loſs of goods uninſured, loſs by accidents, abatement by compoſition, houſehold expenſes, gifts, &c. &c.

Secondly, for money received make caſh debtor to the party that paid it, and the party creditor by caſh.

Thirdly, for money paid make the receiver debtor to caſh, and caſh creditor by the party.

Fourthly, for goods bought for ready money make the goods debtor to caſh, and caſh creditor by the goods.

Fifthly, for goods ſold for ready money make caſh debtor to the goods, and goods creditor by caſh.

Sixthly, goods bought on credit, make the goods debtor to the ſeller, and the ſeller creditor by the goods.

Seventhly, goods ſold upon credit juſt the contrary; that is, the perſon who buys them muſt be debtor to the goods, and the goods creditor by the party.

Eighthly, for goods bought part for ready money and the reſt on credit; firſt make the goods debtor to the party for the whole; ſecondly, make the party debtor to caſh for the money paid him in part of theſe goods, and caſh creditor by the party.

Ninthly, for goods ſold part for ready money and the reſt on credit; firſt make the party debtor to the goods for the whole, and the goods creditor; ſecondly, make caſh debtor to the party for the money received of him in part of theſe goods, and the party creditor.

Tenthly, when diſcount is allowed for money paid before it is due, make the party who receives the money debtor to caſh for ſo much as is paid him, and ſtock debtor to profit and loſs for the diſcount.

To remove an Account full written to another Folio.

Firſt, if the debtor ſide be more than the creditor, make the old account creditor by the new; but if the creditor ſide be more than the debtor, make the old account debtor to the new, and the new account creditor by the old for the remaining ſum.

Secondly, in an account of company, wherein there is placed more received of another than his ſtock amounts to, as much muſt be added to the debtor ſide as there is placed on the creditor ſide, in order that in the new account there may be as much debit as was put in, and as much credit as was received.

Thirdly, in all accounts of merchandiſe, the gain or loſs muſt be entered before the old account is made creditor by the new, and the new debtor to the old.

To balance or clear an Account when full written.

Find the ſum of the debtor and creditor ſide, and the difference place to its oppoſite: thus, if the creditor ſide exceed the debtor, then the line is to be written in the old account to balance on the debtor ſide, to anſwer the line on the creditor ſide in the new account.

How to balance the Account at the Year's End, and thereby to know the State of Affairs.

Firſt, even the account of caſh, and bear the nett remainder to balance debtor.

Secondly, find the value of all goods bought and ſold; and for goods bought which remain unſold, value them at prime coſt, and bear the nett remainder to balance debtor.

Thirdly, find what all the goods coſt which are bought, and alſo how much they were ſold for, and bear the nett gain or loſs to the account of profit and loſs.

Fourthly, even all perſonal accounts with your debtors and creditors in the order in which they are written down, and bear the nett remainder to balance, &c.

Fifthly, even all voyages and factors accounts wherein there is either gain or loſs, and bear the nett gain or loſs to the account of profit and loſs, and the goods unſold to balance.

Sixthly, even the account of profit and loſs, and bear the nett remainder to ſtock.

Seventhly, even the ſtock, and bear the nett remainder to balance creditor.

Laſtly, find the ſum of both the debtor and creditor ſides of the balance, and if they be both alike the account has been kept truly, otherwiſe not.

If the debtor and creditor ſides of the balance be not both alike there is an error, which muſt be found by pricking over the books again, to ſee whether every debtor and creditor is entered in the ledger [*].

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[*] Pricking over the books is comparing every article of the journal with the ſame in the ledger, and marking it thus †; and on the ſecond examination thus ‡, &c.

The

The Form of an Invoice.

March 6, 1808.

Invoice of 100qrs of wheat in ſacks, ſhipped on board the good ſhip Neptune, William Browne maſter, and conſigned to Henry Jones of Jamaica, for the account and riſk of John Dalton and Thomas Edwards of London, merchants, marked and numbered as per margin:

		£.	s.	d.
	100qrs. of wheat, at 12*s.* per quarter	60	0	0
A D W	200 ſacks, at 1*s.* each — —	10	0	0
B O	Meterage — — —	2	0	0
	Porterage, at 1*s.* 6*d.* per quarter —	7	10	0
No. 1	Cartage — — —	5	18	4
to 100	Lighterage — — —	3	0	0
	Freight, at 2*s.* per quarter — —	10	0	0
		98	8	4
	The commiſſion on 98*l.* 8*s.* 4*d.* at 1 per cent.	0	19	0
		£99	7	4

The Form of a Bill of Lading.

Shipped by the grace of God, in good order and well conditioned, by Henry Williams of London, merchant, in and upon the good ſhip called the Mercury of London, whereof is maſter under God for this preſent voyage John Dickſon of London, mariner, and now riding at anchor in the port of London, and by God's grace bound for Antigua; to ſay,

A D No. 3, 4, 7. One bale of Iriſh linen, and one bale of men's ſhoes, and five hundred pair of ſilk ſtockings, contents, &c. as per invoice, being marked and numbered as per margin, and are to be delivered in the like good order at the aforeſaid place (Antigua), the danger of the ſea only excepted, unto Mr. Edward Brown, merchant there, or to his aſſigns, he or they paying freight for the ſaid goods, three pieces of eight per cwt. with primage and average accuſtomed. In witneſs whereof, the maſter or purſer of the ſaid ſhip hath affirmed to three bills of lading, all of this tenour and date, one of which three bills being accompliſhed the other two to ſtand void; and ſo God ſend the good ſhip to her deſired port in ſafety. Amen.

Dated in London, the 6th of March 1808, inſide and contents unknown to *John Dickſon.*

An Account of a Sale.

London, March 6th, 1808.

Account of a ſale of two thouſand four hundred yards of muſlin, two thouſand two hundred yards of dimity, two hogſheads of rum, five cwt. of ſugar, and five hundred pair of cotton ſtockings, received from on board the ſhip Maria, Captain Thomas Jones commander, for account of Edward Wills of Jamaica, merchant, is debtor

	£.	s.	d.	£.	s.	d.
To porterage of ditto —	1	2	6			
To commiſſion of ſales —	15	2	2			
To ſtorage, at [illegible] per cwt. —	6	10	0			
				22	14	8
To the nett product, bad debts excepted —				310	12	0
				333	6	8
Per contra Cr.						
By two thouſand four hundred yards of muſlin, at 1*s.* per yard — — —				120	0	0
By two thouſand two hundred yards of dimity, ſold Henry Brett, at 1*s.* 2*d.* per yard —				128	6	8
By two hogſheads of rum, at 20*l.* per hogſhead				40	0	0
By Henry Jones, for 5 cwt. of ſugar, at 1*l.* 10*s.* per cwt. — — —				7	10	0
By Henry Williams, for five hundred pair of cotton ſtockings, at 1*s.* 6*d.* per pair —				37	10	0
				333	6	8

Errors excepted, London, 6th March 1808,

Per *John Smith.*

Moſt writers on merchants accounts give a variety of examples and rules to inform the learner how to ſtate the ſeveral articles according to debtor and creditor; but the foregoing examples, and a ſtrict attention to the general rules before delivered, will be ſufficient for any ordinary capacity.

Concerning the Exporting and Importing of Goods.

Goods to be exported muſt have a bill of entry, in the following form:—

In the Maria, Captain Thomas Jones, for Jamaica,

John Smith.

Two thouſand four hundred yards of muſlin.
Two thouſand two hundred yards of dimity.
Two hogſheads of rum.
Five cwt. of ſugar.
Five hundred pair of cotton ſtockings.

There muſt be ſeven of theſe bills, one of which muſt be in words at length, the others may be in figures; theſe bills are entered into books by the Clerks of the Cuſtom Houſe. When ſome goods pay cuſtom and others do not, there muſt be two entries made, one for thoſe which pay cuſtom, and the other for thoſe which do not pay.

Importing Goods.

When goods are to be landed from any ſhip arrived, the entry-book of the Cuſtom Houſe muſt be ſearched, to find the names of the ſhip and the captain, as alſo the waiters that are to attend the landing of the goods, &c.

The Form of a Bill of Entry Inwards.

In the Maria, Captain Thomas Jones, from Jamaica.

Fifty hogſheads of ſugar.
Twenty bags of cotton.

Forty hogſheads of rum.
Twenty bales of muſlin, &c.

Of theſe bills of entry inwards there muſt be eight; one of which muſt be expreſſed in words at length; this muſt be ſigned by the perſon in whoſe name the goods are entered, with the mark in the margin: then the cuſtoms and the fee for entry being paid, the land-waiter will grant a warrant for landing the goods.

Of Tare and Trett.

Tare and trett are rules for making a deduction from the value of any goods which are ſold by weight.

Tare is that allowance which is made to the buyer for the box, barrel, bag, &c. which contains the goods, and is either ſo much per box, barrel, bag, &c. at ſo much per cwt. or at ſo much in the groſs weight.

Trett is an allowance of 4lb. in every 104lb. for waſte, duſt, &c.

Cloff is an allowance made to the buyer after tare and trett are deducted, and is generally 2lb. in every 3cwt.

Groſs weight is the whole weight of the goods at firſt, together with the box, barrel, bag, &c.

Suttle is when a part only of the allowance is deducted, as the tare or the trett.

Nett weight is the weight that remains after all the foregoing deductions are made.

Caſe 1. When the tare is at ſo much per box, barrel, bag, &c.

Rule. Multiply the number of boxes, barrels, bags, &c. by the rate per tare, and ſubtract the product from the groſs weight, and the remainder will be the ſuttle.

Example 1. What is the nett weight of 20 hogſheads of tobacco, each weighing 4cwt. 2qrs. 20lb. at 80lb. per hogſhead tare?

		Cwt.	*qrs.*	*lb.*
20 hhds.	The weight of each hhd.	4	2	20
80 lb. tare				5
1600 or 14cwt. 1qr. 4lb. tare		23	1	16
				4
		93	2	8
	Tare	14	1	4
	Anſwer	79	1	4

Qu. 2. What is the nett weight of 28 frails of raiſins, each frail 5cwt. 2qrs. 17lb. groſs, tare 50lb. per frail?—*Anſwer* 145cwt. 3qrs.

Caſe 2. When the tare is at ſo much per cwt.

Rule. Divide the groſs weight by ſuch aliquot parts of a cwt. as the tare conſiſts of, as in practice, and ſubtract the ſum of the quotient from the groſs weight as before, and the remainder will be the anſwer.

Qu. 3. What is the nett weight of 22cwt. 3qrs. 20lb. at 21lb. per cwt.?—*Anſ.* 179cwt. 2qrs. 1lb.

Qu. 4. What is the nett weight of 14 barrels of figs, each barrel weighing 201lb. groſs, tare 10lb. per cwt.?—*Anſwer* 2562lb. 12oz.

Caſe 3. When there is an allowance for both tare and trett.

Rule. Subtract the tare from the groſs weight by the foregoing rule, and the remainder is called ſuttle weight, which divided by 26 gives the trett, and which being ſubtracted from the ſuttle weight gives the nett weight.

Example 5. In 250cwt. 19lb. groſs, how much nett, tare 2qrs. 14lb. trett as uſual?

	Cwt.	*qrs.*	*lb.*	
	250	0	19	groſs
		2	14	tare
26)	249	2	5	ſuttle
	9	2	10	15oz. trett
	239	3	22	1oz. nett weight.

Caſe 4. When there is an allowance made for tare, trett, and cloff.

Rule. After the tare and trett are deducted as before, divide the remainder or ſuttle by 168, and the quotient is the cloff; then ſubtract the cloff from the ſuttle, and the remainder is the nett weight.

Example 6. What is the nett weight of 27hhds. of tobacco, weighing 140cwt. 2qrs. 20lb. groſs, tare 14lb. per cwt. trett as before?

	Cwt.	*qrs.*	*lb.*	
For 14lb. per cwt. tare, divide by 8)	140	2	20	groſs
	17	2	9	tare
26)	123	0	11	
	4	2	25	trett
168)	118	1	14	ſuttle
		2	22	
	117	2	20	nett weight.

Qu. 7. What is the value of 58hhds. of ſugar, each hhd. weighing 6cwt. 1qr. groſs, at a guinea and a half per cwt. tare 8lb. per cwt. trett and cloff as uſual?—*Anſ.* 505*l.* 14*s.* 7¼*d.*

Commiſſion, Brokerage, and Inſurance.

Commiſſion is an allowance of a certain ſum per cent. to an agent abroad, for buying or ſelling goods for his principal.

Brokerage is an allowance to a broker for aſſiſting a perſon in procuring or diſpoſing of goods, as auctioneers, ſtock-brokers, &c. And the allowance is generally at a certain rate per cent.

Inſurance is a premium at ſo much per cent. given to certain offices or individuals who engage to make good the loſs of any ſhip or merchandiſe, which may happen from ſtorms, fire, &c. &c.

Rule. Queſtions concerning theſe rules are reſolved in the ſame manner as thoſe in ſimple intereſt, viz. by multiplying the principal ſum by the rate per cent. and dividing the product by 100.

Example 1. How much is the commiſſion of 540*l.* 12*s.* 6*d.* at 4½ per cent.?

£.	*s.*	*d.*	
540	12	6	principal ſum
		4½	rate per cent. commiſſion
2162	10	0	
270	6	3	
1,00)24,32	16	3	
20			
1,00)6,56			
12			
1,00)6,75			
4			
1,00)3,00			*Anſwer* 24*l.* 6*s.* 6¾*d.*

Qu. 2. What is the commiſſion of 1059*l.* 16*s.* 10*d.* at 2¼ per cent.?—*Anſ.* 23*l.* 16*s.* 11*d.*

Qu. 3. What is the brokerage of goods ſold to the amount of 1017*l.* 5*s.* 8*d.* at 1½ per cent.?—*Anſ.* 15*l.* 5*s.* 2*d.*

Qu. 4. What is the inſurance of a ſhip and her cargo, valued at 17863*l.* 18*s.* 9*d.* at 17⅞ per cent.?—*Anſ.* 3193*l.* 3*s.* 6¾*d.*

SECT. XXI.

PRECEDENTS OF RECEIPTS, PROMISSORY NOTES, BILLS OF EXCHANGE, BILLS OF DEBT, BILLS OF LADING, BILLS OF PARCELS, &C. WITH THE LAWS CONCERNING THEM.

Bills of Parcels.

It is uſual in ſome caſes for ſhopkeepers to deliver to their cuſtomers a bill of the articles ſold them, with the total value caſt up at the bottom; theſe are called bills of parcels.

A Linen-

A Linen-draper's Bill.

Mr. Francis, March 10, 1808.

Bought of John Green.

		s.	d.		£.	s.	d.
25 yards of dowlaſs	at	1	6	per yd.	1	17	6
30 yards of diaper	—	1	6	do.	2	5	0
20 yards of Holland	—	5	6	do.	5	10	0
40 yards of Iriſh cloth	—	2	6	do.	5	0	0
10 yards of muſlin	—	7	0	do.	3	10	0
5 yards of cambric	—	12	0	do.	3	0	0
20 yards of printed cotton		2	6	do.	2	10	0
					£23	12	6

A Woollen-draper's Bill.

Mr. W. Smith, March 12, 1808.

Bought of Henry Jones.

		s.	d.		£.	s.	d.
17 yards of fine ſerge	at	4	9	per yd.	3	8	0
18 yards of drugget	—	8	0	do.	7	4	0
15 yards of ſuperfine ſcarlet		1	6	do.	1	2	6
16 yards of ſuper black cloth		18	0	do.	14	8	0
25 yards of ſhalloon	—	2	0	do.	2	10	0
17 yards of drab	—	18	0	do.	15	6	0
					£43	18	6

A Hoſier's Bill.

Mr. Holmes, March 14, 1808.

Bought of James Richards.

		s.	d.		£.	s.	d.
16 pair of worſted ſtockings	at	6	6	per pair	5	4	0
16 pair of do.	—	3	2	do.	2	10	8
6 pair of black ſilk do.	—	14	0	do.	4	4	0
12 pair of black worſted do.	—	6	0	do.	3	12	0
8 pair of cotton do.	—	8	0	do.	3	4	0
4 yards of fine flannel	—	1	8	per yd.	0	6	8
					£19.	1	4

A Wine-

A Wine merchant's Bill.

Mr. Harriſon, March 16, 1808.

Bought of James Simpſon.

		£.	s.	d.		£.	s.	d.
12 gallons of palm ſack	at	0	8	0	per gall.	4	16	0
24 gallons of red port	—	0	12	0	do.	14	8	0
20 gallons of claret	—	1	0	0	do.	20	0	0
30 gallons of white Liſbon		0	6	0	do.	9	0	0
22½ gallons of rheniſh	—	1	10	0	do.	33	15	0
30 gallons of ſherry	—	0	10	0	do.	15	0	0
						£96	19	0

RECEIPTS.

Received, January 19th, 1808, of Mr. John Smith, the ſum of ten pounds ten ſhillings, for a quarter's rent, due at Chriſtmas laſt. Per me,

£10 10 0 *Thomas Jones.*

Received, Jan[illegible]ry 10th, 1808, of Mr. Charles Hamilton, fifty pounds, in part of indigo ſold him the 8th inſtant. Per

£50 0 0 *Henry Holmes.*

A Receipt given in a Receipt Book.

Received, January 11th, 1808, the ſum of forty-five pounds, by the order, and for the account of William Johnſon, Eſq. Per me,

£45 0 0 *John Lock.*

Received, January 12th, 1808, of Meſſrs. James Williams and Co. one hundred pounds, on account of ſelf and partner.

£100 0 0 Per me, *William Martin.*

Received, January 13th, 1808, of Mr. George Goodman, fifty-five pounds ſixteen ſhillings and nine pence, in part for tobacco ſold him the 24th of December laſt. Per

£55 16 9 *Samuel Thompſon.*

Received, January 14, 1808, of the Honourable Eaſt India Company, three hundred and fifteen pounds ten ſhillings, per order, and for the account of Philip Fox. Per

£315 10 0 *Benjamin Hamilton.*

Received, January 15th, 1808, of the Governor and Company of the Bank of England, one thouſand ſix hundred pounds ten ſhillings, for ſelf and company. Per

£1600 10 0 *Stephen Barber.*

Received, January 16, 1808, of the Worſhipful Company of Grocers, forty-nine pounds fifteen ſhillings, in full payment, for my father, John Harriſon. Per me,

£49 15 0 *John Harriſon*, jun.

Received, January 17th, 1808, of Richard Clarke, Eſq. chamberlain of London, the ſum of ſixty pounds, for the uſe of the Worſhipful Company of Joiners. Per

£60 0 0 *Richard Stevens*, Clerk.

A Rent-gatherer's Receipt.

Received, February 7th, 1808, of Mr. Stephen Dickſon, in money, eighteen pounds, and allowed him for land-tax five pounds, and for repairs two pounds, in all twenty-five pounds, in full for half a year's rent due at Chriſtmas laſt: I ſay received for the uſe of Thomas Cox, Eſq. by virtue of his letter of attorney. Per

£25 0 0 *Francis Ward.*

Received of Mr. William Morgan, this 9th of February 1808, ſix pounds, for one quarter's rent, due at Chriſtmas laſt, for my maſter, Robert Kerr. Per

£6 0 0 *Richard Heath.*

Received, February 10th, 1808, of Mr. Samuel Wilkinſon, twenty-nine pounds ſix ſhillings, in part of a bill of ſixty pounds, due the 3d of May next, to Mr. John Lewis. Per

£29 6 0 *Nicholas Manſel.*

A Receipt on the Back of a Bill of Exchange.

March 1ſt, 1808. Received the full contents of the within mentioned, being 500 pieces of eight. Per

500 pieces 8 *John Wilſon.*

PROMISSORY NOTES.

I promiſe to pay to Mr. Edward Brown, or order, the ſum of ſixty pounds, on the 29th of this inſtant March. Witneſs my hand this 6th day of March 1808.

£60 0 0 *Edward Jones.*

March 10th, 1808.

I promiſe to pay to the Honourable the Eaſt India Company, or bearer, upon demand, two hundred pounds, for Edward Williams.

£200 0 0 *Henry Smith.*

March 12th, 1808.

I promiſe to pay to Mr. Joſeph Brown, or order, five months after date, ten pounds ten ſhillings, for value received.

£10 10 0 *William Holmes.*

March 14th, 1808.

I promiſe to pay to Meſſrs. John Edwards and Co. or bearer, the ſum of twenty pounds ten ſhillings, on demand.

£20 10 0 *Robert Martin.*

A Note from two to one.

We or either of us promiſe to pay to Mr. Robert Stokes, or his order, on demand, twelve pounds ten ſhillings, for value received. Witneſs our hands,

£12 10 0
Thomas Williams.
Edward Morris.

BILLS OF EXCHANGE.

A Bill payable at Sight.

London, March 5th, 1808.

At ſight hereof pay Mr. Robert Winds, or his order, the ſum of twenty-five pounds, for value received of Edward Mills, and place it to account, as per advice from

To Mr. John Willis, Your humble ſervant,
Merchant, High Street, Briſtol. *John Miller.*

Dover, March 13, 1808.

Ten days after ſight hereof pay to Mr. Thomas Truſt, or his order, fifty pounds ten ſhillings, and place it to account, as per advice from

To Mr. John Francis, *Your humble ſervant,*
Linen-draper, Cheapſide, London. *Henry Hill.*

Foreign Bills of Exchange.

London, March 6, 1808, for 540 crowns,
at 56¼*d.* ſterling per crown.

At uſance* pay this my firſt bill of exchange (my ſecond and third not being yet paid) unto Mr. Henry Brown, or order, five hundred and forty crowns, at 56⅔*d.* per crown, for the value received here of Mr. Edward Wills, and place it to account, as per advice from

To Mr. John Edwards, Your humble ſervant,
Merchant in Amſterdam. *John James.*

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* Uſance between England and France, or England and Holland, is one calendar month; between England and Spain, or Portugal, two months; between England and Italy, three months; and between England and Turkey, four months, &c.

Naples, March 12, 1808, for 640*l.* 10*s.*

At three uſances pay this my firſt bill of exchange, unto Mr. John Stocks, or order, ſix hundred and forty pounds ten ſhillings ſterling, for value received of himſelf, and place it to account, as per advice from

To Meſſrs. Bardy and Edwards, Your humble ſervant,
Merchants in London. *John Smith.*

Liverpool, March 14, 1808, for 6000 pieces of eight, at 53⅜*d.* per piece.

At double uſance pay this my fourth bill of exchange, unto Mr. John Jervis, or order, ſix thouſand pieces of eight, Mexico, at 53⅜*d.* ſterling per piece, for value received of Mr. Joſeph Hunt, and place it to account, as per advice from

To Mr. Matthew Marſh, Your humble ſervant,
Merchant in Leghorn. *Henry Edwards.*

A Bill of Debt.

Know all men by theſe preſents, that I John Briton, of London, merchant, do owe and am indebted unto Henry Haynes, of Southwark, ſaker, in the ſum of four hundred and forty-nine pounds, of lawful money of Great Britain; which ſaid ſum I do hereby promiſe to pay unto the ſaid Henry Haynes, his heirs, executors, adminiſtrators, and aſſigns, on or before the twenty-fourth day of September next enſuing the date hereof. Witneſs my hand and ſeal, this fourteenth day of March 1808. *John Briton,* (L S).

Sealed and delivered } *William Smith.*
in the preſence of us, } *Thomas Harriſon.*

A Bond.

Know all men by theſe preſents, that I John Drew, of the pariſh of Saint Giles, in the county of Middleſex, carpenter, am held and firmly bound to Samuel Price, of the city of Weſtminſter, in the county aforeſaid, vintner, in the ſum of one hundred pounds, of good and lawful money of Great Britain, to be paid to the ſaid Samuel Price, or to his certain attorney or attornies, heirs, executors, adminiſtrators, or aſſigns; for which payment well and truly to be made, I do hereby bind myſelf, my heirs, executors, adminiſtrators, or aſſigns, firmly by theſe preſents. In witneſs whereof I have hereunto ſet my hand and ſeal this eighth day of March, in the forty-eighth year of the reign of our ſovereign Lord George the Third, &c. and in the year of our Lord 1808.

John Drew, (J D).

The *condition* of the above written obligation is ſuch, that if the above bound John Drew, his heirs, executors, adminiſtrators, or aſſigns, do well and truly pay, or cauſe to be paid, unto the aforeſaid Samuel Price, his heirs, executors, adminiſtrators, or aſſigns, the full ſum of forty pounds, of lawful money of Great Britain, on or before the twenty-ſixth day of December next enſuing the date hereof, then this obligation to be void, otherwiſe to remain in full force and virtue.

A Bond from two to one.

Know all men by theſe preſents, that we Edward Willis and Joſeph Jenks, of the city of London, merchants, are held and firmly bound to Thomas Smith, of Liverpool, merchant, in the ſum of five thouſand pounds, of good and lawful money of Great Britain, to be paid to the ſaid Thomas Smith, his heirs, executors, adminiſtrators, or aſſigns, and for which payment well and truly to be made, we and either of us do jointly and ſeverally bind ourſelves, our joint and ſeveral heirs, executors, adminiſtrators, or aſſigns, firmly by theſe preſents. In witneſs whereof we have hereunto ſet our hands and ſeals this twelfth day of March, in the forty-eighth year of the reign of our ſovereign Lord George the Third, &c. and in the year of our Lord 1808.

Edward Willis, (E W).
Joſeph Jenks, (J J).

The *condition* of this obligation is ſuch, that if the above bounden Edward Willis and Joſeph Jenks, each or either of them, or each or either of their heirs, executors, adminiſtrators, or aſſigns, ſhall well and truly pay, or cauſe to be paid unto the above-named Thomas Smith, his executors, adminiſtrators, or aſſigns, the ſum of one thouſand pounds, of lawful money of *Great Britain*, on or before the twenty-fifth day of December next enſuing the date hereof, then this obligation to be void, otherwiſe to remain in full force and virtue.

A Penal Bill.

Know all men by theſe preſents, that I Andrew Jones, of the borough of Southwark, do owe unto Edward Mills, of the pariſh of St. Clement Danes, Weſtminſter, the ſum of one hundred pounds, of lawful money of Great Britain, to be paid unto the ſame Edward Mills, his executors, adminiſtrators, or aſſigns, on or before the fifth day of November next enſuing the date hereof; for which payment well and truly to be made, I do hereby bind myſelf, my heirs, executors, adminiſtrators, or aſſigns, to the ſaid Edward Mills, his executors, adminiſtrators, or aſſigns, in two hundred pounds, of like lawful money of Great Britain, firmly by theſe preſents. In witneſs whereof, &c.

Andrew Jones, (L. S.)

Of Receipts.

A receipt is a diſcharge for money owing, and is a bar to all ſuits in law and equity. When given *in full of all demands*, it diſcharges all debts between the two parties prior to the date thereof.

If a receipt be given, it is good, though no money be paid, except it be obtained by fraud, in which caſe relief may be had in equity.

A ſervant may give a receipt in his own name for his maſter, if he be accuſtomed to receive or pay money for his maſter's uſe; or a wife may give a receipt for her huſband in her own name, and it is equally valid.

If a receipt be given *in full*, though there be a condition annexed to it ſpecifying a former debt between the parties, yet the receipt ſtands as a receipt in full.

Of Promiſſory Notes and Bills.

A promiſſory note is an engagement for a ſum of money to be paid at a certain time, or on demand; when the ſum is to be paid on demand, it is due preſently, and there needs no actual demand; but it is otherwiſe where the money is to be paid to a third perſon, or where there is a penalty, for in that caſe there muſt a demand.

A bill is a ſingle bond, without any condition. If a perſon acknowledges himſelf indebted to another by bill in the ſum of fifty pounds, and by the ſame bill binds himſelf and his heirs for the payment of the ſaid ſum, in a hundred pounds, without inſerting the perſon's name to whom he is bound in the penalty, it ſhall nevertheleſs be taken as a bond againſt him for the ſaid ſum and penalty.

If a man inſert in a bill theſe words: *I do owe and promiſe to pay to A B* 50*l.* &c. *for the payment whereof I bind myſelf to C D*, &c. (another perſon), yet this is a good bill by the words of the firſt part, and the words obligatory to another perſon, C D, are void.

If a man ſays in a bill, *That I A B had received of C D the ſum of twenty pounds, which I promiſe to pay to E F*; or if the bill be, *I ſhall pay to C D* 20*l.*; or if it be, *I owe to C D* 20*l. to be paid at*, &c.; or, "*I had of C D* 20*l.* &c.; *to be repaid him again*; or, *I A B do bind myſelf to C D, that he ſhall receive* 20*l.*—either of theſe phraſes binds the drawer of the bill, and conſtitutes a good bill.

Of Bills of Exchange.

A bill of exchange is a writing given among merchants and traders for money, and by the credit of the drawer generally paſſes as money; theſe bills are drawn ſometimes payable at ſight, ſometimes at a certain time. An inland bill of exchange is of the nature of a letter, and is drawn by one merchant upon another in the ſame country, as, by A B of Briſtol upon C D of London; but a foreign bill of exchange is drawn by a merchant of one country upon a merchant of another country. Theſe foreign bills are of more conſequence in the eye of the law; a foreign bill being refuſed to be accepted, an action may be had againſt the drawer, and every perſon who indorſes a foreign bill becomes liable to the payment as if he were the drawer, becauſe every indorſement is in the nature of a new bill; but the indorſer is not liable to pay if the drawer can be found; but if the drawer's hand cannot be proved, the acceptor may come upon the laſt indorſer. If a perſon only ſigns his name at the back of the bill, the acceptor of the bill may fill up the indorſement, or the bare ſignature will paſs in law for an indorſement.

There is a material difference between a bill of exchange payable to a perſon *or bearer*, and one payable to a perſon *or order*. A bill payable to a perſon or bearer is not aſſignable, ſo as to enable the acceptor to bring an action if the drawer refuſes payment. In a bill payable to A B or bearer, the acceptor may ſue in the name of him to whom it is made payable. If a bank bill payable to A B or bearer be loſt, and found by a ſtranger, payment to the ſtranger will indemnify the Bank, yet A B may have an action of trover againſt the finder, but not againſt any perſon to whom the finder has paid it for valuable conſideration.—A bill of exchange payable to a perſon or order may always be aſſigned, and the acceptor can bring his action in his own name. A bill payable to order is within the cuſtom of merchants, and may be negotiated and aſſigned by cuſtom.

If a perſon accepts a bill, though the date of payment be paſt, he cannot return it, but he may bring an action for the money, if the bill be tendered in time.

A ſervant cannot accept a bill for his maſter, except there be good evidence that he has authority to do ſo, as where a maſter allows a ſervant to draw bills, &c.

If one partner in company draws a bill of exchange for ſelf and company, all the other partners are bound by it.

If the acceptor of a bill makes no demand when the bill becomes due, or in reaſonable time, and the party on whom it is drawn fails, the acceptor muſt ſuſtain the loſs, as it is his negligence.

If a bill of exchange be loſt, notice muſt be given by a notary public before witneſſes, that the bill is loſt, or miſlaid, requiring that payment be made to no perſon without his knowledge.

All inland bills of exchange of the ſum of 5*l.* or upwards, in which the value is ſaid to be received, and which are payable at a certain time after the date thereof, may, three days after they become due, be proteſted by a notary public, or any other ſubſtantial perſon of the place, if there be no notary public, and before two witneſſes, on the refuſal or neglect of payment; which proteſt ſhall be made under a copy of the ſaid bill, and be notified within fourteen days after to the party from whom the bill was received, who, upon producing the ſame proteſt, is to pay the bill with intereſt, and the charges of proteſting; but if the acceptor neglects to make a proteſt, and give notice, he ſhall be liable to the loſs.

Of Bonds.

A bond is an obligatory deed in writing, whereby one party binds himſelf to another to pay a ſum of money, or to perform any other lawful act, as, to make a releaſe, ſurrender an eſtate, to perform covenants, &c. &c. and it contains a penalty of non-performance; it may be made either on parchment or paper, though it is uſually on paper; and the condition to the bond may be either in the ſame deed or in another; the condition muſt be to do a thing which is lawful, otherwiſe it is not good; thus a condition not to uſe a trade, till or ſow ground, &c. is unlawful, being againſt the public good, and therefore void. When a bond depends upon a deed, and the deed becomes void, the bond is void alſo; a condition to indemnify a perſon from any legal proſecution is void, being againſt law. Conditions of bonds are to be not only lawful, but poſſible: if no time is limited in a bond for payment of money, it is due preſently, and payable on demand.

In a bond where ſeveral perſons are bound *ſeverally*, the obligee has his option, either to ſue all the bondmen together, or all of them apart, and have ſeveral judgments and executions, but he can have ſatisfaction but once; but if a bond be made to three to pay money to one of them, they muſt all join in the action. An heir is not bound by a bond, unleſs he be expreſsly named, but the adminiſtrators and executors are.

A bond may be made from one to any number of perſons, or from any number of perſons to one. If the ſurname only of the drawer be ſubſcribed, it is ſufficient, though there be a blank for his Chriſtian name. If a bond has no date, or a falſe date, or an impoſſible date (as the thirtieth day of February), yet if it be ſealed and delivered it is good; a perſon cannot be charged by a bond without delivery of the bond to a creditor.

Where a bond is made for the payment of a ſum of money by inſtalments, the holder of the bond in ſome caſes may come upon the drawer for failure of the firſt payment, but in other caſes he cannot ſue for the money till the laſt payment becomes due: as for inſtance, if A gives a bond to B to pay 20l. in the following manner; viz. 10l. to be paid at a certain day, and 10l. at another certain day; in this caſe B cannot ſue for the debt till the laſt payment becomes due, as the 20l. is mentioned as an entire debt; but if it be expreſſed to pay 10l. at a certain day, and 10l. at another certain day, *making in all* 20l. the creditor may ſue upon the bond on failure of the firſt payment.

A bond may be good though it contains falſe Latin or falſe English, or though it be of a doubtful interpretation; as, if A binds himſelf to B to pay a ſum of money to A (whereas it ſhould be to B), here the obligation is good, and the *ſolœcidum* void. If a bond be interlined in a place not material, it will not hurt the bond; but if it be in a material part, it will make the bond void. In ſhort, all bonds, obligations, bills, promiſſory notes, &c. when any doubt ariſes, are always interpreted in favour of the party to whom the money is due.

These are the principal legal precedents uſed by mercantile men, to which I ſhall add a power or letter of attorney, as being in moſt general uſe by all deſcriptions of people.

A Letter of Attorney from one, or two, to ſettle Accounts and receive Money.

Know all men by theſe preſents, that I Edward Stokes, of the pariſh of St. Andrew, Holborn, London, chinaman [or Edward Smith and John James, upholſterers, of the pariſh of St. Luke, Middleſex] have made, ordained, conſtituted, and appointed, and do by theſe preſents make, ordain, conſtitute, and appoint my friend Henry Brown, of the pariſh of St. James, Clerkenwell, in the ſaid county of Middleſex, watchmaker, my [or our] true and lawful attorney for me, in my name, and on my behalf [or for us, in our names, and on our behalf], to adjuſt and ſettle all and every account and accounts with all and every perſon and perſons with whom I [or we] have had, or ſhall or may have any tranſactions or dealings whatſoever, and to compromiſe, agree, and determine all diſputes and differences that have or ſhall ariſe between me [or us] and any other perſon or perſons whomſoever, and to execute all ſuch deeds, inſtruments, and writings, as he the ſaid Henry Brown ſhall judge neceſſary, and to aſk, demand, ſue for, recover, and receive, to and for my [or our] uſe, of and from all and every perſon or perſons, that now is, are, or hereafter ſhall or may become indebted to me [or us] by any ways or means whatſoever, all and every the debt and debts, ſum and ſums of money by them reſpectively due and owing, and to compound for any ſuch debt or debts, and to take leſs than the whole for the ſame, or otherwiſe to adjuſt and ſettle the ſame in ſuch manner and upon ſuch terms as he the ſaid Henry Brown ſhall in his diſcretion think fit; and for non-payment thereof, or of any part thereof, to take ſuch courſe for recovering the ſame as to my [or our] ſaid attorney ſhall ſeem meet; and upon receipt of the ſaid debt or debts, ſum or ſums of money reſpectively, or any part thereof, acquittances or other ſufficient diſcharges for me, and in my name [or for us, in our names], or in his own name, to make and give for what he ſhall ſo receive; and generally to do, negotiate, tranſact, and perform all ſuch other acts, matters, and things, for me, and in my behalf [or for us, and on our behalf], in and about the premiſes, as fully in every reſpect as I [or we] might or could do if perſonally preſent; hereby ratifying and confirming, and agreeing further to ratify and confirm all and whatſoever my [or our] ſaid attorney ſhall lawfully do, or cauſe to be done, in and about the ſaid premiſes, by virtue of theſe preſents. In witneſs whereof, I [or we] have hereunto ſet my hand and ſeal [or our hands and ſeals] this twentieth day of March, in the year of our Lord 1808, and in the forty-eighth year of the reign of our Sovereign Lord George the Third, &c.*

Edward Stokes, (E. S.)
[or, *Edward Smith*, (E. S.)
John James,] (J. J.)

A letter of attorney may be executed by any perſon, if of full age; and a man may give a power of attorney to his wife, or a woman to her huſband, in caſes that do not belong to him. The nature of it is, to give the party who is to act the full power and authority of the maker, to accompliſh the act intended.

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* The words included in crotchets belong to the power when given by Edward Smith and John James; in other reſpects the power is the ſame as when given by Edward Stokes.

In theſe inſtruments the authority muſt be ſtrictly purſued; where the party acting does leſs than the authority mentions, the act is in moſt caſes void; but where he doth more than his authority, it may be good for ſo much as he had power to do, if he does nothing unlawful.

If a letter of attorney be made to three jointly, two cannot execute it; or if it be made to two jointly, one cannot execute it. The death of the maker generally determines the power.

All the foregoing precedents muſt be ſtamped according to the ſeveral rates impoſed by the late acts for increaſing the ſtamp duty, or they are void by ſtatute.

CHAP. VII.

OF MENSURATION.

SECT. I.

OF SUPERFICIAL MEASURE, WITH THE METHOD OF MEASURING MASONS, BRICKLAYERS, CARPENTERS, SAWYERS, PLASTERERS, PLUMBERS, PAINTERS, AND GLAZIERS WORK, AND THE PRICE OF EACH; ALSO THE PRICE OF EACH COMMODITY, AND THE WAGES OF JOURNEYMEN.

BEFORE the learner proceeds to menſuration, it is neceſſary that he ſhould underſtand duodecimals, or, as it is generally called, croſs multiplication.

This rule is called duodecimals, becauſe the numbers decreaſe from the left hand in a twelve-fold proportion; the firſt number being feet, the next number inches, and the next number the twelfth parts of an inch, &c.

By this rule workmen and artificers caſt up the contents of their work, and multiply feet, inches, and parts, by feet, inches, and parts, without reducing them to one denomination, as in common arithmetic. In this rule inches are ſometimes called *primes*, the parts are called *ſeconds*, the next diviſion *thirds*, &c.

Rule 1. Under the multiplicand, write the correſponding denominations of the multiplier: viz. feet under feet, inches under inches, &c.

2. Multiply every term in the multiplicand, beginning at the loweſt, by the feet in the multiplier, and write the reſult of each under its reſpective term, obſerving to carry 1 for every 12 in the product to the next higher denomination.

3. In the ſame manner multiply every term in the multiplicand by the inches in the multiplier, and ſet the product of each term one place further to the right hand of that term in the multiplicand, obſerving to carry 1 to the next higher denomination for every 12 as before.

4. Proceed in the ſame manner with the ſeconds, and the other denominations, if there be any more, and the ſum of the products will be the product required.

Example 1. Multiply 10 feet 4 inches and 5 parts by 7 feet 8 inches and 6 parts.

Feet.	Inches.	Parts.			
10	4	5			
7	8	6			
0	2	11			
2	4	0			
70	0	0			
0	0	3	4		
0	2	8			
6	8	0			
0	0	0	2	6	
0	0	2			
0	5	0			
79	11	0	6	6	*Anſwer.*

In this example the 5 parts are firſt multiplied by 7 feet, and the product is 2 inches 11 parts; then the 4 inches are multiplied by the 7 feet, and the product is 2 feet 4 inches; then the 10 feet are multiplied by the 7 feet, and the product is 70 feet. Then I multiply by the 8 inches, ſaying 5 times 8 is 40, which is 3 twelves and 4 over, the 4 I ſet one place further towards the right hand than the multiplicand 5, and the 3 I place under the ſeconds; then I multiply the 4 inches, ſaying 4 times 8 is 32, which is 2 twelves and 8, the 8 I ſet under the parts and the 2 under the inches; then I ſay 8 times 10 is 80, which is 6 twelves and 8, the 8 I ſet under the inches and the 6 under the feet. In the ſame manner I multiply by the third and laſt figure 6, ſaying 6 times 5 is 30, which is 2 twelves and 6, the 6 I place one place further towards the right hand than the firſt figure of the laſt product, and proceed in the ſame manner as before, and the ſum of all the products will be the anſwer.

I have given this example at ſomewhat an unneceſſary length, in order to inſtruct the learner more fully in the nature of the rule; but it is generally performed in three lines only when conſiſting of feet, inches, and parts, as in the following example:

Example 2. Multiply 6 feet 4 inches and 5 parts, by 4 feet 6 inches and 9 parts.

Feet.	Inches.	Parts.			
6	4	5			
4	6	9			
25	5	8			
3	2	2	6		
0	4	9	3	9	
29	0	5	9	9	*Anſwer.*

Note. As the foot is divided into 12 inches, ſo each inch is ſubdivided into 12 parts, called ſeconds, and each ſecond is again ſubdivided into 12 thirds, and each third into 12 fourths, &c.

Qu. 3. What is the ſuperficial content of a ſquare piece of ground, meaſuring 39 feet 10 inches and 7 parts in length, and in breadth 18 feet 8 inches and 4 parts?—*Anſ.* 745 feet 6 inches 10 ſeconds 2 thirds and 4 fourths.

Qu. 4. What is the area of a floor in length 24 feet 10 inches 8 ſeconds 7 thirds and 5 fourths, and in breadth 9 feet 4 inches and 6 ſeconds?—*Anſ.* 233 feet 4 inches 5 ſeconds 9 thirds 6 fourths 4 fifths and 6 ſixths.

Seconds, thirds, &c. are uſually marked by two or three ſhort ſtrokes placed over the figures: thus, the anſwer to the laſt queſtion is expreſſed, 233 feet $\overset{|}{4}\;\overset{||}{5}\;\overset{|||}{9}\;\overset{||||}{6}\;\overset{|||||}{4}\;\overset{||||||}{6}$

Superficial Meaſure.

Menſuration, in general, is the art of meaſuring and eſtimating the magnitude and dimenſions of bodies or figures, and is divided principally into three parts, called *lineal meaſure*, *ſuperficial meaſure*, and *ſolid meaſure*.

Definitions.

1. *Lineal meaſure* is ſimple meaſure in length, without breadth or depth.

2. *Superficial meaſure* conſiſts of length and breadth taken together.

3. *Solid meaſure* conſiſts of length, breadth, and depth; of which hereafter.

4. A point has no parts nor dimenſions, neither in length nor breadth.

5. A line has only length, without any other dimenſions.

6. A right line lies all in the ſame direction, and is the ſhorteſt way between its two extremities.

7. A curve line continually changes its direction. See *fig.* 1.

8. Parallel lines are always at the ſame diſtance from each other. See *fig.* 2.

9. Oblique right lines change their diſtance and meet in an angle. See *fig.* 3.

10. An angle is the meeting of two lines. See *fig.* 5.

11. If the two lines which form an angle be perpendicular to each other, they then form a right angle. See *fig.* 6.

12. But if the two lines be not perpendicular to each other they form what is called an oblique angle, which is either greater or leſs than a right angle. See *fig.* 9 and 10.

13. If an angle be leſs than a right angle, it is called an acute angle. See *fig.* 7.

14. An angle that is greater than a right angle is called an obtuſe angle. See *fig.* 8.

15. A triangle is a figure contained under three lines, and has various names, according to the ſize of its angles.

16. An equilateral triangle has its three ſides, and conſequently its three angles, equal to each other. See *fig.* 9.

17. An iſoſceles triangle has only two ſides equal. See *fig.* 10.

18. A ſcalene triangle has its three ſides and angles unequal to each other. See *fig.* 11.

19. A right-angled triangle has one right angle. See *fig.* 12.

20. An obtuſe-angled triangle has one obtuſe angle. See *fig.* 13.

21. An acute-angled triangle has all its angles acute. See *fig.* 14.

22. A figure of four ſides is called a quadrangle, or a quadrilateral figure, and is either a parallelogram, a ſquare, a rhomboid, a rhombus, a trapezium, or a trapezoid.

23. A ſquare is an equilateral rectangle, having all its ſides equal, and all its angles right angles. See *fig.* 16.

24. A rhomboid is an oblique-angled parallelogram. See *fig.* 17.

25. A rhombus is an equilateral figure, having all it ſides equal, but its angles are oblique. See *fig.* 18.

26. A trapezium is a quadrilateral figure, but its oppoſite ſides are not parallel. See *fig.* 19.

27. A trapezoid has only two oppoſite ſides parallel. See *fig.* 20.

28. Plane figures having more than four ſides, are generally called polygons, but they receive particular names, according

to the number of their ſides: thus a polygon of five ſides is called a *pentagon*, a figure of ſix ſides an *hexagon*, a figure of ſeven ſides an *heptagon*, eight ſides an *octagon*, nine ſides a *nonagon*, ten ſides a *decagon*, eleven ſides an *endecagon*, and twelve ſides a *dodecagon*.

29. A circle is a plain figure bounded by one circular line, called the circumference, which is every where equally diſtant from the centre. See *fig.* 21.

30. The radius of a circle is a right line drawn from the centre to the circumference.

31. The diameter of a circle is a right line drawn through the centre, and bounded at each end by the circumference.

32. An arc of a circle is any part of the circumference.

33. A chord is a right line joining the two extremities of an arc.

34. A ſegment of a circle is any part thereof.

35. A ſemicircle is half a circle.

36. A ſector is a part of a circle contained under part of the arc, and two radii drawn to the centre. See *fig.* 22.

37. A quadrant is a ſector of a circle, having one quarter of the circumference for its arc, and its two radii are perpendicular to each other. See *fig.* 23.

38. The circumference of every circle in geometry is ſuppoſed to be divided into 360 equal parts, called degrees, and each degree ſubdivided into 60 minutes, each minute into 60 ſeconds, and ſo on; hence a ſemicircle contains 180 degrees, and a quadrant 90 degrees, which form a right angle, and half a quadrant, called an octant, contains 45 degrees; for the meaſure of every right-line angle is an arc of a circle contained between the two lines which form the angle, the point of the angle being in the centre of the circle, and the number of degrees contained in the arc of the circle gives the meaſure of the angle. See *fig.* 25.

39. In every right-angled triangle the ſide oppoſite the right angle is called the hypothenuſe, and the other two ſides, the legs of the triangle.

40.

40. The height or altitude of a figure, is a line drawn from the uppermoſt ſide or angle, perpendicular to the baſe.

41. An angle is generally deſcribed by three letters: that letter which ſtands in the middle ſhews the angle intended. *Thus, in figure* 24, E A C repreſents an angle formed by the line E A, and the line A C; and the letters D A E in the ſame figure ſhew the angle formed by the two lines D A and E A.

Problems.

PROBLEM I.

TO DIVIDE A GIVEN LINE INTO TWO EQUAL PARTS.

Fig. 26. From B as a centre with the compaſſes opened to any greater length than B C deſcribe the arcs at m and n; then (the compaſſes being ſtill at the ſame width) from A, as the other centre, deſcribe two arcs cutting the former arcs m n; from the interſections of theſe arcs draw the ſtraight line m C n, and it will divide the given line A B into two equal parts, at the point C.

PROBLEM II.

TO DIVIDE A GIVEN ANGLE INTO TWO EQUAL PARTS.

Fig. 27. From B as the centre with the compaſſes deſcribe the arc A C; and from A and C with the radius deſcribe two arcs interſecting each other at m, then draw the line m B, and it will divide the angle A B C into two equal parts.

PROBLEM III.

TO DIVIDE A RIGHT ANGLE INTO THREE EQUAL PARTS.

Fig. 28. From B as the centre deſcribe the arc A C. Then with the ſame extent of the compaſſes, from A as the centre, croſs the arc A C in n, and with the ſame radius from C, as the centre, croſs the arc in m: from the points of in-

terſection

terſection m and n, draw the right lines m B and n B, and they will divide the right angle A B C as required.

PROBLEM IV.

TO DRAW A LINE PARALLEL TO ANOTHER GIVEN LINE, AND AT A GIVEN DISTANCE.

Fig. 29. Let the given line be A B: then from the points m and n with a radius equal to C, the given diſtance, deſcribe the arcs r and o, upon which draw the line C D, and it will be parallel to the line A B.

PROBLEM V.

TO ERECT A PERPENDICULAR FROM A GIVEN POINT IN A GIVEN LINE.

Fig. 30. Let the given line be B C, and the given point A. Take two equal diſtances A m and A n, and with a radius greater than A m, from m and n as centres, deſcribe two arcs, cutting each other at r: from A draw A r, and it will be perpendicular to the line B C; thus this perpendicular forms a right angle with each part of the line B C. There are many other ways of drawing perpendiculars to a given line, but the beſt way is to draw them by a ſquare, or other mathematical inſtrument for that purpoſe: I ſhall, however, give another Problem, to draw a perpendicular from a given point out of the line.

PROBLEM VI.

TO LET FALL A PERPENDICULAR FROM A GIVEN POINT, NEARLY OPPOSITE TO THE END OF A GIVEN LINE.

Fig. 31. Let the given point be A, and the given line B D; then from A draw any line A m, to meet B C: biſect A m at n, and from n as a centre, with the radius A n, deſcribe the arc A D m, cutting B C in D; then draw a line A D, and it will be the perpendicular required.

PROBLEM VII.

TO DIVIDE A GIVEN LINE INTO ANY NUMBER OF EQUAL PARTS.

Fig. 32. Let the given line be A B, then from A draw a line A C, and from B draw B D parallel to A C: on each of theſe parallel lines point off as many parts as the given line is to be divided into, beginning at A in the firſt line, and at B in the ſecond line; then join the oppoſite points of diviſion in each line, by the lines A 5, 1 4, 2 3, &c. and they will divide the line A B as required.

PROBLEM VIII.

AT A GIVEN POINT IN A GIVEN LINE, TO MAKE AN ANGLE OF ANY PROPOSED NUMBER OF DEGREES.

Fig. 33. Let the given point be A, and the given line A B; then from A, as a centre, with a radius of 60 degrees taken from a line of chords, deſcribe an arc m n, cutting A B in m; then take the propoſed number of degrees with the compaſſes from the ſame line of chords, and apply the compaſſes from m to n; through the point n, draw the right line A n, and it will form an angle with the line A B, of the number of degrees required *.

Angles of more than 90 degrees ſhould be taken off the line of chords at twice.

Or the angle may be made with a divided arch, by laying the centre to the point A, and its radius on the line A B, and mark at n, the number of degrees propoſed from the arch, and draw the line A n, as before.

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* A line of chords is always deſcribed upon the plain ſcale, which is an inſtrument of braſs, ivory, or wood, about twelve inches long, to be had at the mathematical inſtrument makers: there are alſo other lines upon the ſcale of uſe in Geometry, as a line of ſines, of tangents, ſecants, ſemitangents, rhombs, and a line of equal parts; I ſhall give the figure of one of theſe ſcales, and the method of conſtructing the ſeveral lines upon it, with the uſes for which they are applied, when I come to treat of Trigonometry.

PROBLEM IX.

TO MEASURE A GIVEN ANGLE.

Fig. 33. Deſcribe the arc m n, with the compaſſes opened to the radius of 60 degrees in the line of chords, take the width of the arc m n, and apply that diſtance to the line of chords, and it will ſhew the number of degrees in the angle.

If the arc of the angle exceed 90 degrees, it muſt be taken off at twice, as before.

Or, the angle may be meaſured by means of a divided arch, as the angle in the laſt problem was conſtructed.

PROBLEM X.

TO FIND THE CENTRE OF A CIRCLE.

Fig. 34. In the circle A B C, deſcribe any line, as A B, which is called a chord, from the middle of which line draw the perpendicular C D, which will be the diameter of the circle, and biſect C D in the point O, which will be the centre.

PROBLEM XI.

TO DESCRIBE THE CIRCUMFERENCE OF A CIRCLE, THROUGH THREE GIVEN POINTS.

Fig. 35. Let the given points be A B C: then from the middle point B, draw two chords, or two right lines to the other two points A C; biſect theſe two lines by perpendicular lines meeting each other in the point O; O will then be the centre of the circle; and with the radius O A, or O B, deſcribe the circle A B C, and it will paſs through the three given points.

PROBLEM XII.

TO MAKE A REGULAR PENTAGON ON A GIVEN LINE.

Fig. 36. Let the given line be A B. Draw B m perpendicular to A B, and equal to half of it. Draw A m, and produce it till m n be equal to B m. With a radius equal to B n deſcribe two arcs, interſecting each other in a point O from A and B, as centres. With the ſame radius, from the point of interſection O, deſcribe the circumſcribing circle A B n, and about the circumference of this circle apply the diſtance A B five times, and it will deſcribe the figure required.

PROBLEM XIII.

TO DESCRIBE AN HEXAGON UPON A GIVEN LINE.

Fig. 37. Let the given line be A B. Then with a radius equal to A B, from A and B as centres deſcribe two interſecting arcs. From the point of interſection O, with the ſame radius deſcribe the circle A B C; then apply the line A B ſix times round the circumference, and it will form the figure required.

If each ſide of the above figure be divided into two parts, a figure of twelve ſides may be formed; and if each ſide be divided into three parts, a figure of eighteen ſides may be formed; thus it appears that the radius of a circle will divide the circumference into ſix equal parts; half the radius will divide it into twelve equal parts; and twice the radius will divide it into three equal parts.

PROBLEM XIV.

TO FIND A RIGHT LINE EQUAL TO ANY GIVEN ARC OF A CIRCLE.

Fig. 38. Let the given arc be A B: through the point A, and the centre of the circle, draw A m, making m n equal to ¾ of the radius n O. Draw a line A P perpendicular to A m, then through m B draw m P; then A P will be equal to the arc A B.

PROBLEM XV.

TO MAKE ANY REGULAR POLYGON ON A GIVEN LINE.

Fig. 36. Let the given line be C D: draw C O and D O, making the angles at C and D, each equal to half the angle of the polygon: from the centre O with the radius O D deſcribe the circle A B C D, then apply the line C D continually round the circumference, and it is done.

Note. The angles of a polygon are found in this manner; divide the whole 360 degrees, by the number of ſides in the polygon, and the quotient will be the angle at the centre, which angle ſubtracted from 180 degrees, the remainder will be the angle of the polygon.

A TABLE

Containing the Number of Degrees in the Angle, and at the Centre of every regular Polygon, from three Sides to twelve.

No. of Sides.	Names of the Polygon.	Angle at the Centre.	Angle of the Polygon.	Angle O D C or O C D.
3	Trigon	120°	60°	30°
4	Tetragon	90	90	45
5	Pentagon	72	108	54
6	Hexagon	60	120	60
7	Heptagon	$51\frac{3}{7}$	$128\frac{4}{7}$	$64\frac{2}{7}$
8	Octagon	45	135	$67\frac{1}{2}$
9	Nonagon	40	140	70
10	Decagon	36	144	72
11	Endecagon	$32\frac{8}{11}$	$147\frac{3}{11}$	$73\frac{7}{11}$
12	Dodecagon	30	150	75

PROBLEM

PROBLEM XVI.

TO MAKE A FIGURE SIMILAR TO ANY OTHER GIVEN FIGURE.

Fig. 39. From any angle, as ſuppoſe A, draw diagonals to each of the other angles; then to b c, one ſide of the figure, draw a parallel line B C; and C D parallel to c d, and E D parallel to e d; and it will be the figure required.

PROBLEM XVII.

TO REDUCE A COMPLEX FIGURE FROM ONE SIZE TO ANOTHER, BY MEANS OF A SCALE.

Fig. 40 and 41. Divide the given figure into ſquares, by croſs-lines, then divide another paper on which you intend to draw your figure, into the ſame number of ſquares, and obſerve what ſquares the ſeveral parts of the figure fall in, and draw ſimilar parts in the correſponding ſquare of the other figure.

PROBLEM XVIII.

TO DRAW A TRIANGLE EQUAL TO A GIVEN TRAPEZIUM.

Fig. 42. Let the trapezium be A B C D; draw the diagonal B D and C E parallel to B D, meeting A B, produced in the point E; join D E: then the triangle A D E will be equal to the trapezium A B C D.

PROBLEM XIX.

TO MAKE A TRIANGLE EQUAL TO A FIGURE OF FIVE SIDES.

Fig. 43. Let the figure be A B C D E A; draw the two diagonals D A, D B, and the lines E F, C G, parallel to them, and produced till they meet the baſe A B, produced to F and G; join D F and D G, and D F G will be the triangle required.

In the ſame manner a triangle may be made, equal to any right-lined figure whatever.

PROBLEM

PROBLEM XX.

TO MAKE A TRIANGLE EQUAL TO A GIVEN CIRCLE.

Fig. 44. Let the given circle be A, draw the radius A O, and the tangent A B, perpendicular to it; on the line A B, take A B equal to the circumference of the circle; join B O: then will the triangle A B O be equal to the circle.

PROBLEM XXI.

TO MAKE A RECTANGLE, OR A PARALLELOGRAM, EQUAL TO A GIVEN TRIANGLE.

Fig. 45. Let the given triangle be A B C, biſect the baſe A B in m, through C; draw C n o parallel to the baſe of the triangle A B; draw the line m n and B O parallel to each other; and the rectangle m n O B will be equal to the triangle A B C.

PROBLEM XXII.

TO MAKE A SQUARE EQUAL TO A GIVEN RECTANGLE.

Fig. 46. Let the given rectangle be A B C D: produce the ſide A B, till B E be equal to B C: biſect A E in the point O, on which as a centre, with the radius O A, deſcribe a ſemicircle A G F E, and produce B C to F, on which deſcribe the ſquare B F G H, and it will be equal to the rectangle.

In this manner any right-lined figure may be reduced to a ſquare.

PROBLEM XXIII.

TO MAKE A FIGURE EQUAL TO TWO OTHER SIMILAR FIGURES.

Fig. 47. Let the two ſimilar figures be P and Q, which are ſquares: let the two ſides A B and B C be perpendicular to each other; join their extremities A C by a right line, on which deſcribe the ſquare R, which will be equal to the two ſquares P and Q taken together.

All ſimilar figures may be added together in the ſame manner: for any two ſimilar figures, conſtructed upon the legs of any right angle, are equal to a ſimilar figure conſtructed upon the hypothenuſe.

PROBLEM XXIV.

TO MAKE A SQUARE EQUAL TO ANY NUMBER OF SQUARES TAKEN TOGETHER.

Fig. 48. Draw two lines A m, A n, perpendicular to each other: on one line mark A B equal to the ſide of one of the given ſquares, and on the other line mark A C equal to the ſide of another given ſquare: then draw the line B C, which will be equal to the ſide of a ſquare, equal to the two former given ſquares taken together: then mark A D equal to B C, and A E equal to the ſide of a third given ſquare; then D E will be the ſide of a ſquare, equal to the ſum of the three given ſquares taken together.

Thus *any number* of ſimilar figures may be added together.

PROBLEM XXV.

TO MAKE PLAIN DIAGONAL SCALES.

Fig. 49. Draw a line of a convenient length, as A B, and divided into eleven equal parts *; form each of theſe parts into a rectangle of a ſufficient height, by drawing parallel and perpendicular lines; divide the altitude into ten equal parts, if it be for a decimal ſcale—but if it be for feet and inches, divide into twelve parts; through theſe points of diviſion draw parallel lines the whole length of the ſcale; then divide the length of the firſt large diviſion A C into ten equal parts, both at the top and the bottom of the ſcale, and connect theſe points of diviſion by diagonal lines as ſhown in the figure, and the ſcale is finiſhed.

---

* Only four parts are laid down in this ſcale for want of room.

The uſe of theſe diagonal ſcales is, to take off the dimenſions of three figures; and if the firſt large diviſions in the ſcale from B to C be units, the ſecond ſet of diviſions from C to A will be the tenth parts of an unit, and the diviſions in the altitude from D to A will be hundredth parts: if B C be tens, A C will be units, and A D tenth parts: if C B be thouſands, A C will be hundreds, and A D tens: for example—if it were required to take off 243 from the ſcale, fix one foot of the compaſſes, at the figure 2 of the largeſt diviſions at the bottom of the ſcale, and extend the other foot to 4 of the ſecond ſmaller diviſions, and on the bottom of the ſcale; then for the three units ſlide up both points of the compaſſes in a perpendicular line, till they fall upon the third longitudinal line, and on that line, with one foot of the compaſſes remaining fixed, extend the other foot to the third diagonal line, and you will have the extent of the three figures as required.

To meaſure the Length of any Line.

Take the length of the line, between the compaſſes, and apply it to the ſcale: ſuppoſe it contains above three of the large diviſions, then ſet one foot of the compaſſes on the point 3 of the large diviſions, and ſuppoſe the other foot of the compaſſes to fall between 4 and 5 of the ſecond diviſions; ſlide up the compaſſes by a perpendicular motion, keeping one foot on the line of the large diviſion 3, till the other foot fall on the interſection of one of the diagonal lines, which ſuppoſe to be four; this ſhows that the length of the line meaſured is 344.

PROBLEM XXVI.

TO MAKE PLAIN SCALES FOR TWO FIGURES.

FIG. 50. If the ſcale be a decimal one, after dividing it into any propoſed number of large diviſions, one of thoſe diviſions muſt be ſubdivided into ten parts: but, if it be a duodecimal ſcale, one of the large diviſions muſt be divided into twelve parts, and ſerve to take dimenſions for feet and inches.

The moſt proper form for a ſcale of equal parts is that where the diviſions are marked on the very edge; and are generally made of ivory, the edge being made thin for the purpoſe of pricking off diviſions on the paper, without the help of the compaſſes.

PROBLEM XXVII.

TO MAKE A RIGHT LINE, THAT SHALL BE A MEAN PROPORTIONAL BETWEEN TWO GIVEN LINES.

Fig. 51. Let the two given lines be A C, C B: join them to each other ſo as to form a right angle at the point C: join the points A B, which will be the diameter of a circle; then deſcribe the circle B C A E, draw C D perpendicular to A B, and it will be the mean proportion required.

Note. The angle A C B, deſcribed in a ſemicircle, is always a right angle.

The chord A C is a mean proportional between A D and A B, and the chord B C is a mean proportional between B D and A B.

The ſquare of the hypothenuſe of a right-angled triangle is equal to the ſquares of both the ſides, as ſeen in Problem XXIII.

The three inward angles of a triangle are equal to two right angles.

Triangles that have each angle equal to each other are called ſimilar triangles.

To find the Area of ſuperficial Figures.

The area of a figure is the meaſure of the ſurface, without any regard to its thickneſs or depth.

The area of a ſuperficial figure is the number of ſquare inches, feet, yards, &c. contained on its ſurface.

Square yards, feet, inches, &c. differ from the ſame meaſures in length, as ſeen in the following table:—

Lineal Meaſures.

12 Inches	1 Foot
3 Feet	1 Yard
6 Feet	1 Fathom
16½ Feet or 5½ Yards	1 Pole or Rod
40 Poles	1 Furlong
8 Furlongs	1 Mile

Square Meaſures.

144 Inches	1 Foot
9 Feet	1 Yard
36 Feet	1 Fathom
272¼ Feet or 30¼ Yards	1 Pole or Rod
1600 Poles	1 Furlong
64 Furlongs	1 Mile

PROBLEM XXVIII.

TO FIND THE AREA OF A PARALLELOGRAM, A SQUARE, A RECTANGLE, A RHOMBUS, OR A RHOMBOID.

The area of any of theſe figures is found by the following rule:

Rule. Multiply the length by the breadth (or, if it be a rhombus or a rhomboid, by the perpendicular height *), and the product will be the area.

Fig. 40. What is the area of a ſquare, one ſide whereof is 6 feet, and the other ſide 7 feet? Anſwer 42 feet.

$$\begin{array}{r} 6 \\ \underline{7} \\ \underline{\underline{42}} \text{ feet.} \end{array}$$

PROBLEM XXIX.

Fig. 18. What is the area of a rhombus, the length whereof is 12 feet, 5 inches, and the perpendicular height 6 feet, 10 inches?

$$\begin{array}{rrrr} \text{Feet} & 12 & 5 & \text{inches} \\ & 6 & 10 & \\ \hline & 74 & 6 & \\ & 10 & 4 & 2 \\ \hline \text{Anſwer} & 84 & 10 & 2 \end{array}$$

---

* The perpendicular height of a parallelogram, a rhombus, &c. is a line drawn from the uppermoſt ſide, perpendicular to the baſe, and the perpendicular height of a triangle is alſo a line drawn from the uppermoſt angle, perpendicular to the baſe.

PROBLEM XXX.

FIG. 17. What is the area of a rhomboid, whoſe length is 5 chains and 20 links; and perpendicular height 20 chains 5 links?

In this Problem the chains and links are multiplied by each other as decimals, as the links are always decimal parts of the chain, as will be ſhown hereafter, and the product is the anſwer in ſquare chains and links, which is divided by 10 to bring them into acres, as there are 10 ſquare chains in an acre; and the decimals are multiplied by 4 for roods, and thoſe decimals again multiplied by 40 for perches, becauſe 4 roods make one acre, and 40 perches one rood.

```
       5,20
      20,05
       2600
     1040
 10)104,2600
     10,4260
           4
      1,7040
          40
     28,1600
```

Acres.	Rood.	Perches.
10	1	28,1600 Anſ.

PROBLEM XXXI.

TO FIND THE AREA OF A TRIANGLE.

FIG. 9. The area of all triangles is found by the following rules:—

RULE 1. Multiply the baſe by the perpendicular height, and half the product will be the area. But when only the three ſides of the triangle are given, the area is found by the following rule:—

RULE 2. Add the three ſides together, and from half the ſum ſubtract each ſide ſeparately; multiply the half ſum by the remainders of the three ſides continually together, and the ſquare root of the product will be the area of the triangle.

Example 1. What is the area of a triangle, whoſe baſe is 40 feet, and perpendicular height 20 feet?

```
     40
     20
  2)800   Anſwer 400 feet.
```

Example 2. What is the area of a triangle whoſe 3 ſides are 20, 18, and 16 feet?

```
  20        27       27 27 27
  18         7       20 18 16
  16       189        7  9 11
  54         9
          1701
            11
         18711

           · · ·
          18711(136.78
          1
     23 | 87
      3 | 69
    266 | 1811
      6 | 1596
   2727 | 21500
      7 | 19089
  27348 | 241100
      8 | 218784
            22316
```

Anſwer 136.78 feet.

PROBLEM XXXII.

TO FIND ANY ONE SIDE OF A RIGHT-ANGLED TRIANGLE, HAVING THE OTHER TWO SIDES.

In every right-angled triangle, the ſquare of the hypothenuſe is equal to both the ſquares of the two legs, as ſhown in Problem XXIII.

Therefore, to find the hypothenuſe, add the ſquares of the two legs together, and extract the ſquare root of the ſum.

And to find one leg, ſubtract the ſquare of the other leg from the ſquare of the hypothenuſe, and the ſquare root of the difference is the leg required.

Example 1. What is the hypothenuſe of a right-angled triangle, whoſe baſe is 33 feet, and perpendicular 56 feet?

$$\begin{array}{r} 56\\ \underline{56}\\ 336\\ 280\\ \hline 3136\\ \underline{1089}\\ \underline{\underline{4225}} \end{array} \text{(65 feet. Anſwer.} \qquad\qquad \begin{array}{r} 33\\ \underline{33}\\ 99\\ 99\\ \hline \underline{\underline{1089}} \end{array}$$

Example 2. What is the perpendicular of a right-angled triangle, whoſe baſe is 40 yards, and hypothenuſe 50 yards?

$$\begin{array}{r} 40\\ \underline{40}\\ \underline{\underline{1600}} \end{array} \qquad\qquad \begin{array}{r} 50\\ \underline{50}\\ 2500\\ \underline{1600}\\ 900 \end{array} \text{ yards. Anſwer.}$$

Qu. 3. What is the height of a ſcaling-ladder to reach the top of a wall 28 feet in height, and acroſs a ditch 45 feet in breadth? Anſwer 50 feet.

Queſtions of this nature are reſolved by the foregoing Problem; the height of the ladder being conſidered as a hypothenuſe of a right-angled triangle, and the height of the wall and breadth of the ditch as the two other legs of the triangle.

PROBLEM XXXIII.

TO FIND THE AREA OF A TRAPEZOID.

Rule. Multiply the ſum of the two parallel ſides by half the perpendicular diſtance between them, and the product will be the area.

Example 1. What is the area of a trapezoid, whoſe two parallel ſides are 8 feet and 12 feet, and perpendicular diſtance 10 feet?

$$\begin{array}{r} 8 \\ \underline{12} \\ 20 \\ \underline{5} \\ \text{Anſwer } \underline{\underline{100}} \text{ feet.} \end{array}$$

PROBLEM XXXIV.

TO FIND THE AREA OF A TRAPEZIUM.

Rule. Divide it into two triangles, by a diagonal line drawn from any one angle to the oppoſite angle, then find the area of theſe two triangles, by Problem XXXII. and add them together.

Note. Two perpendiculars drawn from the diagonal line to the other two oppoſite angles will be the perpendiculars of the triangle.

PROBLEM XXXV.

TO FIND THE AREA OF ANY IRREGULAR FIGURE OF ANY NUMBER OF SIDES.

All irregular figures conſiſting of more than three ſides are to be divided into triangles or trapeziums, then find the area of each of theſe triangles or trapeziums ſeparately, and add them together, and the ſum will be the area of the whole figure.

Note. Every figure may be divided into as many triangles, except two, as the figure has ſides.

Thus, a figure of five ſides may be divided into three triangles; a figure of ſix ſides, into four triangles; a figure of eight ſides, into ſix triangles, &c.

PROBLEM XXXVI.

TO FIND THE AREA OF A REGULAR POLYGON.

Case 1. When the centre can be found.

Rule. Multiply the perimeter of the figure, or ſum of its ſides, by half the perpendicular, drawn from the middle or one of its ſides to the centre; and the product will be the anſwer.

Case 2. When the centre cannot be found readily, as in pieces of water, where buildings are erected, &c.

Rule 2. Multiply the ſquare of one ſide of the polygon, by the multiplier ſet againſt it, in the following Table; and the product will be the anſwer.

TABLE.

No. of Sides of the Pol.	*Names of the Polygon.*	*Multipliers.*
3	Trigon, or equi. tran.	0.4330127
4	Tetragon, or ſquare	1.0000000
5	Pentagon	1.7204774
6	Hexagon	2.5980762
7	Heptagon	3.6339124
8	Octagon	4.8284271
9	Nonagon	6.1818242
10	Decagon	7.6942088
11	Undecagon	9.3656399
12	Dodecagon	11.1961524

Example. What is the area of a regular pentagon, whoſe ſide is 25 feet, and perpendicular 17.204774 feet?

By the firſt rule,

8.602387 ½ per.
125 perim.
43 011935
172 04774
860 2387
1075.298375 Anſ.

By the ſecond rule,

1.7204774
625
8 6023870
34 409548
1032 28644
1075.2983750 Anſ.

PROBLEM XXXVII.

TO FIND THE DIAMETER OR THE CIRCUMFERENCE OF A CIRCLE, THE ONE FROM THE OTHER.

RULE 1. As 7 is to 22, ſo is the diameter to the circumference.—As 22 is to 7, ſo is the circumference to the diameter.

RULE 2. As 113 is to 355, ſo is the diameter to the circumference.—As 355 is to 113, ſo is the circumference to the diameter.

RULE 3. As 1 is to 3.1416, ſo is the diameter to the circumference.—As 3.1416 is to 1, ſo is the circumference to the diameter.

Qu. 1. If the diameter of the earth be 7958 miles, as it nearly is, what is the circumference, ſuppoſing it were exactly round? Anſwer 25000.8528 miles.

Qu. 2. What is the diameter of the earth, ſuppoſing the circumference 25000 miles? Anſwer 7957¾ miles.

PROBLEM XXXVIII.

TO FIND THE AREA OF A CIRCLE.

RULE 1. Multiply half the circumference by *half* the diameter, and the product is the area.

RULE 2. Multiply the ſquare of the diameter by .7854.

RULE 3. Multiply the ſquare of the circumference by .07958.

RULE 4. As 14 is to 11, ſo is the ſquare of the diameter to the area.

RULE 5. As 88 is to 7, ſo is the ſquare of the circumference to the area.

Qu. 1. What is the area of a circle, whoſe diameter is 7 feet? Anſwer 38.4846 feet.

Qu. 2. How many ſquare feet does a circle contain, the circumference of which is 10.9956 yards? Anſwer 86.19266.

The operation of theſe problems is purpoſely omitted for the exerciſe of the learner.

PROBLEM XXXIX.

TO FIND THE LENGTH OF ANY ARC OF A CIRCLE.

Rule 1. As 180 is to the number of degrees in the arc, ſo is the product of the radius multipled by 3.1416 to the length of the arc.

Rule 2. As 3 is to the number of degrees in the arc, ſo is the product of the radius multiplied by .05236 to the length of the arc.

Qu. 1. What is the length of an arc of 12 degrees 10 minutes, the radius being 10 feet? Anſwer $2.1234\frac{8}{9}$ feet.

Qu. 2. What is the length of an arc of 30 degrees, the radius being 9 feet? Anſwer 4.7124 feet.

PROBLEM XL.

TO FIND THE AREA OF A SECTOR OF A CIRCLE.

Fig. 22. Rule 1. Multiply the radius by half the arc of the ſector, and the product is the area.

Rule 2. As 360 is to the degrees in the arc of the ſector, ſo is the whole area of the circle to the area of the ſector.

Note. For the ſemicircle take one half of the whole circle; for a quadrant, one quarter; for the ſextant, one ſixth, &c.

Example. What is the area of a ſector of a circle, whoſe radius is 15 feet, and the arc 30 feet?

$$\begin{array}{r} 15 \\ \underline{15} \\ 75 \\ \underline{15} \\ \text{Anſwer } 225 \text{ feet.} \end{array}$$

Qu. 2. What is the area of a quadrant, and a ſemicircle, whoſe radius is 13 feet each? Anſwer, the quadrant 132.7326 feet, and ſemicircle 265.4652 feet.

PROBLEM XLI.

TO FIND THE AREA OF A SEGMENT OF A CIRCLE.

Rule. Find the area of the ſector of the circle by the laſt Problem.

Find the area of the triangle formed by the chord of the ſegment, and the radius of the ſector.

Then the ſum of theſe two will be the anſwer, if the ſegment be greater than the ſemicircle; but if the ſegment be leſs than a ſemicircle, the difference will be the anſwer.

The foregoing Problems will be found fully ſufficient to inſtruct the learner in the method of meaſuring every kind of ſuperficies whatever, as there is no figure but may be reduced to one or more of the foregoing figures.

OF MEASURING ARTIFICERS' WORK.

Maſons' Work.

To maſonry belong all ſorts of ſtone-work, and the meaſure of it is by the foot, either ſuperficial or ſolid. Stone walls, columns, blocks of ſtone, &c. are meaſured by the ſolid foot; but ſlabs, chimney-pieces, pavement, &c. are meaſured by the ſuperficial foot.

The Prices of Maſons' Work.

	£.	s.	d.
Newcaſtle ſtone, *per foot cubic*	0	1	10
Workmanſhip to ditto, *per ſuperficial foot*, from 6*d.* to	0	1	6
Portland ſtone, *per cube foot*	0	4	6
Workmanſhip to ditto, *per foot ſuperficial*, from 1*s.* to	0	2	8

Paving

	£.	s.	d.
Paving in Portland ſtones ſtraight, 1½ inch thick, *per foot ſuperficial*	0	2	0
Ditto 2 inch thick	0	2	2
Purbeck ſtone, paving, *per foot ſuperficial*, from 11*d.* to	0	1	4
Bremen ſtone, paving in terraces, *per foot ſuperficial*	0	1	6
Portland ſtone, chimney-pieces and ſlabs, from 1½ inch thick to 3 inches, *per foot ſuperficial*, from 1*s.* 11*d.* to	0	2	9
Marble chimney-pieces veined, *per foot cube*	1	15	0
Workmanſhip to ditto, *per foot ſuperficial*, from 3*s.* 6*d.* to	0	8	6
Blue and white marble, an inch thick, *per foot ſuperficial*	0	5	6

Bricklayers' Work.

Brick-work is meaſured by the ſquare rod of 16 feet and a half, and the brick-work eſtimated at a brick and a half thick; if a wall be more or leſs than this ſtandard thickneſs, it muſt be reduced to it, by the following rule:

Rule. Multiply the ſuperficial content of the wall by the number of half bricks in the thickneſs, and divide the product by 3, and the quotient will be the ſuperficial contents of the wall, at the true ſtandard thickneſs of a brick and a half.

To take the dimenſions of a building, it is uſual to meaſure half round the outſide, and alſo half round the inſide; the ſum of theſe two meaſures multiplied by the height, gives the ſuperficial content, which muſt be reduced to the ſtandard meaſure, in order to aſcertain the price.

Chimnies are ſometimes meaſured as if they were ſolid, deducting only the vacuity from the hearth to the mantle; and ſometimes they are meaſured round for their breadth, and the height of the ſtory is their height, and the depth of the jambs is their thickneſs; in this caſe no deduction is made for the fire-place.

The chimney-ſhafts, which appear above the building, are meaſured round for their breadth, and multiplied by their height for the ſuperficial content; and the thickeſt is accounted half a brick more than it really is, on account of the ſcaffolding, &c.

All windows, door-ways, &c. are to be deducted in meaſuring for the materials, but not in meaſuring for workmanſhip. There are alſo ſome other allowances, as double meaſure for feathered gable ends, &c.

All the following brick-work is meaſured by the rod ſquare of 16½ feet, which contains 272¼ ſquare feet; hence, when the contents of brick-work is found in ſquare feet, it is divided by 272 (the fraction being neglected), and the quotient is the number of ſquare rods; which is to be reduced to ſtandard meaſure by the former rule.

Tiling is meaſured by the ſquare of 10 feet, making one hundred ſquare feet to each ſquare.

Prices.

Brick-work at *per rod.*	£.	s.	d.
New place bricks, laid dry, without mortar, in wells, ceſs-pools, &c.	10	10	0
Rough greyſtock brick-work, in party-walls, &c. from 10*l.* to	12	0	0
Labour and mortar only, to brick-work, from 3*l.* to	3	15	0
Labour only to ditto, from 1*l.* 8*s.* to . .	1	10	0
Plain tiling, at *per ſquare.*			
New plain tiling, from 2*l.* 12*s.* to	2	15	0
Labour only, from 6*s.* to	0	7	0

Price of Materials.

Lime, *per hundred*	0	14	0
Sand, *per load*	0	6	0
Mortar, *per load*, 16*s.*—*per hod*	0	0	7

Pointing

	£.	s.	d.
Pointing mortar, *per load*	1	12	0
Bricks, *per hundred*, according to the quality, from 3*s*. 6*d*. to	0	16	6
Bricklayer, *per day*, in ſummer	0	4	8
Ditto, in winter	0	4	3
Labourer, *per day*, in ſummer	0	3	4
Ditto, in winter	0	3	0

Carpenters' and Joiners' Work.

Moſt large and plain articles are meaſured by the ſquare of 10 feet, each ſquare being 100 feet; but mouldings and other ornaments are generally meaſured by the foot, running meaſure.

In meaſuring flooring, no deductions are made for hearths, on account of the trouble.

Partitions are meaſured from wall to wall, and from floor to floor, and no deductions made for door-ways.

In meaſuring roofs, the length of the houſe in the inſide with two thirds of the thickneſs of one gable, is conſidered as the length; and the breadth is equal to twice the length from the ridge to the end of the rafters.

For ſtaircaſes, take the breadth of all the ſteps by a ſtring from the top of the ſtaircaſe to the bottom, which multiply by the length of one ſtep.

For wainſcoting, take the compaſs of the whole room for the length, and the height from the floor to the ceiling for the breadth, deducting door-ways, windows, &c. from the materials, but not from the workmanſhip.

For doors, ſhutters, &c. Theſe are meaſured by the foot ſquare, allowing for their thickneſs, the number of pannels, &c.

Prices.

Flooring, at *per ſquare ſuperficial*,	£.	s.	d.
Labour and nails only, from 6*s*. 6*d*. to . .	0	12	6

Roofs,

Windows are done by the piece, and ſaſh-ſquares by the dozen.

Chimney-pieces, variegated or fancied colours, &c. by the foot ſquare.

Ornamental figures; mouldings, &c. by the foot running.

Prices.

Common painting in oil	£.	s.	d.
Once, *per yard*	0	0	3½
Ditto, twice	0	0	5¼
Ditto, three times	0	0	7½
Saſh frames, once in oil, *each*	0	0	9
Ditto, twice in oil	0	1	2
Saſh ſquares once in oil, *per dozen*	0	0	8
Ditto, twice	0	1	0
Caſements	0	0	6

Glaziers' Work.

Glaziers take their dimenſions in feet, inches, and parts, and compute their work by the ſquare foot.

In taking the length and breadth of a window, the croſs-bars between the ſquares are included; and windows of an oval or circular form are meaſured as if they were ſquare, taking the greateſt length and breadth, on account of the waſte in cutting the glaſs.

Prices.

	£.	s.	d.
Plate-glaſs, cut from 1 foot to 2 feet, *per foot ſuperficial*	0	5	6
Ditto from 2 feet to 3 feet	0	6	6
Beſt Ratcliffe crown glaſs *per foot ſuperficial*	0	2	4

SECT.

SECT. II.

OF MEASURING LAND; OR SURVEYING.

Before I proceed to the practice of ſurveying, it is neceſſary to mention the inſtruments uſed in meaſuring land.

1. *Of the Chain.*

The chain, called Gunter's chain, conſiſts of 100 equal links, each link being $\frac{22}{100}$ of a yard, or $\frac{66}{100}$ of a foot, that is 7.92 inches long; and the whole chain is 4 poles or 22 yards in length.

An acre of land is equal to 10 ſquare chains.

An acre is divided into 4 equal parts, called roods, and each rood into 40 parts, called perches or poles.

The length of lines meaſured with the chain is ſet down in links, as integers, every chain being 100 links in length; therefore, when the content of a piece of land is found in ſquare links, cut off 5 of the figures on the right hand for decimals, and the reſt will be acres: thoſe decimals are then multiplied by 4, to bring them into roods, and the decimals of the product multiplied by 40, for perches, as in the following example:

Example. What is the area of a rectangular piece of ground, the length of which is 1300 links, and the breadth 240?

```
          1300
           240
         ------
         52000
         2600
        -------
        3.12000
              4
        -------
         .48000
             40
        -------
Anſwer  19.20000   3 Acres, 0 Roods, 19 Perches.
```

2. *Of the Plain Table.*

This inſtrument conſiſts of a plain rectangular board of any convenient ſize, fixed to a ſtand of three legs, by means of a ball and ſocket, by which the table is inclined in any direction: it has alſo a frame of wood to fit round its edges, which can be taken off, for the purpoſe of fixing a ſheet of paper on the table. One ſide of the frame is divided into ſeveral equal parts, for the purpoſe of drawing lines acroſs the paper, and the other ſide of the frame is divided into 360 degrees from a centre in the middle of the table, for the purpoſe of taking angles, &c. There is alſo a needle and a compaſs on one ſide of the table, to point out the directions, and fix the table in the ſame poſition with regard to the points of the compaſs at every remove.

There is an index alſo, which is a braſs two-foot ſcale, with two open ſights, one at each end: theſe ſights, and one edge of the ſcale, are in the ſame plane, and that edge is called the fiducial edge of the index.

In uſing this table, a ſheet of paper is to be wetted and ſpread ſmooth, with the frame of the table preſſed down cloſe, to keep the paper ſteady, which is to remain till it is dry, that it may ſtretch itſelf ſmooth: upon this paper the plan is to be drawn.

In taking a ſurvey of any place, a point is to be made on the paper to denote that ſpot where the table is fixed, which is called the *ſtation*; then in that point fix a pin, or one foot of the compaſſes, and to it apply the fiducial edge of the index, moving the other part of the index about, till through the ſights you perceive one angle of the field you ſurvey, or ſome other remarkable object; and from the ſtation point draw a line along the fiducial edge of the index. Then turn the index about upon the ſtation point as a centre, till you perceive another angle, or ſome other object, and draw a line from the ſtation point along the edge of the index, as before: continue to do the ſame, till you have drawn lines from the ſtation, to repreſent the bearings of as many angles or objects as may be neceſſary, and no more; then meaſure from the ſtation where the table is fixed to every object which you have viewed, and lay the meaſures down upon their reſpective lines on the paper.

If it be required, as it ſometimes is, to take a ſurvey from more than one ſtation; remove the table, and by the help of the compaſs fix the table in the ſame poſition as before, and mark another point on the paper, for this ſecond ſtation point; and from thence draw lines to as many objects as may be neceſſary, marking the diſtance from the ſtation as before.

In uſing the plain table, chooſe ſuch a point for your ſtation as ſhall have an object both before and behind the ſtation point, if it be poſſible: and in moving the table from one ſtation to another, it will be neceſſary to prove that it be ſtraight in the line towards the object, and alſo that the diſtance be rightly laid down on the paper. To know whether the table be ſet ſtraight in the line, move the table about till through the ſights of the index you can perceive either the fore or back object; then go round the table, and look through the ſights at the other end of the index, to ſee if the other object can be perceived; if it can, the table is in the line, but if not, the table muſt be ſhifted according to your judgment.

To know if the table be in the right part of the line, that is, if the diſtance has been rightly meaſured, fix the table in the ſame poſition as at firſt, and lay the index along the ſtation line; then turn the table about, till the fore and back objects appear through the ſights, and the needle will point to the ſame degree as at firſt; then lay the index over the ſtation point, and any other point on the paper repreſenting an object which is ſeen from the ſtation, and if the ſame object appears ſtraight through the ſights, the ſtation may be depended upon as right, but if not, the diſtance muſt be examined and corrected till the ſaid object can be ſeen.

Of

Of ſhifting the Paper on the plain Table.

When the paper is full written, and it is neceſſary to continue the plan upon another ſheet, draw a line through the fartheſt point of the laſt ſtation line, then take the ſheet off the table, and fix another ſheet on, drawing a line upon it in the moſt convenient part for the reſt of the work: then fold the old ſheet back, cloſe by the line drawn upon it; apply the edge to the line on the new ſheet of paper, and as they lie in that poſition continue the laſt ſtation line upon the new ſheet, and alſo the reſt of the meaſures, beginning at where the old ſheet left off; and ſo on from one ſheet to another. When the work is done, the ſheets are to be faſtened together into one piece, and the lines in each ſheet to be accurately joined together.

It muſt here be noted, that the ſaid joining lines upon the old and new ſheets muſt have the ſame inclination with regard to the points of the compaſs.

3. *Of the Theodolite.*

This is a braſs circular ring divided into 360 degrees, with an index, with open ſights, or a teleſcope moveable upon the centre; alſo a compaſs to point out the bearings, &c. The whole is fixed by the centre upon a ſtand.

When this inſtrument is uſed, a field-book is neceſſary, to note down all meaſures, angles, &c. to be remembered when the plan is drawn.

In uſing this inſtrument, any ſtation may be taken as is judged moſt convenient, but it is beſt to take a ſtation from which moſt objects can be ſeen; and it is neceſſary at every new ſtation to fix the Theodolite in the ſame poſition by means of viewing the fore and back objects, and the compaſs, as in uſing the plain table; regiſtering in the field-book the number of degrees cut off by the index in viewing each object.

The beſt method of uſing this inſtrument is to draw a large circle, quarter it, and mark upon it the number of degrees cut by the index, in viewing each object; then by a parallel ruler draw from ſtation to ſtation, lines parallel to the lines drawn from the centre to the reſpective points of the circumference.

4. *Of the Croſs.*

This inſtrument is only two pair of ſights, ſet at right angles to each other upon a ſtaff with a ſharp point to ſtick in the ground.

The croſs is very uſeful to meaſure ſmall and crooked pieces of ground. The method of meaſuring them is this: meaſure a baſe or chief line in the longeſt direction of the piece, from corner to corner, and, while meaſuring it, find the places where perpendicular lines ſhould fall upon this line, from the ſeveral corners and bends in the piece, with the croſs, by fixing it upon ſuch parts of the line that through one pair of the ſights both ends of the line may be ſeen, and through the other pair of ſights the correſponding angle or angles may be ſeen, and then meaſure the length, &c.

There are alſo ſeveral other inſtruments uſed in ſurveying; as the Circumferentor, which reſembles the Theodolite both in ſhape and uſe; the Semicircle, uſed for taking angles, &c.; and the Perambulator, for meaſuring roads and other great diſtances, on level ground; it has a wheel of 8¼ feet in circumference, upon which the machine turns, and the diſtance meaſured is pointed out by an index moved round by clock-work.

Levels are inſtruments with teleſcopes or other ſights, uſed to find levels, or how much one place is higher than another.

The offset-ſtaff is ten links in length, and uſeful for meaſuring the offſets and other ſhort diſtances.

There are alſo various ſcales uſed in protracting, and meaſuring on the paper; as plain ſcales, protractor, line of chords, line of equal parts, reducing ſcale, parallel and perpendicular rules, &c.

But of all the inſtruments uſed in ſurveying, the plain table is by much the beſt, both in point of accuracy and expedition; for by planning every part immediately upon the ſpot, while the objects are in view, the work is more correct, and a deal of writing is ſaved in the field-book; it alſo affords an opportunity of proving the work at every ſtation: but there are many caſes in which ſome of the other inſtruments will be found more proper; and ſome caſes in which no inſtrument at all is neceſſary, but the chain itſelf, particularly in large open fields lying together.

In ſurveying with any inſtrument, except the plain table, ſome ſort of a field-book is neceſſary to note down the *angles*, diſtances, &c.; but in uſing the plain table, a book is not requiſite, as the whole plan is drawn upon the ſpot.

This book every perſon contrives for himſelf; I ſhall, however, give the form of one as it is generally uſed.

Form of the Field-Book.

Offſets and Remarks on the left.	*Stations, Bearings, and Diſtances.*	*Offſets and Remarks on the right.*
	⊙ 1	21
	10° 20′	
21 —	0 —	Day's hedge.
	24	40
Red Houſe 12	50	0
Offset 20	240	
	⊙ 2	
	870° 14′	
	0	
	30	20 Brown's pond.
A tree.	90	
A ſtile 20	450	10 Sheep pens.

Here, in this ſyſtem, the book is ruled in three columns; the middle one is for the ſtations, angles, bearings, diſtances meaſured, &c. and thoſe columns on the right and left are for the offsets on the right and left ſides, which are ſet againſt their correſponding figures in the middle column, and alſo for ſuch remarks as may be neceſſary to note.

Thus ⊙ 1 in the middle column ſtands for the firſt ſtation where the angle or bearing is 10° 20′. On the left ſide againſt 50 links is an offset of 12 links to the red houſe. On the left at 0, or the beginning of the principal line, is an offset of 20 links, and on the right of the ſame Day's hedge begins; at 24 on the right an offset to the ſame hedge of 40 links, and at 50 the ſame hedge terminates, and at 240 an offset of 20 links on the left.

A line is to be drawn under the work, at the end of every ſtation line, to prevent confuſion.

Notwithſtanding the field-book is in very general uſe, yet a more ready method is to draw by hand a rough figure reſembling that which is to be meaſured, and write the dimenſions as they are found againſt the correſponding parts of the figure. This method may be practiſed in moſt caſes, where the ſurvey is not exceedingly large and intricate.

PROBLEM I.

TO MEASURE A LINE, OR DISTANCE, WITH THE CHAIN.

After having ſet up a picket or ſtation-ſtaff at the end of the line to be meaſured, two perſons are to begin the meaſure at the other end of the line. One perſon is called the leader, and the other the follower.

The leader, having one end of the chain, proceeds forwards in a ſtraight line towards the ſtation-ſtaff, or object, till he has advanced the length of the whole chain: the follower all the while ſtanding ſtill at the beginning of the line, holding the ring, at the other end of the chain, in his hand. When the leader has arrived to the end of the chain, he is directed by the follower waving his hand to the right, or left, till the follower ſees him in the ſame line with the mark to be meaſured to; then both of them ſtooping, and ſtretching the chain, the leader ſticks an arrow in the ground at the end of the chain (he being provided with ten ſmall arrows for that purpoſe). The leader leaving the arrow in the ground, as a mark for the follower to come to, advances another chain forward, being directed by the follower, waving his hand as before, and alſo by moving himſelf from ſide to ſide, till he brings the follower and the firſt mark both in one line; having then ſtretched the chain, and ſtuck down another arrow, as before, the follower takes up his arrow, and they both advance another chain length: thus they continue to do, till all the ten arrows are in the hands of the follower, and the leader is advanced to the end of the eleventh chain without an arrow: the follower then ſends or brings the ten arrows to the leader, and they proceed on as before. Thus, the arrows are changed from the one to the other at the end of every ten chains.

PROBLEM II.

TO TAKE ANGLES AND BEARINGS.

(See Fig. 1. in Surveying.)

Let C D be two objects, and let it be required to take their bearings, or the angle at the ſtation G.

1. *With the Plain Table.*

The table being covered with a paper, and fixed on its ſtand, plant it at the ſtation G, and fix a pin, or the point of the compaſſes, in a point of the paper, to repreſent the point G: cloſe by the ſide of this point lay the fiducial edge of the index, and turn it about, ſtill touching the point G, till one object, D, can be ſeen through the ſights, then by the fiducial edge of the index draw the line G D. In the very ſame manner draw the line C G from the object ſeen at C, and the angle C G D on the paper, will be equal to the angle meaſured.

2. *With the Theodolite.*

Turn the inſtrument about till you ſee the object D through the ſights, and there ſcrew the inſtrument faſt: then turn the index about, till, through the ſights, you ſee the object C; then the degrees cut by the index upon the ring of the inſtrument will ſhow the meaſure of the angle.

3. *With the Chain only.*

Meaſure one chain length, or any other length from the angle, to both objects C and D, then meaſure the diſtance D C, and it is done. This muſt be transferred to paper, by making a triangle G C D, with lengths proportional to the correſponding lengths in the figure.

PROBLEM III.

TO SURVEY A TRIANGULAR FIELD.

1. *By the Chain only.*

Let the field to be meaſured be repreſented by the triangle A B C, (fig. 1.) then having ſet up marks at the corners (which is always to be done, when there are no objects to ſerve as marks) as you meaſure along the line from A to C, when you have arrived about n, where you judge a perpendicular will fall from the angle B, plant the croſs, or any other fit inſtrument; moving it from one place to another, till, through one pair of ſights, you can perceive the marks A C, and through the other ſights the mark B. Then meaſure the remainder of the line from n to C, and alſo the perpendicular n B; thus, having the baſe A C, and the perpendicular n B, the area is eaſily found.

Or the area may be found by meaſuring the three ſides of the triangle, as taught in the foregoing Section.

2. *By taking one or more Angles.*

This is done by meaſuring any two ſides, and taking the angle between them, as A B, B C, and the angle B; or, meaſuring one ſide, and the two adjacent angles, as A B, and the angles A and B.

PROBLEM IV.

TO SURVEY A FIELD OF FOUR SIDES BY THE CHAIN.

Let the field be repreſented by the four-ſided figure A B C G (fig. 1); meaſure from A to C, which will be a diagonal, dividing the field into two triangles, A B C, A C G. In meaſuring along this diagonal line, draw the perpendiculars n B, m G, as directed in the laſt Problem; then find the areas of the two triangles A B C and A G C, and add them together for the area of the trapezium A B C G.

PROBLEM V.

TO SURVEY A FIELD OF ANY NUMBER OF SIDES BY THE CHAIN.

Let the field be repreſented by the figure A B C D E F G (fig. 1.) Take a view of the field, and conſider how it may beſt be divided into triangles and trapeziums; and divide it into as many trapeziums, and as few triangles, as poſſible. Thus, this figure is divided into two trapeziums, A B C G G D E F, and the triangle C G D. Then the areas of the two trapeziums are found by means of the diagonals A C, F D, and perpendiculars B n, m G, G o, p E, as in the laſt Problem: and the area of the triangle, by the baſe C G, and perpendicular q D, as in Problem III. The ſum of theſe two trapeziums, and triangle, is the area of the whole field.

Note. The meaſures of each line may be wrote againſt the correſponding ſides of a ſimilar rough figure, drawn upon paper, or in any other manner.

Figures of this ſort are, in fact, divided into triangles; as each trapezium is divided into two triangles.

PROBLEM VI.

TO SURVEY ANY FIELD WITH THE PLAIN TABLE.

FIG. 2. Plant the table at any one angle, as at C, from whence all the other angles can be ſeen; turn the table about till the needle points to the flower de luce, and there ſcrew it faſt: make a point on the paper for C, and then lay the edge of the index to C, turning it about the point C, till through the ſights you ſee the mark D; and by the edge of the index draw a line D C; then meaſure the diſtance from C, where the table is fixed, to the mark D, and lay that diſtance down on the paper, on the line C D. Then turn the index about the point C, till the mark E can be ſeen through the ſights, and draw the line C E, and meaſure the diſtance from the ſtation C to the mark E, laying the diſtance down on the paper, on the line C E: in the ſame manner find the poſitions of the points A and B; and lay the lengths of theſe lines down on the paper, on the line C A, C B: then join the points by the lines C D, D E, E A, A B, B C.

PROBLEM VII.

TO SURVEY A FIELD BY MEASURING ROUND IT.

Let the field be repreſented by figure A B C D E F (fig. 3.) Having ſet up marks at the angles A B, &c. plant the inſtrument at any point A, and turn it till the fixed index be in the direction A B, and there ſcrew it faſt; then turn the moveable index to the direction A F, and the degrees cut off will be the meaſure of the angle A: meaſure the line A B, planting the inſtrument at B; and there in the ſame manner obtain the angle B; then meaſure B C, and obſerve the angle C; then meaſure the diſtance C D, and take the angle D; then meaſure D E, and take the angle E; then meaſure E F, and take the angle F; and laſtly, meaſure the diſtance F A.

To prove the work;—add all the inward angles A B C, &c. together; and if the work be right, the ſum will be equal to twice as many right angles as the figure has ſides, except four right angles; but when there is an angle that bends inwards, as at F, and you meaſure the outward angles, ſubtract it from four right angles, or 360 degrees, and the remainder will be the inward angle.

Otherwiſe;

Inſtead of taking the inward angles, you may take the outward angles, by producing one ſide of each angle further out, as ſeen in the figure: and if the ſum of all the outward angles be equal to 360 degrees, or four right angles, the work is right: but when one of the angles runs inwards, as F, ſubtract it from the ſum of the reſt, and the remainder will be 4 right angles.

PROBLEM VIII.

TO SURVEY A FIELD WITH CROOKED HEDGES.

Let A B C D E be the ſhape of the field, (fig. 4.) Set up marks within the field, at a b c d, dividing it into as few ſides as poſſible. Then begin at any ſtation, as at a, and meaſure along the imaginary lines a b, b c, c d, d a, taking the offsets as you meaſure along the lines, at a, m, n, o, p, &c. either with the offset ſtaff, or any other inſtrument; which are to be added to the area of the figure. The area is found either by meaſuring a diagonal from b to d, with perpendiculars to a, &c. or by taking the angles at the four corners a, b, c, d, as before directed.

Note. The area of the offsets is found either by the rule for finding the area of a triangle; or taking the mean meaſure of the perpendiculars, for the mean diſtance of the hedge; thus, the offset from o to B forms a triangle whoſe perpendicular is p. But that from B to D muſt be meaſured by adding the two perpendiculars together, and taking half the ſum for the mean diſtance of the hedge from the line b c, which is to be multiplied thereby.

If the meaſurement be taken outſide the offsets, they muſt then be ſubtracted from the area.

PROBLEM IX.

TO SURVEY ANY PLACE BY TWO STATIONS.

Let the place to be meaſured be repreſented by the heptagonal figures A B C D E F G (fig. 5.); plant the plain table at m, for the firſt ſtation; then turn the table about, till through the ſights you percieve n, the other ſtation. Draw the line m n on the paper, along which lay the fiducial edge of the index, and then ſcrew the table faſt; turn the ſights round m, to all the objects A B C, &c. ſucceſſively, drawing a dry line by the edge of the index to each, as m A, m B, &c. Next, meaſure the diſtance on the ground from m, the firſt ſtation, to n, the other ſtation, and there plant the table, and lay that diſtance down on the paper, on the line m n; then lay the index by the ſtation line m n, and turn the table about, till through the ſights you perceive the other ſtation m, and there ſcrew it faſt. Laſtly, direct the ſights ſucceſſively to all the objects A B C, &c. as before, drawing lines from each, as n A, n B, &c. and the interſections of theſe lines with the former at the points A B C D, &c. give the place of the objects, or bounds of the figure.

The ſame method muſt be purſued in ſurveying with the theodolite, or any other inſtrument for taking angles; namely, meaſuring the diſtance m n, planting the inſtrument firſt at one ſtation, then at another; placing the fixed ſights in the direction m n, and directing the moveable ſights to every object, noting the degrees, cut off at each time.

Theſe obſervations being planned, the interſections of the lines give the bounds of the figure, as before.

If all the objects cannot be ſeen from two ſtations, then more ſtations may be uſed; always meaſuring the diſtance from one ſtation to another, placing the inſtrument in the ſame poſition at every ſtation, and from each ſtation obſerving every object that can be ſeen from it, by taking its angular poſition; till the place of every object be determined by the interſections of two or more lines—the more lines the better.

The ſtations may be taken either within the bounds of the figure, or in one ſide, or at a diſtance, and without the bounds of the figure.

In this manner very extenſive ſurveys may be taken, of large eſtates, without entering them; or a ſurvey may be taken of a country, or any parts of a country, by taking two ſtations, on two towers or eminences.

Remarks.

1. Take all the angles between the ſtations that are neceſſary, always meaſuring the diſtance from ſtation to ſtation in a right line. In meaſuring any of the ſtation diſtances, mark where theſe lines meet with any ditches, hedges, roads, rivulets, or other remarkable objects, by meaſuring the diſtance from the ſtation line, and where the perpendicular from it cuts that line; and as you meaſure along any main ſtation line, take the offsets to the ends of all hedges, and to any pond, houſe, rivulet, &c. and be careful to ſet up marks at the interſections of all hedges of the ſtation line, that you may know where to meaſure from when you come to ſurvey theſe particular places. By ſuch means all your ſtation lines are to be meaſured, and the ſituation of all places adjoining to them determined.

2. The inner parts of the ſurvey muſt be determined in like manner, by new ſtation lines; taking inner ſtations at thoſe places where you have the greateſt command or view. Meaſure theſe ſtation lines as before, and note all the interſections with hedges or other objects. Thus you may ſurvey the adjoining fields, by taking the angles that the ſides make with the ſtation line at the interſections; and meaſure the diſtances to each corner from the interſections; for every ſtation line will be a baſis to all the future operations, the ſituation of all parts being entirely dependant upon them; therefore the ſtation lines ſhould be as long as poſſible, and ſhould be ſo contrived as to run along the hedge or bound of ſome field, or to paſs through ſome angle. Theſe things being adjuſted, you muſt take more inner ſtations, and divide and ſubdivide till you come to ſingle fields, repeating the ſame work at the inner ſtations as at the outer ones; and take notice how one field lies by another, to place them properly in the draft.

3. When an eſtate is ſo ſituated that one part cannot be ſeen from another, it muſt be divided into two or more parts, and each part ſurveyed ſeparately, as if they belonged to different owners; and at laſt join them together in the plan.

4. In laying the lines of diſtances down upon the paper, it is neceſſary to have a ſcale of a proper length. For this purpoſe you muſt take the length of the whole eſtate in chains, and conſider how many inches in length the map is to be, and from theſe you may know how many chains there muſt be in an inch on the ſcale: thus, if the length of the eſtate be 200 chains, and the map is to be 20 inches in length, a decimal ſcale ſhould be uſed; and each inch on the map will contain ten chains in length, a tenth of an inch, 1 chain, &c.

5. When the ſtations are long, a good inſtrument ſhould be uſed for taking angles; and the plain table may be uſed for the inner ſtations, as it is a quick and ready inſtrument.

6. When

6. When the draft is finiſhed, place all the trees, and every other remarkable object, in their proper ſituation.

PROBLEM X.

TO SURVEY A LARGE TRACT OF LAND; OR A COUNTY.

1. In chooſing your ſtations, let them be on the top of ſome eminences; as, high hills, mountains, towers, church ſteeples, &c. and ſuch as command the greateſt view, and can be ſeen from each other; and as far diſtant from each other as poſſible. Raiſe a beacon, or flag, of a different colour, at each, to be viſible from the other ſtations.

2. Plant flags alſo at each of thoſe places you ſhould diſtinguiſh in your plan, fixing them upon the tops of church ſteeples, houſes, &c. and changing them to other places when neceſſary.

3. Then from one ſtation take all the angles between the other ſtation and each of thoſe marks, obſerving the different colours of the flags, and note them down in the plan; from the other ſtation take all the angles between the firſt ſtation and each of the former marks, and note them down in the plan alſo againſt the ſame coloured mark; if it be neceſſary, the angles may alſo be taken from a third ſtation, which will ſerve to prove the work, if the three lines interſect each other in the ſame point. The inſtrument for taking angles muſt be an exceedingly good one. A circumferentor is the beſt of four or five feet radius with teleſcope ſights.

4. It is not neceſſary to meaſure every diſtance, becauſe any ſtationary line being exactly laid down from any ſcale, all the other lines will be proportional to it: but ſometimes it is neceſſary to meaſure the diſtance of a place in miles: and in meaſuring long diſtances, it will not be exact enough to meaſure along the high roads, by reaſon of their windings; but the beſt way is, to meaſure in a ſtraight line, over hills and dales, and all obſtacles, if poſſible; but where it is not

poſſible, ſuch parts muſt be meaſured by the method of meaſuring inacceſſible diſtances. A good compaſs is alſo neceſſary, to ſhow the bearing of the two ſtations, and to direct you to go ſtraight when you do not ſee the ſtations.

5. From all your ſtations, and in the whole progreſs, be very careful in obſerving all remarkable objects; as, towns, caſtles, churches, windmills, rivulets, bridges, &c. &c.

6. Having taken your obſervations from the main ſtations which command the whole country, then proceed to take inner ſtations, as in ſurveying large eſtates; and from theſe new ſtations determine the places of as many of the remaining towns as you can; and if any remain, the ſituation of which you cannot take, take more inner ſtations; and thus proceed through all the parts, taking one ſtation after another as long as is neceſſary.

7. Laſtly, take the poſition of the ſtation line with regard to the points of the compaſs, by an aſtronomical proceſs, as follows: hang a plummet in the ſun, over ſome part of the ſtation line, and when the ſhadow of the plummet line runs along the ſtation line, at that moment take the ſun's altitude; then, having his declination and the altitude of the place, the azimuth will be found by ſpherical trigonometry; for the azimuth is the angle which the ſtation line makes with the meridian: and thus the meridian may be drawn through the map.

PROBLEM XI.

TO SURVEY A TOWN OR CITY.

The proper inſtrument for this purpoſe is the plain table; as the diſtances are not very great, and every part ſhould be laid down while in ſight.

Take the firſt ſtation at the meeting of two or more of the principal ſtreets, in order to get the longeſt ſtation lines, then draw the lines of direction to repreſent theſe ſtreets: having choſen objects, meaſure the ſtreets, and lay the meaſures down from a ſcale, on the lines on the paper, taking offsets with the ſtaff, to all bendings and windings in the ſtreets, and to all remarkable objects; then take another ſtation in one of the foregoing lines, and repeat the proceſs.

Thus, (fig. 6.) having choſen A for the firſt ſtation, draw two lines in the direction of the two ſtreets meeting there; and meaſure A B, marking the ſtreet on the left, and lay the ſame meaſure down on the plan: let B be the ſecond ſtation, from which draw the directions of the ſtreets meeting there; meaſure from B to C, noting the ends of the ſtreets at n and o as you paſs them; and lay the ſame meaſure down on the plan. At C, the third ſtation, take the direction of the ſtreets meeting there, and meaſure from C to D. At D, the next ſtation, do the ſame, and meaſure D E, noting the croſs ſtreet at p; and in the ſame manner take the directions of all the principal ſtreets, laying the meaſures down upon the plan. Then proceed to the next ſmaller ſtreets; and laſtly, to the lanes, alleys, courts, &c. uſing the ſame proceſs throughout the whole.

Note. In taking the ſurvey of a town there is no neceſſity for having objects fixed up, as there are *generally* marks enough to ſerve as objects; ſuch as the doors or windows of houſes, &c.

OF PLANNING, CASTING UP, AND DIVIDING.

PROBLEM I.

TO LAY DOWN THE PLAN OF A SURVEY.

When the ſurvey is taken by the plain table, a rough draft of the plan is already drawn on the paper; but where the ſurvey is taken with any other inſtrument, the plan is to be drawn from the meaſures taken in the ſurvey: firſt, a rough draft is to be drawn on paper, in which all the lines and angles muſt be laid down in the ſame order in which they were taken in the ſurvey; laying down firſt the angles, then the lengths of the lines, with the places of the offsets; then the offsets themſelves. All theſe ſhould be done with dry or obſcure lines; then a black line drawn through the extremities of all the lines, and the offsets will be the bounding line of the whole plan.

After the principal lines and bounds are laid down, proceed to mark the ſmaller objects, till every thing be laid down that is neceſſary.

The north ſide of the map or plan is commonly placed uppermoſt. And in ſome vacant part a ſcale of equal parts muſt be drawn. All hills muſt be ſhadowed, and all hedges coloured with different colours; all hilly grounds ſhould be drawn like broken hills and valleys: foot-paths are repreſented by ſingle dotted lines, and roads by double ones.

In taking large ſurveys, all oblique lines, ſuch as are meaſured up and down hills, muſt be reduced to ſtraight lines, by making a proper allowance; for which purpoſe there is generally a table engraved on ſome of the inſtruments.

PROBLEM II.

TO CAST UP THE CONTENTS OF A FIELD.

This is no more than what has been taught in the firſt ſection of this chapter, for finding the contents of figures, whether they be triangular, ſquare, four-ſided, polygonal, or circular figures, by the proper rules there delivered; then, having found the area in links, after you have cut off five figures on the right hand, the remainder is acres. The five ſeparated figures are to be multiplied by 4 for roods, and five figures cut off from this product are to be multiplied by 40 for perches.

In pieces of land bounded by winding hedges, in meaſuring the offsets, all the parts between the offsets are accurately meaſured ſeparately, like ſmall triangles or trapezoids; but ſometimes ſuch pieces are meaſured by finding a mean breadth, which is done by dividing the ſum of the offsets by the number of them, accounting that for one where the boundary meets the ſtation line, and the quotient is the mean breadth: but this laſt method is not very exact.

But in very large ſurveys, and where there are many fields, the beſt way of finding the contents of the whole is to make a rough plan of the whole, and divide it into ſeveral trapeziums and triangles, and find the area of each ſeparately, and add them together; then, to prove the work, divide the whole eſtate into as few triangles or trapeziums as poſſible, by drawing new lines in the plan: and if the contents found by this laſt method be equal to the contents found by the former method, the work is right; but if they differ, the work muſt be examined and recomputed till they nearly agree.

But the chief difficulty in caſting up the contents ariſes from the ſides of the figure being curved, or of any other irregular ſhape; in which caſe ſuch bounds of the figure muſt be reduced to ſtraight lines, in the following manner: make a ſmall bow with a piece of horſehair, and whalebone, or cane, or any other elaſtic ſubſtance, which will keep the horſehair extended at full ſtretch; then apply the horſehair to the crooked bounds of the figure, in ſuch a manner, that thoſe parts of the curve which are on one ſide the hair may be equal to the parts of the curve on the other ſide the hair; then, by making two points, draw a ſtraight line in the direction of the horſehair, which will be a ſtraight ſide to the figure, equal to the curve one. Do the ſame by every curve ſide of the figure; thus the figure may be reduced into a right-lined figure.

PROBLEM III.

TO TRANSFER A PLAN FROM ONE PAPER TO ANOTHER.

There are ſeveral methods of performing this; four of which I ſhall mention:

Firſt. Rub the back of the plan over with black lead powder, and then lay the black ſide upon the ſheet of paper on which the plan is to be copied, keeping it ſteady: then with a blunt point of a tracer, trace over all the lines in the plan, preſſing the tracer ſo that the black lead on the back of the paper may be transferred to the clean paper; then take off the plan, and you will ſee all the marks on the clean paper in black lead, which muſt be traced over with a pen and ink, &c.

Secondly. The plan may alſo be transferred to another paper, by dividing both ends and ſides of the plan into any convenient number of equal parts, connecting the correſponding points of diviſion with lines; which will divide the whole plan into a number of ſquares, or parallelograms: then divide the paper upon which the plan is to be drawn into the ſame number of ſquares or parallelograms; next copy the parts contained in the ſquares of the old plan, in the correſponding ſquares of the new ones. (See figures 40 and 41.)

Thirdly. Another method is by the inſtrument called a pentagraph, which will copy the plan in any ſize required.

Fourthly. The beſt method of any is the following: fix the old plan on the front of a copying frame of glaſs, with the face uppermoſt (which is a large ſquare of the beſt window glaſs, ſet in a broad frame of wood, and conſtructed ſo as to be raiſed up to any angle whatever), the clean paper on the face of the old plan being fixed to the frame by ſeveral pins; then the frame being raiſed up facing the window, by means of the light ſhining through the paper, you will perceive every line of the plan through the clean paper, which is to be drawn thereon with a pencil: having copied that part which covers the glaſs, the other part is to be brought over the glaſs, and copied as before, and ſo on throughout the whole.

Then take them aſunder, and trace the lines with pen and ink; and finiſh the piece, writing ſuch names as are neceſſary.

Thus the fineſt plan may be copied without the leaſt injury.

The foregoing rules will be found fully ſufficient to inſtruct any ordinary capacity in all the practical parts of Menſuration and Surveying.—I have avoided every thing that ſavours more of curioſity than real utility.

SECT. III.

OF SOLID MEASURE, MEASURING TIMBER, DIGGING, AND GAUGING.

Definitions.

1. Solids are ſuch bodies as have length, breadth, and thickneſs.

2. A priſm is a ſolid body whoſe ends are ſimilar and equal plain figures, and its ſides are parallelograms. It is called a triangular priſm when its ends are triangles (as fig. 1. in the ſolids); when its ends are ſquares, a ſquare priſm; when pentagons, a pentagonal priſm, &c.

3. A cube is a ſquare priſm, having ſix equal ſquare ſides perpendicular to each other. (Fig. 2.)

4. A parallelopipedon is a figure having ſix rectangular ſides; each two oppoſite ſides being equal, and parallel. (Fig. 3.)

5. A cylinder is a round priſm, having two equal circles for its ends. (Fig. 4.)

PROBLEM X.

TO FIND THE SUPERFICIAL CONTENT OF A BOARD OR PLANK.

Rule. Multiply the length by the mean breadth.

Note. When the board is tapering, add the breadths of the two ends together, and take half the ſum for the mean breadth.

Example 1. What is the ſuperficial content of a board, whoſe length is 20 feet 3 inches, and mean breadth 10 inches?

By Decimals.	*By Duodecimals.*
	Feet. Inches.
20.25	20 .. 3
10	10
12)202.50	Anſwer 16 .. 10 .. 6
16.875 Anſwer.	

When the board does not run regularly tapering, but is broader in ſome parts than others; take ſeveral breadths and add them together, and divide the ſum by the number of them, and the quotient will be the mean breadth.

PROBLEM XI.

TO FIND THE SOLID CONTENT OF SQUARE OR FOUR-SIDED TIMBER.

Rule. Multiply the mean breadth by the mean thickneſs, and that product by the length; and the laſt product will be the ſolid content.

Note. If the timber taper regularly from one end to the other, the mean breadth and thickneſs is found either by meaſuring the middle of the piece, or by taking half the ſum of the two ends.

But if the timber does not taper regularly, take ſeveral dimenſions, and divide their ſum by the number of them, for the mean breadth or thickneſs.

Example 1. What is the ſolid content of a piece of timber, the length of which is 20 feet 6 inches, the breadth at the greater end 2 feet 3 inches, and at the leſs end 1 foot 6 inches; and thickneſs at the greater end 2 feet, and at the leſs end 1 foot 9 inches?

Decimals.		*Duodecimals.*
2.25		2 3
1.5		1 6
2)3.75		2)3 9
1.875	mean breadth	1 10 6
2.0		2 0
1.75		1 9
2)3.75		2)3 9
1.875	mean thickneſs	1 10 6
1.875	mean breadth	1 10 6
9375		1 10 6
13125		1 6 9
15000		11 3
1875		
3.515625		3 6 2 3
20.5	length	20 6
17578125		70 3 9 0
70312500		1 9 1 1
72.0703125	Anſwer	72 0 10 1

Qu. 2. What is the content of a piece of timber, the length of which is 20.38 feet, and its ends unequal ſquares, the ſide of the greater being 10⅛, and the ſide of the leſs 9⅞ feet? Anſwer 29.838 feet.

PROBLEM XII.

TO FIND THE SOLIDITY OF ROUND TIMBER.

The common rule is to multiply the ſquare of the quarter girt, or one quarter of the mean circumference, by the length; and the product will be the content.

The quarter girt is a geometrical mean proportional, between the mean breadth, and thickneſs; that is, the ſquare root of their product: unſkilful meaſurers uſe the arithmetical mean inſtead of it, that is, half their ſum; but this occaſions a very great error in the contents, and particularly when the breadth and depth differ much from each other.

When the round timber or tree is tapering, take the mean dimenſions from the dimenſions of each end, as in the foregoing Problem, either by girting it in the middle for the mean girt, or half the ſum of the two ends; but when the tree is irregular, find the content of each part ſeparately.

Note. This rule gives the contents about ¼ leſs than the true quantity, ſo that it makes an allowance for hewing the tree ſquare. But when the true quantity is required, it is found by the next rule.

Example 1. What is the content of a piece of round timber, being 9 feet 6 inches long, and its mean quarter girt 3 feet 6 inches?

Decimals.		*Duodecimals.*
3.5	quarter girt	3 6
3.5		3 6
175		10 6
105		1 9
12.25		12 3
9.5	length	9 6
6125		110 3
11025		6 1 6
116.375	content	116 4 6

PROBLEM XIII.

TO FIND THE CONTENT OF A TREE, OR ROUND PIECE OF TIMBER, MORE EXACTLY.

Rule. Multiply the ſquare of $\frac{1}{5}$ of the mean girt by twice the length, and the product will be the contents.

Example. What is the content of a tree, the length of which is 9 feet 6 inches, and mean girt 14 feet?

Decimals.		*Duodecimals.*
2.8	$\frac{1}{5}$ of mean girt	2 9 7
2.8		2 9 7
22 4		5 7 2
56		2 1 2
		1 7
7.84	ſquare of $\frac{1}{5}$ of mean girt	7 9 11
19	- twice the length -	19
70 56	Anſwer	148 8 5*
78 4		
148.96	Anſwer.	

PROBLEM XIV.

TO FIND THE DEPTH, BREADTH, OR LENGTH OF ANY VAULT, CELLAR, RESERVOIR, &c. TO BE MADE.

Rule. Divide the ſolid content by any one of the three dimenſions (viz. length, breadth, or depth), and the quotient will be the product of the two other dimenſions.

Example. What will be the length and breadth of a ſquare reſervoir of water, to contain 5000 cubic feet of water, and the depth to be 20 feet?

---

* The anſwer by duodecimals is leſs in this queſtion than the truth, as all the parts could not conveniently be included in the operation.

2,0)500,0

```
 2,0) 500,0
      ──────
      250.00(15.8   Anſwer.
      1
      ─────
 25)  150
      125
      ─────
308)   25.00
      ─────
         36
      ═════
```

Qu. 2. A bowling-green, 300 feet long and 200 feet broad, is to be raiſed one foot higher by the earth to be dug out of a ditch that encompaſſes the green, and is 8 feet in breadth; to what depth muſt the ditch be dug? Anſwer $7\frac{23}{86}$ feet.

Of Gauging.

Gauging is moſt commonly performed by the ſliding rule, or gauging rod. But I ſhall here give the practice of it, as wrought by calculation of the common meaſures only.

By gauging is always underſtood the method of finding the content of any hollow veſſel, and is moſtly uſed for finding the content of caſks, coppers, &c.

The rule is to find the content of the veſſel in inches, and divide them by 282, if for ale gallons; and by 231 for wine gallons; becauſe 282 cubic inches contains an ale gallon, and 231 cubic inches is a wine gallon. But if any prefer multiplication to diviſion, when the ſolid content is found, it may be multiplied by .003546 for ale gallons, inſtead of being divided by 282; and by .004329 for wine gallons, inſtead of being divided by 231; for dividing by 282 or 231 is the ſame as multiplying by theſe vulgar fractions $\frac{1}{282}$ or $\frac{1}{231}$; and the above multipliers are the decimal fractions, equal to theſe vulgar ones.

In veſſels of a circular or elliptical form, to obtain the content in inches, it is neceſſary to multiply by .7854, and then to reduce them to ale or wine gallons. Therefore to multiply by .7854, and then divide by 282, or 231, is the ſame as to multiply by the fraction $\frac{785.4}{282}$ or $\frac{7851}{231}$, and the decimal fractions equal to theſe vulgar ones are .0027851 for ale, and .00339999 for wine.

PROBLEM XV.

TO GAUGE A CASK.

There are ſeveral methods of gauging caſks according to their ſhapes, or the curvature of the ſides, of which there are four general forms, but the content of any of them may be found by the following general rule:

Rule. To the ſquare of twice the middle diameter add the ſquares of the head and bung diameters. Multiplying the ſum by $\frac{1}{6}$ of the length, and that product multiplied or divided by the circular multipliers or diviſors, will give the contents: or multiply the ſquares of the diameters by the length of the caſk, and that product again by .0004$\frac{2}{3}$ for ale gallons, or by .0005$\frac{2}{3}$ for wine gallons.

Example 1. Required the contents of any caſk, whoſe length is 40 inches, the bung diameter being 32, the head diameter 24, that between the bung and head 28$\frac{3}{4}$ inches.

```
     57.5         24          32
     57.5         24          32
    28875         96          64
   40250         48          96
  28875          576        1024
  3306.25
  1024
   576
  4906.25
       40
  1962 50              196250
  .000 4⅔              .0005⅔
  785000               981250
  1308 33              130833
  91.5833 Gallons.     111.2083 Wine.
```

PROBLEM

PROBLEM XVI.

TO FIND THE CONTENT OF ANY CASK, FROM THREE DIMENSIONS ONLY.

Rule. Add into one ſum 39 times the ſquare of the bung diameter, 25 times the ſquare of the head diameter, and 26 times the product of the two diameters: then multiply the ſum by the length, and that product by .00034, and divide the laſt product by 9 for wine gallons, and divide by 11 for ale gallons, or multiply by .00003$\frac{1}{11}$ for ale.

Example. What is the content of a caſk, whoſe length is 40, the bung diameter 32, and head diameter 24 inches?

32	24	32
32	24	24
64	96	128
96	48	64
1024	576	768
39	25	26
9216	2880	4608
3072	1152	1536
39936	14400	19968
	39936	
	19968	
	74304	
	40	
	2972160	2972160
	.00034	.00003$\frac{1}{11}$
	11888640	8916480
	8916480	270196
	9)1010.53440	Ale 91.86676 gallons.
Wine gallons	112.2816	

PROBLEM XVII.

TO ULLAGE A STANDING CASK *.

Rule, Add together the ſquare of the diameter at the ſurface of the liquor, the ſquare of the diameter of the neareſt end, and the ſquare of double the diameter taken in the midway between the other two; then multiply this ſum by the length between the ſurface of the liquor and the neareſt end, and multiply the product again by $.0004\frac{2}{3}$ for ale gallons, or by $.0005\frac{2}{3}$ for wine gallons, in the leſs part of the caſk, whether empty or filled.

---

* The ullage of a caſk is what it contains when only partly filled, and it is conſidered in two poſitions; namely, either ſtanding on its end with its axis perpendicular to the horizon, or as lying on its ſide with the axis parallel to the horizon.

Example. The three diameters being 24, 27, and 29, what is the ullage for ten wet inches?

24
24
96
48
576

29
29
261
58
841

54
54
216
270
2916

2916
841
576
4333
10
43330
$.0005\frac{2}{3}$
216650
28886
24.5536 Wine.

43330
$.0004\frac{2}{3}$
173320
28886
Ale 20.2206

PROBLEM XVIII.

TO ULLAGE A LYING CASK.

Rule. Divide the wet inches by the bung diameter, and find the quotient in the column of verſed ſines in the table of circular ſegments at the end of this Section, taking out its correſpondent ſegment; then multiply this ſegment by the whole content of the caſk, and the product again by $1\frac{1}{4}$ for the ullage required.

Example. If the bung diameter be 32 inches, and content 92 ale gallons, what is the ullage of 8 wet inches?

32)

6. A pyramid is a figure that has a right-lined figure for its baſe, and each of its ſides is a triangle, whoſe vertices meet in a point at the top, which is called the vertex of the pyramid, as A (fig. 5.) The pyramid takes its name from the figure of its baſe, like the priſm.

7. A cone is a round pyramid, having a circular baſe. (Fig. 6.)

8. A ſphere is that ſolid body bounded by one continued convex ſurface, every part of which is equally diſtant from the centre. It may be ſuppoſed to be formed by the revolution of a ſemicircle about its diameter. (Fig. 7.)

9. The axis of any ſolid is a line drawn from the middle of one end to the middle of the oppoſite end. Thus the axis of a pyramid is a line drawn from the top, or vertex, to the middle of the baſe; and the axis of a ſphere is a line drawn through the centre. When the axis is perpendicular to the baſe, it is a right priſm, or pyramid; otherwiſe it is oblique.

10. The height or altitude of a ſolid is a line drawn from the vertex, or top, perpendicular to the baſe.

11. If the baſe of a priſm or pyramid be a regular figure, it is called a regular priſm or pyramid: but if the baſe be irregular, the priſm or pyramid is called irregular.

12. The ſegment of a pyramid is a part cut off the top by a plane, parallel to the baſe, as C E, (fig. 6.)

13. A fruſtum, or trunk, is the part that remains after the ſegment is cut off, as E A, (fig. 6.)

14. A zone of a ſphere is a part intercepted between two parallel lines.

15. The ſegment of a ſphere is a ſegment leſs than a hemiſphere, or half ſphere; like a cone, whoſe baſe is the ſame as the baſe of the ſegment, and whoſe vertex paſſes through the centre of the ſphere.

16. A circular ſpindle is formed by the revolution of the ſegment of a circle, about its chord, which remains fixed. (Fig. 8.)

17. A regular body is a ſolid, contained under a certain number of equal and regular plain figures.

18. The faces of a ſolid are the plain figures under which it is contained.

19. There are only five ſolid bodies which are regular, *i. e.* which have each ſide equal and ſimilar: namely, firſt, the tetraedon, which is a regular pyramid, having four triangular faces. Second, the hexaedron, or cube, which has ſix equal ſquare faces. Third, the octaedron, which has eight triangular faces. Fourth, the dodecaedron, which has twelve pentagonal faces. Fifth, the icoſaedron, which has twenty triangular faces.

Note. As ſquare meaſure is greater than running meaſure, ſo cubic meaſure is greater than ſquare meaſure.

The Table of Cubic Meaſure.

1728	Inches make	1	Foot
27	Feet........	1	Yard
$166\frac{3}{8}$	Yards........	1	Pole
64000	Poles........	1	Furlong
512	Furlongs.....	1	Mile.

PROBLEM I.

TO FIND THE SOLIDITY OF A CUBE, PARALLELOPIPEDON, PRISM, OR CYLINDER.

The ſolidity of any of theſe figures is found by the following rule:—

Rule. Find the area of one end of the figure by the rules for finding the area of a ſuperficies in the foregoing Section, and multiply ſuch area by the length of the figure, and the product is the ſolid contents.

Example 1. What is the content of a triangular priſm, whoſe 3 ſides are 3, 4, and 5 feet, and length 10 feet?

3	6	6	6	3	10
4	3	4	5	2	6
5	3	2	1	6	Anſ. 60 feet.
12				6	
				)36(6	

Here the three ſides of the triangular end are added together, and from the half ſum each ſide is ſubtracted ſeparately, and the remainders and half ſum multiplied continually together: then the ſquare root of the product is the ſuperficial content, as taught in the foregoing Section; which, multiplied by the length, gives the ſolid content.

Cubes, parallelopipedons, and cylinders, have their ſolid contents found in the ſame manner.

Qu. 2. What is the ſolid content of a cylinder, whoſe length is 20 feet, and circumference 5½ feet? Anſwer 48.1459.

PROBLEM II.

TO FIND THE SOLIDITY OF A CONE OR PYRAMID.

Rule. Multiply the area of the baſe by the height, and one third of the product will be the content.

Example 1. What is the ſolidity of a cone, whoſe height C D is 20 feet, and diameter A B of the baſe 3 feet? (Fig. 6.)

.7854
9
7.0686 Area of the baſe.
20
3)141.3720
47.1240 Anſwer.

Example 2. What is the ſolid content of a pentagonal pyramid, the height whereof is 12 feet, and each ſide of the baſe 2 feet?

1.720477 Tab. area.
4
6.881908 Area baſe.
4 One third of the height.
27.527632 Anſwer.

PROBLEM III.

TO FIND THE SOLIDITY OF A SPHERE OR GLOBE.

Rule. Multiply the cube of the axis by .5236.

Example 1. What is the ſolidity of a ſphere or globe, whoſe axis or diameter is 12 feet?

12
12
144
12
1728
.5236
10368
5184
3456
8640
904.7808 Anſwer.

PROBLEM IV.

TO FIND THE SOLIDITY OR SUPERFICIES OF ANY REGULAR BODY.

1. *For the Superficies.*

Rule. Multiply the tabular area taken from the following table of ſurfaces by the ſquare of the linear edge of the ſolid.

2. *For*

2. *For the ſolid Content.*

Multiply the tabular ſolidity by the cube of the linear edge.

Surfaces and Solidities of regular Bodies.

No. of Sides.	*Names.*	*Surfaces.*	*Solidities.*
4	Tetraedron	1.73205	0.11785
6	Hexaedron	6.00000	1.00000
8	Octaedron	3.46410	0.47140
12	Dodecaedron	20.64573	7.66312
20	Icoſaedron	2.66025	2.18169

Example 1. If the linear edge of the tetraedron be 3 feet, what is its ſurface and ſolidity?

0.11785	1.73205 Square of the edge.
27 Cube of the edge.	9
82495	15.58845
23570	
3.18195 Anſwer.	

Qu. 2. What are the ſuperficies and ſolidity of the octaedron, whoſe linear ſide is 2 feet? Anſwer, ſuperficies 13.85640, and ſolidity 3.77120 feet.

Qu. 3. What are the ſuperficies and ſolidity of the icoſaedron, whoſe linear ſide is 2 inches? Anſwer, ſuperficies 10.64100, and ſolidity 17.45352 inches.

PROBLEM V.

TO FIND THE CONVEX SURFACE OF A CYLINDER, OR THE UPRIGHT SURFACE OF ANY PRISM.

Rule. Multiply the circumference by the height, and the product will be the anſwer.

Qu. 1. What is the convex ſurface of the cylinder whoſe length is 20 feet, and diameter 2 feet? Anſwer 125.664 feet.

PROBLEM VI.

TO FIND THE CONVEX SURFACE OF A RIGHT CONE.

Rule. Multiply the circumference of the baſe by the ſlant height, or length of one ſide; and half the product will be the ſurface.

Example 1. If the diameter of the baſe be 3 feet, and the ſide of the cone 10 feet, what is the convex ſurface?

3.1416	
3	
9.4248	Circumference.
5	Half of the height.
47.1240	Anſwer.

PROBLEM VII.

TO FIND THE CONVEX SURFACE OF A SPHERE OR GLOBE.

Rule. Multiply the diameter by the circumference, and the product will be the anſwer.

Example 1. What is the ſurface of a globe, whoſe diameter is 12 inches?

3.1416	
12	Diameter.
37.6992	Circumference.
12	
452.3894	Anſwer.

In the ſame manner the convex ſurface of any zone, or ſegment of a ſphere, is found: viz. by multiplying the height of the zone or ſegment by the whole circumference of the ſphere.

Qu. 2. What is the ſuperficies of the earth, its diameter being 7957¾ miles, and the circumference 25,000 miles? Anſwer 198,943,750 ſquare miles.

PROBLEM VIII.

TO FIND THE SOLIDITY OF THE SEGMENT OF A CONE OR PYRAMID.

The ſegment of a cone, as C E (fig. 6.), or of a pyramid, may be conſidered as an entire cone or pyramid; and the ſolidity or ſuperficies is found in the ſame manner, and therefore need not be repeated.

PROBLEM IX.

TO FIND THE SOLIDITY OF THE FRUSTUM OF A CONE OR PYRAMID.

Rule 1. Multiply the mean area of the end by the height of the fruſtum, and the product will be the ſolid content.

2. The mean area of the ends is found by one of the following rules:—

Add the areas of the two ends, and the mean proportional between them, together, and one third of the ſum will be the mean area.

Or, when the ends of the pyramid are regular plain figures, the mean area is found by multiplying one third of the tabular number belonging to the polygon (in the table of multipliers, in the ſuperficial meaſure) by the ſum ariſing by adding together the ſquare of a ſide of each end, and the product of the 2 ſides.

In the fruſtum of a cone, the mean area is found by multiplying the ſum of the ſquares of the two diameters, added to the product of the diameters, by .2618, or one third of .7854.

But if the circumference be taken inſtead of the diameter, the multiplier will be .02654.

EXAMPLE 1. What is the content of the fruſtum of a cone, whoſe height is 20 inches, and the diameters of the two ends 12 and 10 inches?

Greater diameter	12		leſs diameter	10	12
	12			10	10
Square of greater diameter	144			100	120
Square of leſs diameter	100				
Product of the two diameters	120				
	364				
	.2618				
	2912				
	364				
	21 84				
	72 8				
	95.2952	Mean area.			
	20	Height.			
	1905.9040	Anſwer.			

EXAMPLE 2. What is the content of a pentagon fruſtum whoſe height is 50 feet, each ſide of the baſe 3, and each ſide of the leſs end 2 feet?

⅓ of tabular area	.57349	3	2	3
	19	3	2	2
	5 16141	9	4	6
	5 7349	4		
Mean area	10.89631	6		
	50	19		
Anſwer.	544.81550			

Qu. 3. What is the ſolidity of the fruſtum of a cone, the height of which is 25 feet, the circumference at the greater end 20, and the leſs end 10 feet? Anſwer 464.205 feet.

Qu. 4. What is the ſolidity of an hexagonal fruſtum, the height being 6 feet, the ſide of the greater end 18 inches, and that of the leſs 12 inches? Anſwer 24.681722.

32)8(.25, whoſe tab. ſeg. is .153546
92
307092
1381914
14.126232
¼ is 3.531558
Anſwer 17.657790

A TABLE

Of the Areas of the Segments of a Circle,

Whoſe Diameter is Unity, and ſuppoſed to be divided into One Thouſand equal Parts.

Height.	*Area Seg.*	*Height.*	*Area Seg.*	*Height.*	*Area Seg.*
·001	·000042	·027	·005867	·053	·016007
·002	·000119	·028	·006194	·054	·016457
·003	·000219	·029	·006527	·055	·016911
·004	·000337	·030	·006865	·056	·017369
·005	·000470	·031	·007209	·057	·017831
·006	·000618	·032	·007558	·058	·018296
·007	·000779	·033	·007913	·059	·018766
·008	·000951	·034	·008273	·060	·019239
·009	·001135	·035	·008638	·061	·019716
·010	·001329	·036	·009008	·062	·020196
·011	·001533	·037	·009383	·063	·020680
·012	·001746	·038	·009763	·064	·021168
·013	·001963	·039	·010148	·065	·021659
·014	·002199	·040	·010537	·066	·022154
·015	·002438	·041	·010931	·067	·022652
·016	·002685	·042	·011330	·068	·023154
·017	·002940	·043	·011734	·069	·023659
·018	·003202	·044	·012142	·070	·024168
·019	·003471	·045	·012554	·071	·024680
·020	·003748	·046	·012971	·072	·025195
·021	·004031	·047	·013392	·073	·025714
·022	·004322	·048	·013818	·074	·026236
·023	·004618	·049	·014247	·075	·026761
·024	·004921	·050	·014681	·076	·027289
·025	·005230	·051	·015119	·077	·027821
·026	·005546	·052	·015561	·078	·028356

THE AREAS OF THE

Height.	*Area Seg.*	*Height.*	*Area Seg.*	*Height.*	*Area Seg.*
·079	·028894	·114	·049528	·149	·073161
·080	·029435	·115	·050165	·150	·073874
·081	·029979	·116	·050804	·151	·074589
·082	·030526	·117	·051446	·152	·075306
·083	·031076	·118	·052090	·153	·076026
·084	·031629	·119	·052736	·154	·076747
·085	·032186	·120	·053385	·155	·077469
·086	·032745	·121	·054036	·156	·078194
·087	·033307	·122	·054689	·157	·078921
·088	·033872	·123	·055345	·158	·079649
·089	·034441	·124	·056003	·159	·080380
·090	·035011	·125	·056663	·160	·081112
·091	·035585	·126	·057326	·161	·081846
·092	·036162	·127	·057991	·162	·082582
·093	·036741	·128	·058658	·163	·083320
·094	·037323	·129	·059327	·164	·084059
·095	·037909	·130	·059999	·165	·084801
·096	·038496	·131	·060672	·166	·085544
·097	·039087	·132	·061348	·167	·086289
·098	·039680	·133	·062026	·168	·087036
·099	·040276	·134	·062707	·169	·087785
·100	·040875	·135	·063389	·170	·088535
·101	·041476	·136	·064074	·171	·089287
·102	·042080	·137	·064760	·172	·090041
·103	·042687	·138	·065449	·173	·090797
·104	·043296	·139	·066140	·174	·091554
·105	·043908	·140	·066833	·175	·092313
·106	·044522	·141	·067528	·176	·093074
·107	·045139	·142	·068225	·177	·093836
·108	·045759	·143	·068924	·178	·094601
·109	·046381	·144	·069625	·179	·095366
·110	·047005	·145	·070328	·180	·096134
·111	·047632	·146	·071033	·181	·096903
·112	·048262	·147	·071741	·182	·097674
·113	·048894	·148	·072450	·183	·098447

SEGMENTS OF A CIRCLE.

Height.	*Area Seg.*	*Height.*	*Area Seg.*	*Height.*	*Area Seg.*
·184	·099221	·219	·127285	·254	·157019
·185	·099997	·220	·128113	·255	·157890
·186	·100774	·221	·128942	·256	·158762
·187	·101553	·222	·129773	·257	·159636
·188	·102334	·223	·130605	·258	·160510
·189	·103116	·224	·131438	·259	·161386
·190	·103900	·225	·132272	·260	·162263
·191	·104685	·226	·133108	·261	·163140
·192	·105472	·227	·133945	·262	·164019
·193	·106261	·228	·134784	·263	·164899
·194	·107051	·229	·135624	·264	·165780
·195	·107842	·230	·136465	·265	·166663
·196	·108636	·231	·137307	·266	·167546
·197	·109430	·232	·138150	·267	·168430
·198	·110226	·233	·138995	·268	·169315
·199	·111024	·234	·139841	·269	·170202
·200	·111823	·235	·140688	·270	·171089
·201	·112624	·236	·141537	·271	·171978
·202	·113426	·237	·142387	·272	·172867
·203	·114230	·238	·143238	·273	·173758
·204	·115035	·239	·144091	·274	·174649
·205	·115842	·240	·114944	·275	·175542
·206	·116650	·241	·145799	·276	·176435
·207	·117460	·242	·146655	·277	·177330
·208	·118271	·243	·147512	·278	·178225
·209	·119083	·244	·148371	·279	·179122
·210	·119897	·245	·149230	·280	·180019
·211	·120712	·246	·150091	·281	·180918
·212	·121529	·247	·150953	·282	·181817
·213	·122347	·248	·151816	·283	·182718
·214	·123167	·249	·152680	·284	·183619
·215	·123988	·250	·153546	·285	·184521
·216	·124810	·251	·154412	·286	·185425
·217	·125634	·252	·155280	·287	·186329
·218	·126459	·253	·156149	·288	·187234

THE AREAS OF THE

Height.	*Area Seg.*	*Height.*	*Area Seg.*	*Height.*	*Area Seg.*
·289	·188140	·324	·220404	·359	·253590
·290	·189047	·325	·221340	·360	·254550
·291	·189955	·326	·222277	·361	·255510
·292	·190864	·327	·223215	·362	·256471
·293	·191775	·328	·224154	·363	·257433
·294	·192684	·329	·225093	·364	·258395
·295	·193596	·330	·226033	·365	·259357
·296	·194509	·331	·226974	·366	·260320
·297	·195422	·332	·227915	·367	·261284
·298	·196337	·333	·228858	·368	·262248
·299	·197252	·334	·229801	·369	·263213
·300	·198168	·335	·230745	·370	·264178
·301	·199085	·336	·231689	·371	·265144
·302	·200003	·337	232634	·372	·262111
·303	·200922	·338	·233580	·373	·267078
·304	·201841	·339	·234526	·374	·268045
·305	·202761	·340	·235473	·375	·269013
·306	·203683	·341	·236421	·376	·269982
·307	·204605	·342	·237369	·377	·270951
·308	·205527	·343	·238318	·378	·271920
·309	·206451	·344	·239268	·379	·272890
·310	·207376	·345	·240218	·380	·273861
·311	·208301	·346	·241169	·381	·274832
·312	·209227	·347	·242121	·382	·275803
·313	·210154	·348	·243074	·383	·276775
·314	·211082	·349	·244026	·384	·277748
·315	·212011	·350	·244980	·385	·278721
·316	·212940	·351	·245934	·386	·279694
·317	·213871	·352	·246889	·387	·280668
·318	·214802	·353	·247845	·388	·281642
·319	·215733	·354	·248801	·389	·282617
·320	·216666	·355	·249757	·390	·283592
·321	·217599	·356	·250715	·391	·284568
·322	·218533	·357	·251673	·392	·285544
·323	·219408	·358	·252631	·393	·286521

SEGMENTS OF A CIRCLE.

Height.	*Area Seg.*	*Height.*	*Area Seg.*	*Height.*	*Area Seg.*
·394	·287498	·430	·322928	·466	·358725
·395	·288476	·431	·323918	·467	·359723
·396	·289453	·432	·324909	·468	·360721
·397	·290432	·433	·325900	·469	·361719
·398	·291411	·434	·326892	·470	·362717
·399	·292390	·435	·327882	·471	·363715
·400	·293369	·436	·328874	·472	·364713
·401	·294349	·437	·329866	·473	·365712
·402	·295330	·438	·330858	·474	·366710
·403	·296311	·439	·331850	·475	·367709
·404	·297292	·440	·332843	·476	·368708
·405	·298273	·441	·333836	·477	·369707
·406	·299255	·442	·334829	·478	·370706
·407	·300238	·443	·335822	·479	·371705
·408	·301220	·444	·336816	·480	·372704
·409	·302203	·445	·337810	·481	·373703
·410	·303187	·446	·338804	·482	·374702
·411	·304171	·447	·339798	·483	·375702
·412	·305155	·448	·340793	·484	·376702
·413	·306140	·449	·341787	·485	·377701
·414	·307125	·450	·342782	·486	·378701
·415	·308110	·451	·343777	·487	·379700
·416	·309095	·452	·344772	·488	·380700
·417	·310081	·453	·345768	·489	·381699
·418	·311068	·454	·346764	·490	·382699
·419	·312054	·455	·347759	·491	·383699
·420	·313041	·456	·348755	·492	·384699
·421	·314029	·457	·349752	·493	·385699
·422	·315016	·458	·350748	·494	·386699
·423	·316004	·459	·351745	·495	·387699
·424	·316992	·460	·352742	·496	·388699
·425	·317981	·461	·353739	·497	·389699
·426	·318970	·462	·354736	·498	·390699
·427	·319959	·463	·355732	·499	·391699
·428	·320948	·464	·356730	·500	·392699
·429	·321938	·465	·357727		

In this table the diameter of the circle is 1, and the whole area ·785398; the figures in the columns of heights are the height of the ſegment, or the verſed ſine of half its arc; and thoſe in the columns of area ſegments are the areas of the circular ſegments, whoſe heights ſtand on the left hand in the column of heights.

The uſe of this table is to find the area of any ſegment of a circle, by the following rule:

Rule. Divide the height of the given ſegment by the diameter of the circle, and the quotient is the height of the ſegment: and oppoſite this height, in the table, is the tabular area, which is to be multiplied by the ſquare of the diameter, and the product will be the area of the ſegment.

Example. What is the area of a ſegment of a circle, whoſe height is 5 inches, and the diameter 30 inches?

30)5.000(.166 Tab. height
·085544 Tab. ſegment
900 Square of the diameter.
Anſwer. 76.989600

30
30
900

Note. In dividing the given height by the diameter, if the quotient does not terminate in three places of decimals, then the area for that fractional part ought to be proportioned for as follows:

Having found the tabular area anſwering to the firſt three decimals of the quotient, take the difference between it and the next following tabular area; which difference multiplied by the fractional part remaining, the product will be the correſpondent proportional part, to be added to the firſt tabular area.

Thus in the laſt example the quotient is .166, and there is a remainder of $\frac{2}{3}$, the tabular area of the ſegment is

.085544

.085544
.086289

⅔745 The difference
248
2

496 ⅔ of the difference
.085544

.086040 The true tab. area.

SECT. IV.

OF INSTRUMENTAL ARITHMETIC.

Inſtrumental arithmetic is the method of calculation by inſtruments made for that purpoſe, for a quicker diſpatch of buſineſs, and for the help of thoſe who are deficient in common arithmetic. The moſt common inſtruments are, the *carpenter's rule, Coggeſhall's ſliding rule, Gunter's line, the gauging rule, and the diagonal rod.*

1. The carpenter's rule conſiſts of two equal pieces, each a foot in length, connected together by a joint: one ſide of this rule is divided into inches, and eighths of an inch; on the ſame ſide are alſo ſeveral plane ſcales, divided into twelfth parts by diagonal lines, for planning dimenſions in feet and inches.

2. On the other ſide of this rule, on one piece, are ſet all the principal lines which are on Coggeſhall's ſliding rule: namely, four lines marked A B C, and D reſpectively; the two middle ones being on a ſlider, which runs in a groove.

Theſe four lines are logarithmic lines; the three marked A, B, C, are all equal; and are called double lines, becauſe the numbers ſet upon them run from 1 to 10, twice: the loweſt line D is a ſingle line, the numbers running from 4 to 40. It is called the girt line, from its uſe in caſting up the contents of trees, and timber. This line alſo may be uſed for gauging; as upon it, at 17.15, is marked W. G. and at 18.95, A. G. for the wine and ale gauge points.

Upon the other part of this ſide is a table of pounds, ſhillings, and pence, ſhowing the value of a load, or 50 cubic feet of timber, at all prices from ſixpence to two ſhillings a foot.

In the uſe of thoſe lines it muſt be ſtrictly obſerved, that when 1 at the beginning of a line is accounted unity, then the 1 in the middle of the line will ſtand for 10, and the 10 at the end of the line will ſtand for 100: and when the 1 at the beginning of the line ſtands for 10, the 10 in the middle of the line will ſtand for 100, and the 10 at the end of the line 1000, &c. and all the intermediate diviſions are altered in proportion.

The edge of the rule is divided decimally, each foot being divided into an hundred equal parts: by this meaſure dimenſions are taken in feet, and decimals of a foot, which is by much the beſt way.

3. Gunter's line is a line of figures, exactly the ſame as the three ſingle lines on the carpenter's rule, and therefore needs no further deſcription. Its uſe is to multiply, or divide numbers, to perform the rule of three direct, &c. It was formerly ſet by itſelf on the carpenter's rule.

The figures 1, 2, 3, 4, &c. ſometimes ſtand ſimply for themſelves, at other times they ſignify 10, 20, 30, &c. again at other times, 100, 200, &c. or 1000, 2000, &c.

4. The gauging rule ſerves to compute the contents of caſks, or any other veſſels, after the dimenſions have been taken. It is a ſquare rule, with logarithmic lines on the ſides, having three ſliders running in grooves in three of the ſides.

Upon the firſt face of this rule are three lines: two, marked A and B, for multiplying and dividing; and one, marked M D, for malt depth, as it ſerves to gauge malt. The middle line B is upon a ſlider, and is a double line, marked at each edge of the ſlider, like that on the carpenter's rule: theſe three lines are all of the ſame radius, or diſtance from 1 to 10, each line containing twice the length of the radius. A and B are numbered exactly alike, and the numbers may be increaſed or decreaſed at pleaſure: thus the firſt number 1 may either ſtand for an unit, 10, or 100, &c. or 1, or 01, or 001, &c.: but whatever the firſt number is, the middle number will be ten times as much, and the laſt number 100 times as much, as on the carpenter's rule.

The 1 on the line M D is oppoſite 215 on the other lines, which are the cubic inches in a malt buſhel; and the diviſions on this line are numbered retrograde to thoſe of A and B.

Upon the lines A and B are ſeveral other marks and letters: thus, on the line A are M B, for malt buſhel, oppoſite the aforeſaid number 215: and A, for ale, oppoſite 282, the cubic inches in an ale gallon; and upon the line B, oppoſite 231, is W, for wine, the cubic inches in a wine gallon; alſo S I, for ſquare inſcribed at 707, which is the ſide of the ſquare inſcribed in a circle, whoſe diameter is 1; and S E, for ſquare equal, at .886, being the ſide of a ſquare equal to the ſame circle: alſo C, for circumference, at 3.1416, which is the circumference of the ſame circle.

Upon the ſecond face, which is that oppoſite the firſt, are a ſlider, and four lines marked D, C, D, E, at one end, and at the other end marked *root*, *ſquare*, *root*, *cubic*; the lines C and E, containing reſpectively the ſquares and cubes of the oppoſite numbers of the lines D, D: the radius of the line D is double to that of C, and triple to that of E; ſo that whatever the firſt one on D denotes, the firſt on C is the ſquare of it, and the firſt on E the cube of it; thus, if D begins with 1, C and E will begin with 1; but if D begins with 10, C will begin with 100, which is the ſquare of 10, and E with 1000, the cube of 10, and ſo on. Upon the line C, at .0796, is marked a c for the area of a circle, whoſe circumference is 1; and a d at .7854, for the area of a circle, whoſe diameter is 1. Alſo upon the line D are W G for wine gauge, at 17.15, and A G for ale gauge, at 18.95; and M R for malt round, at 52.32; theſe three being the gauge points of round and circular meaſure, and are found by dividing the ſquare roots of 231, 282, and 2150.4 by the ſquare root of .7854; alſo M S for malt ſquare is marked at 46.37, the malt gauge point for ſquare meaſure, being the ſquare root of 2150.4.

Upon the third face of this rule are three lines, one marked N upon the ſlider; and two upon the ſtock, marked S S and S L, for ſegment ſtanding and ſegment lying, which ſerve to ullage ſtanding and lying caſks.

Upon the fourth face are a ſcale of inches, and three other ſcales, marked *firſt variety*, *ſecond variety*, and *third variety*; the ſcale for the *fourth variety* being on the inſide of the ſlider on the third face: the uſe of theſe lines is to find the mean diameter of caſks of each of the four different forms or ſhapes of the ſides.

Beſides thoſe lines, there are two others on the inſide of the two firſt ſliders, being continued from one ſlider to the other: one of theſe is the ſcale of inches, from 12½ to 36, and the other a ſcale of ale gallons between the correſponding numbers 435 and 3.61, which forms a table to ſhow, in ale gallons, the contents of all cylinders, whoſe diameters are from 12½ to 36 inches, their common altitude being 1 inch.

5. The diagonal rod is a ſquare rule, commonly four feet long, and folding together by joints; this inſtrument is uſed both for gauging caſks, and computing their contents; and this is performed by taking one dimenſion only; namely, the diagonal of the caſk, that is, the length from the middle of the bung-hole to the meeting of the head of the caſk, with the ſtave oppoſite the bung-hole; and is the longeſt line within the caſk from the middle of the bung. On one face of this rule is a ſcale of inches to take this diagonal by, oppoſite

which are placed the areas, in ale gallons, of circles to the correſponding diameters, in like manner as the lines on the under ſide of the three ſliders in the ſliding rule.

On the oppoſite face are the ſcales of ale and wine gallons, to ſhow the contents of caſks, from having the diagonal. All the other lines on this inſtrument are the ſame as thoſe on the ſliding rule, and to be uſed in the ſame manner.

The Uſe of the Carpenter's Rule.

PROBLEM I.

TO MULTIPLY NUMBERS TOGETHER.

Suppoſe the two numbers 24 and 12; draw the ſlider out, till 1 on the line B is oppoſite to 12 on the line A; then againſt 24 on the line B ſtands 288 on the line A, which is the product of the two numbers.

Note. In any operations, when a number runs beyond the end of the line, ſeek it on the other radius, that is, the other part of the line, which will be the tenth part of the required number.

PROBLEM II.

TO DIVIDE NUMBERS.

To divide 312 by 24; draw out the ſlider till the diviſor 24 on B be oppoſite to the dividend 312 on A, then againſt 1 on B ſtands 13, the quotient on A.

PROBLEM III.

TO SQUARE ANY NUMBER.

Suppoſe to ſquare 12: ſet 1 on the line B to 12 on A, then againſt 12 on B ſtands 144 on A.

PROBLEM IV.

TO EXTRACT THE SQUARE ROOT.

Set 1 or 100, &c. on C to 1 or 10, &c. on D; then againſt every number found on C ſtands its ſquare root on D.

PROBLEM V.

TO FIND A MEAN PROPORTIONAL BETWEEN TWO NUMBERS,

As, ſuppoſe between 29 and 430. Set 29 on C, to the ſame on D; then againſt 430 on C ſtands their mean proportional 111 on D.

PROBLEM VI.

TO FIND THE THIRD PROPORTIONAL TO TWO NUMBERS.

Suppoſe, to 21 and 32; ſet 21 on B to 32 on A; then againſt 32 on B ſtands 48.8 on A; which is the third proportional required.

PROBLEM VII.

TO FIND THE FOURTH PROPORTIONAL TO THREE NUMBERS; OR, TO PERFORM THE RULE OF THREE DIRECT.

Suppoſe to find a fourth proportional to 12, 28, and 114: ſet 12 on B to the ſecond number 28 on A; then againſt the third number 114 on B ſtands 266 on A; which is the fourth proportional ſought.

The Uſe of Gunter's Line.

PROBLEM VIII.

MULTIPLICATION BY GUNTER'S LINE.

To multiply 5 by 7: ſet one foot of the compaſſes on the 1 towards the left hand, and extend the other foot of the compaſſes to 5 upwards, or towards the right hand; and with the ſame extent place one foot of the compaſſes on 7, in the left hand end of the line, and the other foot will fall upon 35 the required product.

PROBLEM

PROBLEM IX.

TO DIVIDE BY GUNTER'S LINE.

To divide 63 by 3; extend the compaſſes from 3 to 1 downwards, or towards the left hand; and that extent will reach the ſame way, from 63 to 21, the quotient.

Note. In multiplying, you muſt always extend upwards, or towards the right hand; and in dividing, extend the contrary way.

PROBLEM X.

TO PERFORM THE RULE OF THREE DIRECT; OR, TO FIND A FOURTH PROPORTIONAL.

Suppoſe 5 buſhels of barley coſt 15 ſhillings, what will 25 buſhels coſt? Extend the compaſſes from 5 to 15 upwards, and that extent will reach the ſame way from 25 to 75 ſhillings, the anſwer.

The Uſe of the Gauging Rule.

PROBLEM XI.

TO MULTIPLY, DIVIDE, PERFORM THE RULE OF THREE, EXTRACT THE SQUARE ROOT, OR TO FIND A MEAN PROPORTIONAL BETWEEN TWO NUMBERS.

All theſe are wrought on the gauging rule exactly in the ſame manner as on the carpenter's ſliding rule; the lines A, B, C, D, on each rule being uſed in the ſame way.

PROBLEM XII.

TO COMPUTE MALT BUSHELS BY THE LINE M D.

To find the malt buſhels in a floor, whoſe length is 230, breadth 58.2, and depth 5.4 inches.

Set 230 on B to 5.4 on M D, then againſt 58.2 on A ſtands 33.6 buſhels on B, for the anſwer.

PROBLEM

PROBLEM XIII.

TO GAUGE A CASK OF THE FIRST FORM.

There are four different forms of caſks, being diſtinguiſhed from each other according to the greater or leſs curvature of their ſides; namely, firſt, that which reſembles the middle fruſtum of a ſpheroid, whoſe ſides have a greater ſwell than thoſe of any other form. Secondly, that which reſembles the middle fruſtum of a parabolic ſpindle, whoſe ſides have not ſo great a ſwell as thoſe of the former; and whoſe length is leſs in proportion to its diameter. Thirdly, that which reſembles the two equal fruſtums of a paraboloid; and whoſe ſides ſwell the leaſt of all. Fourthly, that which reſembles the two equal fruſtums of a cone, and whoſe ſides have no curve, but run ſtraight from each end to the middle of the caſk, where they form a very obtuſe angle.

To find the contents of a caſk of the firſt form, regard muſt be had to that ſcale on the rule which is marked *firſt variety*; then, having the length of the caſk, which ſuppoſe to be 40 inches, and diameters at the head, and bung, 24, and 32 inches; ſet 40 on the line C to the ale gauge 32.82 on D, and againſt

24 on D ſtands	21.3	on C
32 on D ſtands	38.0	on C
The ſame	38.0	
The ſum	97.3	Ale gallons.

Again, ſet 40 on C to the wine gauge 29.7 on D, and againſt

24 on D ſtands	26.1	on C
32 on D ſtands	46.5	on C
The ſame	46.5	
The ſum	119.1	Wine gallons.

PROBLEM

PROBLEM XIV.

TO FIND THE CONTENT OF A CASK OF THE SECOND FORM.

Let the length, and head, and bung diameters be the ſame as in the laſt Problem: then, having ſet 40 on C to 32.82 on D, againſt 8 on D ſtands 2.4 on C, $\frac{4}{10}$ of which is 0.96. This, ſubtracted from 97.3, the contents of the laſt form, leaves 96.34 ale gallons.

Again, having ſet 40 on C to 29.7 on D, againſt 8 on D ſtands 2.9 on C, $\frac{4}{10}$ of which is 1.16, which, taken from 119.1, in the laſt form, leaves 117.94 wine gallons.

PROBLEM XV.

TO FIND THE CONTENT OF A CASK OF THE THIRD FORM.

The dimenſions being the ſame as in the foregoing Problems, ſet 40 on C to 26.8 on D, then againſt

24 on D ſtands	32.0	on C
32 on D ſtands	57.3	on C
The ſum	89.3	Ale gallons.

Again, ſet 40 on C to 24.25 on D: then againſt

24 on D ſtands	39.1	on C
32 on D ſtands	69.8	on C
The ſum	108.9	Wine gallons.

PROBLEM XVI.

TO FIND THE CONTENT OF A CASK OF THE FOURTH FORM.

The dimenſions being the ſame as before, ſet 40 on C to 65.64 on D, then againſt

8 on D ſtands	0. 6	on C
56 on D ſtands	29. 1	on C
The ſame	29. 1	
Ditto	29. 1	
The ſum	87. 9	Ale gallons.

Again, ſet 40 on C to 59.41 on D; then againſt

8 on D ſtands	0. 7	on C
56 on D ſtands	35. 6	on C
The ſame	35. 6	
Ditto	35. 6	
The ſum	107. 5	Wine gallons.

PROBLEM XVII.

TO FIND THE CONTENT OF A CASK BY THE DIAGONAL ROD.

Apply the rod within the caſk, from the bung-hole to the end of the oppoſite ſtave; then take the length of the diagonal, or the meaſure of the rule from the middle of the bung-hole to the end of the ſaid ſtave, which ſuppoſe to be 34.4 inches; then to 34.4 inches on the rod correſpond $90\frac{3}{4}$ ale gallons, or 111 wine gallons, the content required *.

PROBLEM XVIII.

TO FIND THE ULLAGE OF A CASK BY THE RULE.

By one of the foregoing Problems find the whole content of the caſk; then ſet the length on N to 100, on S S for a ſegment ſtanding, or ſet the bung diameter on N to 100, on S L for a ſegment lying; and then againſt the wet inches on N is a number on S S or S L to be reſerved.

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* The contents thus found by the rod give the true contents of the moſt common form of caſks only; that is, ſuch as fall between the ſecond and third forms; of this form are thoſe chiefly uſed by the London brewers.

Next, ſet 100 on B to the reſerved number on A, and againſt the whole content on B will be found the ullage on A.

There are many ſolid figures beſides the foregoing, the contents of which cannot be found by any actual dimenſions taken on the figure, ſuch as ſtatues, &c. The contents of ſuch bodies are found by immerſing them in water, and meaſuring the riſe of the water occaſioned by ſuch immerſion.

SECT. V.

A VIEW OF THE FIVE ORDERS OF ARCHITECTURE.

The origin of architecture is almoſt contemporary with that of civil ſociety. When mankind felt the inclemencies of the weather, and, conſequently, found the neceſſity of erecting habitations for ſhelter, eaſe and convenience ſoon improved itſelf into ornament and grandeur. A few trees perhaps growing round in a circle, and leaning together at the top, afforded the firſt habitations, being interwoven with twigs, and plaſtered over with mud. This gave riſe, in after-ages, to the idea of columns.

It is probable that the inconvenience of theſe habitations rendered their owners deſirous of inventing ſuch as ſhould anſwer every purpoſe of the former ſort, and poſſeſs ſeveral advantages above thoſe: this improvement naturally gave riſe to the invention of the croſs beams to ſupport the roof; for this purpoſe, they had, no doubt, recourſe to the trunks of trees: thus columns were at once introduced.

The Egyptians were the firſt people from whom the Greeks derived all their knowledge of architecture; as appears from thoſe ancient remains, the pyramids; and from the ruins of a magnificent temple at Luxor, the ancient Thebes, of which there remain more than 130 columns, all of conſiderable magnitude; ſome 10 feet in diameter, with elegant ſculpture: and ſeveral other remains in the neighbourhood of the ſame place.

The fertile genius of the Greeks ſoon improved upon the ponderous architecture of Egypt, whoſe ignorance in conſtructing the arch made it neceſſary for the columns to ſtand very thick, with large and ſtrong architraves.

The Tuſcans (or Etruſcans) were alſo very ſkilful in building; from whom the Romans received great help in architecture, in all their large works: the temple of Jupiter Capitolinus, the Cloaca Maxima, &c. are the works of Tuſcan artiſts.

To produce the authority for every ſpecies of ornament in each of the five orders, would be impoſſible, at the preſent day; I ſhall, therefore, point out the more general parts, with the origin of their introduction.

1. The Plinth was, no doubt, at firſt a ſquare ſtone, or tile, placed under the tree or primitive column, only to prevent its rotting by the conſtant moiſture of the earth, and alſo to ſerve as a ſolid footing to the column.

2. The Torus is the ſwell above the plinth, and originated, no doubt, from the form of the tree, being thicker at the bottom than the part above, by which means it was fixed more firmly on the plinth: by ſome it was conjectured to have been a rope or bandage only, round the trunk, to prevent its ſplitting.

3. The Capital had its origin from the parts of the tree where the arms were cut off: the ſwell of the arms gave the idea of the ſwell of the capital, which is attended with this advantage, by being broader at the top, it is better formed for ſuſtaining the works above.

4. The **Abacus** was no more than a tile or ſtone to throw off the water, and to prevent its ſinking into the column.

5. The **Astragals** and **Fillets** were bandages to bind the column.

6. The **Architrave** was beams neceſſary to hold or unite the columns together.

7. The **Frize** was only an addition, when height was wanted in the inner parts.

8. The **Cornice**, with its ornaments, were only the ends or edges of the timbers, neceſſary to ſupport the roof.

9. The **Denteles**, **Triglyphs**, **Modillions**, &c. were accidental improvements added in after-ages, when ſtructures of wood gave place to thoſe of ſtone.

The five orders of architecture are, the Tuſcan, Doric, Ionic, Corinthian, and the Compoſite.

The **Tuscan** order is diſtinguiſhed by its plain and robuſt appearance: it is uſed only where ſtrength and plainneſs are required. The Trajan column at Rome is an inſtance of its effect and elegance.

The **Doric** has nearly the ſame character for ſtrength as the Tuſcan, but it is enlivened by ſeveral ornaments. It is uſed with good effect where united ſtrength and grandeur are required.

The **Ionic** has more delicacy in the conſtruction than either of the former orders; it partakes of a middle order, between the maſculine Tuſcan and Doric, and the ſlender Corinthian: the capital is diſtinguiſhed by its boldneſs, and the ſhaft by its beauty, which makes it applicable to porticoes, frontiſpieces, entrances to houſes, &c.

The **Corinthian** has more ornament and delicacy than any other order: the capital is diſtinguiſhed by its beauty and richneſs, and the pillar by its delicacy: which renders it proper, where elegance only is required. It is frequently uſed for internal decorations, in ſtate rooms, &c.

The Composite is only the Corinthian, with the modern Ionic volute to the capital, which gives it a bolder projection; the proportions and effects are the ſame as thoſe of the Corinthian, and it is applicable to the ſame uſes.

The meaſurements in architecture are taken in minutes; one half of the lower diameter of the column is uſually divided into thirty parts or minutes. The projections are meaſured from the perpendiculars of the ſuperior and inferior parts of the column.

CHAP. VIII.

Of OPTICS.

Definitions.

1. OPTICS is that ſcience which treats of the nature, properties, and laws of viſion: and in its moſt extenſive ſenſe, comprehends the whole doctrine of *light* and *colours*, and all the phenomena or appearances of viſible objects. Optics conſiſts of three parts: viz. *catoptrics*, *dioptrics*, and *chromatics*.

2. Catoptrics is that part which treats of reflected viſion, or that which relates to viewing objects by reflected light, as in mirrors. This is called *reflex viſion.*

3. Dioptrics treats of viewing objects through a different medium or body; as through *glaſs, water, &c.*

4. Chromatics treats of the different colours of light.

5. Light is that property by which bodies are rendered viſible. It conſiſts of an infinite number of extremely ſmall particles, which flow from every body expoſed to it, and in ſtraight lines.

6. A Ray or Beam of Light is a very ſmall part of it, proceeding from the luminous body.

7. The Radiant is the object from which the rays of light proceed.

8. The Species of an object is the image of it, made by the rays of light, in the *focus* of the reflecting or refracting medium.

9. Parallel Rays are ſuch as proceed from a body, parallel to each other.

10. Converging Rays are thoſe rays, which, proceeding from a body, approach nearer each other in their progreſs, and tend to one certain point: thus the rays, proceeding from L N to the point O, are ſaid to converge to that point. (Fig. 5. plate 5.)

11. Diverging Rays are thoſe which, proceeding from any point, do continually recede from each other in their progreſs: thus the rays proceeding from O towards L N are ſaid to diverge. (Fig. 5.)

12. The Focus of rays is that point to which all converging rays tend, as O (fig. 5): this is called the real focus.

13. The Virtual or Imaginary Focus is a point, as f, to which the rays A B tend, and where they would unite, if they were not intercepted by the obſtacle C D (which ſuppoſe a mirror), by which means they are turned aſide, and made to converge in a real focus, E. (Fig. 6.)

14. The REFLECTION of rays is their regreſs, or returning from the ſurface of ſuch bodies, on which they fall, and cannot enter: thus, the rays A B falling on the ſurface C D, are reflected or turned back again in the direction E. (Fig. 6.)

15. MIRRORS, or Speculums, are thoſe bodies whoſe ſurfaces are ſo very ſmooth, as to be impervious to the rays of light which fall on them; and which, therefore, they reflect, ſo as to repreſent the images of the objects expoſed to them: they are generally made of metal, highly poliſhed, or of glaſs poliſhed on one ſide, and ſilvered on the other; and are either plain, convex, or concave.

16. PLAIN MIRRORS are thoſe whoſe ſurfaces are perfect planes, and whoſe ſection is a right line, as C D (fig. 6): theſe are vulgarly called looking-glaſſes.

17. CONVEX MIRRORS are thoſe whoſe ſurfaces do every way equally riſe above the plane of their baſes; the ſection of theſe ſort of mirrors is a curve, and is either circular, elliptical, parabolical, or hyperbolical. A C D (fig. 4) is a circular ſection; and the mirror is the ſegment of a globe, or ſpherical ſurface, which is that moſtly uſed.

18. CONCAVE MIRRORS are thoſe whoſe ſurfaces ſink down with an uniform hollowneſs below the upper parts; whoſe ſection alſo is a curve, as various as the convex; but the circular form is the moſt common.

19. REFRACTION of rays is their being bent, or turned out of their courſe, in paſſing out of one medium into another: as the ray B C (fig. 2), in paſſing into the denſe medium A I H D, is refracted, or turned out of its natural courſe C K, in the direction C E, which is called the refracted ray.

20. The INCIDENT RAY is that which comes from any object, and falls on the refracting or reflecting ſurface, as B C (fig. 2), or A B (fig. 6).

21. The Angle of Incidence is that which is contained between B C (fig. 4), the incident ray, and F C, perpendicular to the reflecting ſurface, that is, the angle B C F.

22. The Angle of Reflection is that contained between the ſaid perpendicular F C and the reflected ray C E, that is, the angle F C E.

23. The Angle of Incidence in Refraction is the angle B C F (fig. 2), as in reflection F C E (fig. 4) is the angle of reflection.

24. A Lens is a medium, generally of glaſs, made to diſperſe, or collect, the rays of light which paſs through it: of theſe there are various forms, of which the moſt common are the eight following:

Firſt, a *Plano-convex*, which has one ſide plain, the other convex; as A (fig. 3).—Second, *Plano-concave*, which is plain on one ſide, and concave on the other; as B.—Third, a *Double Convex*, which is convex on both ſides; as C.—Fourth, a *Double Concave*, which is concave on both ſides; as D.—Fifth, a *Meniſcus*, which is convex on one ſide, and concave on the other; as E.—Sixth, a *Plain Lens*, which is flat on both ſides, and of equal thickneſs in all its parts; as F.—Seventh, a *Flat Plano-convex*, whoſe convex ſide is ground into ſeveral flat ſurfaces; as G.—Eighth, a *Priſm*, which has three flat ſides; and, when viewed end-ways, appears like an equilateral triangle; as H.

Glaſſes ground into any of the foregoing ſhapes are called *lenſes*: and a line going through the middle, as I K, is called the axis of the lens.

25. The Visual or Optic Angle, is that angle contained under two lines, drawn from the extreme points of an object to the eye; thus A I B (fig. 8) is the optic angle, or the angle under which the object A B appears to the eye at I.

26. A Pencil of Rays is a double cone of rays, as L O N F joined together at the baſe in the lens L N; of

which one cone, L O N, has its vertex O in one point of an object; and the other cone, L F N, has its vertex F in that point of the object. The middle line O F is called the axis of that pencil, (fig. 5.)

SECT. I.

OF VISION.

Direct viſion is the faculty of ſight, and is occaſioned by the rays of light proceeding from an object, and paſſing through the humours of the eye, where they form the image of the object on the back part, or bottom of the eye. In order to underſtand which, it will be neceſſary to explain the figure and conſtruction of the human eye.

ABCE is the eye. It is of a ſpherical figure, by which means it is eaſily moved any way in its ſocket, by muſcles appointed for that purpoſe; the fore part at A (fig. 1) is more convex than any other part. The eye is encloſed in three membranes: the outermoſt is called the *Sclerotica*; the ſecond, the *Tunica Choroides*; the fore part of which is called the *Iris*, which conſiſts of many fibres, like ſo many radii: the third membrane, or innermoſt coat, is called the *Retina*, which is nothing but the optic nerve, ſpread over the bottom of the eye; upon this membrane the images of viſible objects are formed.

In theſe three membranes are contained the three humours of the eye: the firſt, H A I, is called the *aqueous* humour, which is a thin watery liquor; the ſecond, F G O, is the *cryſtalline*, in the form of a double convex lens, and more convex in the back part: behind this is the *vitreous* humour K L.

The cryſtalline is more denſe than the vitreous, and the vitreous more denſe than the aqueous humour: the three humours together form a compound lens, which refracts the rays of light, iſſuing from an object P R, to the bottom of the eye; and there paints its image p r, upon the retina, in an inverted poſition.

The aqueous humour is in the form of a meniſcus; as is alſo the vitreous humour. The fore part of that membrane called the ſclerotica is called the *cornea*, as at A, and that part adjoining is called the white of the eye. Within the cornea is that coat called the *uvea*; in the middle of this is a hole O called the pupil, to let in the rays of light: this pupil is contracted or dilated by ſeveral muſcular fibres, in order to let in more or leſs light, as found convenient.

D is the optic nerve, which, coming from the common ſenſorium in the brain, is expanded all over the concave back ſurface of the eye, and thus forms the retina. This nerve is not ſituated in the middle of the eye, but lies nearer the ſide E, in that part next the noſe.

The cryſtalline F G has a ring of fibres round its edge, by which means it can be drawn more or leſs convex, and the diſtance A C is thereby made greater or leſs, in order to form the image p r, upon the retina, for diſtinct viſion. This ring of fibres is called the *ligamentum ciliare*, the back part of which is black, in order to ſtifle the rays which fall upon it. The eye is moved in the head by ſeveral muſcles in the ſclerotica.

If the image of an object do not fall upon the retina at p r, the viſion will be confuſed; if it fall ſhort, or nearer F G, as is the caſe with ſhort-ſighted people, then a concave lens that makes the rays more diverging will bring it to the retina.

If the rays of light do not unite, ſo as to form the image of the object, till they get beyond the retina, as is the caſe with moſt old people, then a common convex lens of a proper form will make them converge ſooner, and ſo form the image upon the retina; therefore long-ſighted people muſt uſe convex glaſſes; and ſhort-ſighted people concave ones.

The ray of light P p, flowing from the point of the object P, and the ray R r, flowing from the point R, croſs each other at O, and proceeding in the ſame ſtraight lines, paint the image of the object P R on the retina, in an inverted poſition, as p r.

Note. Though the rays of light are in the figure repreſented by ſingle lines, yet it muſt be obſerved that every viſible point of the object ſends forth a *pencil of rays*, which croſs each other at O, and paint the image of the object on the retina.

There are many experiments made by philoſophers to demonſtrate the truth of this theory of viſion; the moſt common of which is the following:—take a bullock's eye, while it is freſh, from a newly killed beaſt, and having cut off the three coats from the back part, quite to the vitreous humour, put a piece of paper behind that part, and hold the front of the eye towards any bright object, and there will be an inverted image of the object upon the white paper; which in this caſe ſerves as a retina to the eye.

Though the image of the object is inverted in the bottom of the eye, yet we judge it to be erect, being always uſed to that poſition of the object. By an attentive peruſal of the figure, the poſition of the object, with regard to that of the image, may be eaſily accounted for. Thus, to view that point of the object P, the pupil of the eye at O muſt be turned upwards towards A, in order that the ray P p may fall on the axis of the eye, oppoſite C, where alone diſtinct viſion is performed. And to view that point of the object R, the pupil of the eye muſt be turned downwards, to take in the rays of light R r, ſo that r may fall on the axis of the eye at C.

The diameters of objects are proportioned to the diameters of the images at the bottom of the eye; thus, the angle P O R is equal to the angle p O r.

Some of the moſt common properties of the eye are the following:—

1. The eye can only ſee a very ſmall part of an object diſtinctly at once; therefore the eye muſt be turned ſucceſſively to the ſeveral parts of the object, that each part may fall in or near the axis of the eye.

2. When an object is ſeen diſtinctly with both eyes, the axes of both eyes are directed to that point; thus the object appears ſingle, though it be ſeen by both eyes at once: but if the axes of both eyes are not directed to the ſame point of the object, it will appear double.

3. Few eyes can diſtinguiſh a particle of matter, that ſubtends, at the eye, an angle leſs than half a minute; and very few can diſtinguiſh it when it ſubtends a minute. If the diſtance of two ſtars in the heavens be not greater than this, they will appear as one.

4. The eyes of young people are more convex than thoſe of old people; and this is the reaſon the former can ſee an object nigher than the latter.

5. The eyes of ſhort-ſighted people are too convex to admit of diſtinct viſion, when the object is placed at the diſtance of ſix or eight inches, which is the common diſtance of an object for diſtinct viſion. And the eyes of long-ſighted people are not convex enough to admit of diſtinct viſion at that diſtance. In the former caſe, the rays of light converging from an object through the humours of the eye, unite too ſoon, and before they reach the retina: to remedy which, concave glaſſes are uſed, which render the rays more diverging. And in the latter caſe the rays of light do not unite ſoon enough, in their paſſage through the eye, to paint the image on the retina; the remedy of which is the convex glaſſes.

6. Whatever light falls upon the optic nerve at D makes no impreſſion; therefore if the image of an object fall thereon it is loſt. This place is not at the bottom of the eye, but on one ſide, and towards the noſe in each eye; ſo that no rays that come from any object can fall upon that part in both the eyes at the ſame time.

7. The dimneſs of ſight which generally attends old people ariſes from two cauſes:—Firſt, by the eye growing more flat, and thereby not uniting the rays of light at the retina. Secondly, by the opacity of the humours of the eye, which, in time, loſe ſomewhat of their tranſparency.

8. As the rays of light flowing from an object are the immediate cauſe of viſion; it muſt neceſſarily follow, that where there is no light there can be no viſion.

Of the Properties of Light in general.

It may be neceſſary to give a general deſcription of the nature and properties of light. Sir Iſaac Newton has demonſtrated, beyond a doubt, that the phenomena of viſion are produced by light; which is a matter emitted from the ſhining body with immenſe velocity, moving uniformly in ſtraight lines, and acted on by other bodies, ſo as to be reflected, refracted, or inflected, in various ways, by means of forces, which act on it in the ſame manner as on other inert matter.

Every viſible body expoſed to the light, emits, or reflects a great number of theſe inconceivably ſmall particles of matter from each point of its ſurface; which iſſue from it continually, in ſtraight lines, and in all directions, forming a ſphere, of which the illuminated body is the centre. Theſe rays of light, as before obſerved, entering the eye, and ſtriking upon the retina, are the cauſe of viſion. And as theſe rays differ in ſubſtance, denſity, velocity, or magnitude, they will produce in the mind the idea of different colours.

That the particles of light are exceedingly ſmall, appears from hence: viz. That if there be a candle lighted, and nothing to obſtruct the progreſs of its rays, it will fill all the ſpace within two miles of it, every way, with luminous rays of light, before the tenth part of a grain of its matter is conſumed.

The velocity with which the rays of light proceed from the ſurface of a body is ſurpriſing. The method by which the ſwiftneſs of the rays of light was determined firſt, was by an obſervation on the eclipſes of Jupiter's ſatellites. Theſe eclipſes appear to us about ſeven minutes ſooner than the preciſe time when the earth is ſituated in that part of its orbit between the ſun and Jupiter; and the eclipſe appears about ſeven minutes later, when the ſun is between him and us: from which it is plain, that the rays of light require about ſeven minutes to paſs through a ſpace equal to the diſtance between the ſun and us; that is, about ninety-five million of miles.

The rays of light proceeding from a viſible body, as they proceed in all directions, and extend themſelves upwards and downwards, as well as ſideways, muſt neceſſarily become thinner and thinner: and that they do, in proportion to the ſquares of their diameters from the luminous body; that is, at the diſtance of twice a certain ſpace they are four times thinner than at the diſtance of one ſuch ſpace; and at the diſtance of three times ſuch a ſpace, nine times thinner; and ſo on.

Of Refraction.

All light proceeding from any luminous body, without being reflected from any opake ſubſtance, or inflected by paſſing very near one, is always found to proceed in ſtraight lines. But if the rays paſs obliquely from one medium to another, they always leave the direction they had before, and aſſume a new one; and this change of direction is called their *refraction*: and after having ſuffered this refraction, they again proceed in ſtraight lines, till they meet with a different medium, when they are again turned out of their courſe.

The phenomena of refraction are occaſioned by an attractive power in the medium through which the light paſſes.

When a ray of light paſſes out of a rarer into a denſer medium (if the latter has a greater attractive force than the former, which is moſtly the caſe), the ray, juſt before its entrance into the denſer medium, will begin to be attracted towards it; and this attraction will continue to act upon the ray till ſome time after it has entered the medium. When a ray enters the denſer medium obliquely, its direction will alſo become leſs oblique to the ſurface of the medium; and approach nearer to a perpendicular, drawn through the ſurface of the medium.

But when the ray of light paſſes from a denſer into a rarer medium obliquely, its direction will be made more oblique to the ſurface of the rarer medium; and depart further from a perpendicular drawn through the ſurface of the rarer medium.

Thus (in fig. 2) let B C be a ray of light paſſing through the air into the denſer medium A D H I; at entering the denſer medium at the point C, it will begin to be attracted towards that medium, which attraction will change its courſe from the direction C K to that of C E. Let F G be a perpendicular drawn through the denſer medium; then L will be the ſine of the angle of refraction, and M N the ſine of the angle of incidence. Now the ſine of the angle of incidence is always proportionate to the ſine of the angle of refraction in the ſame medium.

From this property of refracting rays of light ariſes the magnifying power of the glaſs lenſes uſed in optical inſtruments. For the rays of light proceeding from any object are too diverging to admit of diſtinct viſion at a leſs diſtance than from 6 to 8 inches; if, therefore, by interpoſing a glaſs lens between the eye and the object, the object can be viewed at a leſs diſtance, the rays of light proceeding from the object, will be made to converge to a point ſooner, and by that means the object will be ſeen under a larger angle.

Thus, let O (fig. 10) be an object to be viewed by the eye at F: which object, to be viewed by the naked eye, muſt be removed to the diſtance of ſix inches at leaſt from the eye. But by interpoſing a lens, whoſe focal diſtance is an inch and an half, the eye will have diſtinct viſion of the object at the diſtance F O; conſequently the object will appear magnified four times in diameter, to what it would do to the naked eye. And the object appears under the angle I F M.

If an object A B (fig. 9) be placed in one focus of the lens D E, and the eye in the other focus F, the eye will ſee juſt ſo much of the object as is equal to the diameter of the lens: for the rays A D and B E, which go from the extremities of the object to the extremities of the lens D E, and are united at the focus F, muſt neceſſarily proceed from the object to the lens, parallel to the axis F C, and to each other; therefore, the part of the object A B, ſeen by the rays D F, E F, will be equal to the diameter of the lens. If only one part *de* of the lens be open, then only ſo much of the object A B as is equal thereto, will be ſeen by the eye. Now as A B is equal to D E, or *a b* to *d e*: the angle D F E, or *d* F *e*, is the optic angle; that is, the angle under which the part of the object A B, or *a b*, appears to the eye at F: and as G F is ſuppoſed but half the diſtance of C F; the angle D F E, or *d* F *e*, is double to that under which the part A B, or *a b*, would appear to the naked eye at the diſtance F C; thus the eye ſees the object twice as large in diameter through the lens, as it would do without it.

If it be required to ſee a part of an object larger than the lens, the eye muſt be placed nearer the lens than its focus. Let the lens be D (fig. 8), its two foci F and C. In the focus C, let there be an object A B larger than the lens: ſuppoſe the rays A D and B proceed from the extremities of the object to thoſe of the lens, then by the lens they will be converged into the point K, between the lens and its focus F:

then, if the eye be placed at K, it will ſee all the object A B, which is larger than the lens.

But if the eye be placed further from the lens than its real focus, it can never ſee any object, or part of an object, at once, near ſo large as the lens, but always ſmaller. Let the eye be placed at I, beyond the focus F; it will then only ſee the part of the object G H, which is leſs than the lens.

Thus it is evident that the nature of a convex lens is ſuch, as will render an object diſtinctly viſible to the eye at the diſtance of its focus. The reaſon why theſe lenſes are uſed as microſcopes is exceedingly plain; for if the diſtance F I (fig. 10) be ſix inches, which is the leaſt diſtance an object can be ſeen by the naked eye, and the focal diſtance of the lens c F two inches, then the object O will appear three times magnified in length, as I M: and if it were the ſurface, it would be magnified nine times; and the ſolidity or bulk twenty-ſeven times.

If the focal diſtance F c of the lens be one fourth part of an inch, then will that be but one twenty-fourth part of ſix inches, the diſtance of naked ſight: and ſo the length of an object ſeen through ſuch a lens will be magnified twenty-four times; the ſurface five hundred and ſeventy-ſix times; and the ſolidity thirteen thouſand ſeven hundred and twenty-four times; for thoſe are the ſquare and cube numbers of twenty-four.

To find the principal Focus of any Lens.

Hold the lens perpendicularly againſt the rays of the ſun, or a candle; hold a white paper behind it, to receive the refracted rays, which will make a round white ſpot upon the paper; move the paper backwards or forwards, till this ſpot, which is the image of the ſun or candle, be the leaſt poſſible, there fix the paper; then the diſtance of the paper from the lens is the focal diſtance required, or the burning point.

Or, ſecondly. Cover the ſide of a lens with a paper, having ſeveral ſmall holes made in it with a pin; and placing the lens directly againſt the beams of the ſun, or a candle, the light paſſing through the holes will form ſo many white ſpots upon the paper held behind it. Move the paper backwards and forwards, till all the ſpots coincide in one. That point is the focus; and the diſtance of the paper from the glaſs is the focal diſtance.

I ſhall here give ſome of the moſt general propoſitions and properties in the ſcience of optics, leaving the proceſs and demonſtration for the exerciſe of the reader.

1. Wherever the rays, which come from all the points of an object, meet again in ſo many other points, to which they converge by refraction, there they will make a picture of the object upon any white body on which they fall.

2. Rays flowing from all points of an object, and paſſing through a lens, paint the image of the object in the real focus of the lens, and may be ſeen there upon a white paper: or without the paper, the image may be ſeen hanging pendulous in the air, by an eye placed ſix or eight inches behind the focus.

3. The object, and its image made by a lens, ſubtend equal angles at the lens.

4. The length of an object is to the length of the image made by a lens, as the diſtance of the object from the centre of the lens is to the diſtance of the image from it.

5. A convex lens magnifies an object, when it is nearer than twice the principal focal diſtance; but if further off, it leſſens it.

6. A concave lens always diminiſhes an object.

7. The apparent magnitude of an object is the angle under which the object appears to the eye.

8. If the object and its image be both on the ſame ſide of the lens, the image will be erect; but if they be on different ſides, the image will be inverted.

9. When

9. When the object and its image are on different ſides of the lens, as the object approaches the lens, the image recedes from it: or, if the object recedes from the lens, the image approaches it.

10. If the object and image be both on one ſide of the lens, and the object move towards the lens; the image alſo moves towards the lens: and if the object moves from it, the image alſo moves from it.

11. In a convex lens, if the object be further off than the principal focus, its image will be on the other ſide of the glaſs, inverted. But if the object be nearer than the principal focus, the image will be on the ſame ſide of the glaſs, erect.

12. In a concave lens, the image and object are always on one ſide of the lens.

13. If an object be placed in the principal focus of a lens, its apparent magnitude at any place whatever, beyond the lens, will be invariably the ſame; and equal to the apparent magnitude, when ſeen from the centre of the lens, with the naked eye.

14. The apparent magnitude of a body, placed in the principal focus, will always continue the ſame, however the eye is moved backward or forward, from the lens.

15. The nearer the eye is to the lens, the more of the object appears; and the farther off the eye is from the lens, the leſs of the object is ſeen.

16. If the object be nearer than the principal focus, its apparent magnitude becomes leſs, in going from the glaſs. But if the object be further than the focus, the apparent magnitude will increaſe, in going from it.

17. If the eye be fixed in the principal focus, the apparent magnitude of an object will be invariably the ſame, wherever the object is placed before the glaſs.

18. If the eye and object be fixed; and a convex lens be moved from the object to the eye, the apparent mag-

nitude of the object increaſes to the middle, and then decreaſes to the eye. When it comes into ſuch a poſition that the eye and object are conjugate foci, the object is infinitely confuſed.

19. If the eye and object be fixed, and a concave lens be moved from either of them to the other; the apparent magnitude of the object will decreaſe to the middle, and then increaſe again. The apparent diſtance is reciprocally as the apparent magnitude; and, in general, we judge the apparent diſtance of an object to be the very ſame as we ſuppoſe the real diſtance of an object to be, from which the rays come to our eyes, with the ſame degree of confuſion.

20. Converging rays are made more converging by a convex lens: and diverging rays are made more diverging by a concave.

21. Parallel rays falling on one ſide of a convex lens will be refracted to the focus on the other ſide. But parallel rays falling on a concave lens will, by refraction, diverge from the focus on the ſame ſide.

22. In a convex lens, rays diverging from the focus will emerge parallel on the other ſide.

23. In a concave lens, rays converging to the focus will emerge parallel, going out of the lens on the ſame ſide.

24. In a double convex lens of equal radii, the principal focus is diſtant from the lens, the radius of the ſphere, of which the lens is a ſegment.

25. When the lens is a perfect ſphere, the principal focus is diſtant from the lens half the radius.

26. When the lens is an hemiſphere, the principal focus is diſtant four thirds of the radius, when the convex ſide is expoſed to the rays; and twice the radius, when the plane ſide is expoſed.

27. When the lens is a plano-convex, the diſtance of the focus is equal to the diameter, or twice the radius of the ſphere when the plane ſide is expoſed to the rays. But when

the convex ſide is expoſed to the rays, the focus is diſtant twice the radius, except two fifths of the thickneſs.

28. In double concaves of equal radii, the principal focus is diſtant the radius of the ſphere; and it is virtual.

29. In a plano-concave, the principal focus is diſtant twice the radius; and is virtual.

30. If the radiant point and the focus be equidiſtant from a lens, they will each of them be diſtant twice the principal focal diſtance.

31. If the radiant point be nearer the lens than the principal focus, the rays after refraction will diverge; but if the radiant point be in the principal focus, they will after refraction emerge parallel; and if the point be farther off than the focus, the rays will converge after refraction.

32. The diſtance of the radiant point and its focus, made by a convex lens, is the leaſt poſſible, when they are equidiſtant from the lens.

33. If a convex lens be held directly to the rays of the ſun, and a combuſtible body be held in the principal focus; the heat of the rays of the ſun, converging to a point in the focus, will ſet the body on fire.

34. The heat of the focus of the lens is to the ſun's direct heat, as the area of the glaſs to the area of the image in the focus.

35. The degrees of heat in the foci of different lenſes are as the ſquares of the diameter directly: and the ſquares of the focal diſtances reciprocally.

The foregoing propoſitions contain all the phenomena of refracting lenſes; and may be proved for the moſt part by actual experiments.

It is neceſſary juſt to obſerve, for the ſake of thoſe who may uſe lenſes of a different medium, that different mediums have different refracting powers: I ſhall, therefore, exhibit a table of the refracting power, as given by Sir Iſaac Newton, and proved by many later experiments.

In the firſt column are the names of the bodies; in the ſecond column, the ſines of incidence and refraction; in the third column, the refracting power; in the fourth column, their denſity, or ſpecific gravity; in the fifth column, the ratio.

Bodies.	*Sines of Incidence, and Refraction.*		*Refracting Power.*	*Denſity.*	*Ratio.*
Pſeudo topaz	23	14	1.699	4.27	0.39
Air	3201	3200	0.000625	0.0012	.52
Glaſs antimony	17	9	2.568	5.28	.48
A ſelenitis	61	41	1.213	2.252	.54
Glaſs	31	20	1.4025	2.58	.54
Rock cryſtal	25	16	1.445	2.65	.54
Iſland cryſtal	5	3	1.778	2.72	.65
Sal gemma	17	11	1.388	2.143	.64
Alum	35	24	1.1267	1.714	.65
Borax	22	15	1.1511	1.714	.67
Nitre	32	21	1.345	1.9	.70
Vitriol	303	200	1.295	1.715	.75
Oil vitriol	10	7	1.041	1.7	.61
Rain water	529	396	0.7845	1.	.78
Gum Arabic	31	21	1.179	1.375	.85
Spirit wine	100	73	0.8765	0.866	1.01
Camphire	3	2	1.25	0.996	1.25
Oil olive	22	15	1.1511	0.913	1.26
Linſeed oil	40	27	1.1948	0.932	1.28
Spirit turpentine	25	17	1.1626	0.874	1.32
Amber	14	9	1.42	1.04	1.36
Diamond	100	41	4.049	3.4	1.45

From this Table it appears, that thoſe bodies which contain oily ſulphurous particles differ from the conſtant ratio, and have a greater refractive power.

SECT.

SECT. II.

OF SINGLE AND COMPOUND MICROSCOPES.

SINGLE microſcopes are ſuch as have but one lens; and may be conſtructed in an infinite variety of ways. I ſhall give the forms of a few of the moſt uſeful.

A (fig. 10, plate 5) is a circular piece of wood, metal, or ivory, in the middle of which is a ſmall hole, c. Upon this hole is fixed, by means of a wire, a glaſs ſpherical lens, of one tenth of an inch radius, whoſe focal diſtance is c D. At the focus D is a pair of plyers D E, fixed upon a ſliding ſcrew B, and which opens by means of the two ſtuds a, e: theſe plyers ſerve to hold any ſmall object O, which is to be viewed through the lens by the eye, placed in the other focus F: and according to the focal diſtance of the lens, the object will be more or leſs magnified, as before deſcribed; thus, *in a lens of this radius*, viz. one tenth of an inch, the focal diſtance of which is a radius and an half from the centre thereof, as deſcribed in the foregoing propoſitions; the length of an object, by ſuch a lens, will be magnified forty times; the ſurface, one thouſand ſix hundred; and ſolidity, ſixty-four thouſand times. For a radius and an half of ſuch a ſphere, or $\frac{3}{20}$ of an inch, is only the fortieth part of ſix inches, (the leaſt diſtance of naked viſion;) the ſquare of 40 is 1600, and the cube of 40 64,000. This inſtrument, from its ſize and portability, is very convenient. The beſt lenſes for theſe ſingle microſcopes are, however, thoſe whoſe focal diſtance is three tenths, or four tenths of an inch.

Again, when the diameter of a ſpherical lens is only one twentieth of an inch, the object will ſubtend an angle at the eye as great as if it were only three eightieths of an inch (which is the one hundredth and ſixtieth part of ſix inches) diſtant: therefore, the length of an object ſeen through ſuch a lens will be magnified one hundred and ſixty times; the ſurface, twenty-five thouſand ſix hundred times; and the ſolidity, four million and ninety-ſix thouſand times; which is ſo great a magnifying power as to ſurpaſs conception.

But ſpherule lenſes, of ſo ſmall a diameter as the latter, are of no uſe but in viewing tranſparent objects; for if an opake object were to be viewed by ſuch a lens, the eye muſt be applied almoſt cloſe to the ſurface of the lens, by which the lens and object would be ſo overſhadowed, as to render an opake object too obſcure to be viewed.

Theſe ſpherule lenſes were uſed by the famous Mr. Leeuwenhock, by which he made ſuch wonderful diſcoveries in the minuter parts of nature. And it muſt be by a proper arrangement of theſe, that the particles of matter are to be diſcovered, if ever they can, which Sir Iſaac Newton thought was poſſible.

But there are great difficulties attending the uſe of theſe very ſmall ſpherules: firſt, the difficulty of making very good ones, of a very ſmall diameter. Secondly, the prejudice done to the eyes in uſing them. Thirdly, the trouble of placing objects at the proper focus. And laſtly, the very ſmall part of the object, which can be ſeen at one time. Theſe inconveniences render this ſort of microſcopes of very little ſervice for common uſe; they have therefore been ſuperſeded by others, particularly by the following; in which both tranſparent and opake objects may be viewed with leſs trouble.

In this inſtrument (fig. 11.) F is a piece of braſs turning round in a ſocket, at the end of which is a ſmall ſpring tube moving upon a rivet, through which there runs a ſteel wire, terminated at one end by a ſharp point, G, and the other end hath a pair of plyers, H. The point and plyers are to thruſt into, or take up any object. Either of them may be turned upwards, towards G. I is a ring of braſs with a female ſcrew within it: this ring is fixed upon a piece of braſs which turns round on a rivet near D, that it may be ſet to the proper diſtance, when the leaſt magnifiers are uſed. K (fig. 12.) is a concave ſpeculum, or mirror, generally of ſilver, poliſhed very high, in the centre of which a double convex lens is placed, with a proper aperture, to look through it; on the back of this ſpeculum is a male ſcrew L, which is made to ſcrew into the female ſcrew I (fig. 11).

There are generally four of theſe concave ſpecula, with different lenſes of different magnifying powers, for different objects. The greateſt magnifiers muſt always have the leaſt apertures.

D is a nut adapted to the ſcrew B: one end of this ſcrew is faſtened to the ſide C, and by turning the nut D, the two ſides A and C are gradually brought together, or ſeparated, being held ſteady by the ſteel ſpring E. P is the handle of the inſtrument.

To uſe this inſtrument, the ſpeculum K is to be ſcrewed into the braſs ring I, the magnifying lens being firſt fixed in the ſpeculum. Then place the object either on the needle G, in the plyers H, on the object plate M (fig. 13), or in the braſs box O (fig. 14), as is moſt convenient; then holding up the inſtrument by the handle P, look through the magnifying lens directly againſt the light; and by the nut D, together with the motion of the needle, the object may be turned about, raiſed, or depreſſed; brought nearer to or further from the glaſs, till it be brought to the true focus, and the light is ſtrongly reflected upon it, from the ſpeculum: by theſe means the object will be ſeen very clearly and diſtinctly. The light of a candle will ſerve as well as day-light, for this inſtrument.

O (fig. 14) is a ſmall braſs box, with a glaſs on each ſide to confine any living object, in order to be examined; it ſcrews upon the needle G, by means of a ſmall pipe at bottom.

M (fig. 13) is a round object plate, black on one ſide, and white on the other. N, a ſteel ſpring, turning down on each ſide, to hold any object faſt. A dark-coloured object is to be placed on the white ſide; and a light-coloured object on the black ſide, to render them more diſtinct. This plate alſo has a ſmall pipe to ſcrew on the needle G (fig. 11).

This microſcope is of late invention: and by means of the poliſhed mirror, any opake object may be viewed with a very ſmall magnifier. Tranſparent objects may alſo be viewed by it: but in this caſe, it will not be proper to throw on the object all the light reflected from the ſpeculum; leſt the light tranſmitted through the object, meeting the reflected light, produce too great a glare. For this purpoſe, a bit of paper may be interpoſed between the object and ſome part of the ſpeculum. A little practice will enable a perſon to know how to regulate the light.

As opake objects are more numerous than others, very great diſcoveries may be made in ſuch objects.

A Microſcope on a Stand, which will anſwer all the Ends of the large Double Reflecting Microſcope.

B (fig. 17) is a round frame of wood, on which is fixed the braſs ſcroll A for the purpoſe of holding the microſcope ſteady. C is a braſs ſcrew that paſſes through a hole in the ſcroll, into the microſcope D, and ſerves to ſcrew it faſt to the ſcroll.

E is a concave looking-glaſs, or ſpeculum, of metal, ſet in a braſs box, and hanging in the arch G by two ſmall ſcrews: at the bottom of this arch is a ſmall braſs pin h, which goes through the ſtand of the arch into the wooden frame B, by which means the arch is turned round in an horizontal direction; and the mirror ſwings in a vertical direction. By theſe two motions the ſpeculum may be ſo adjuſted as to reflect the light directly upwards, through the body of the microſcope D, which is fixed perpendicularly over it.

The

The body of the microſcope may alſo be fixed in an horizontal direction, and objects viewed either by the light of a candle, or the common day-light.

To uſe this microſcope, the object is fixed in a ſlider, which is thruſt in between the plates, through a ſlit on one ſide of the microſcope.

This microſcope may be uſed in a variety of ways. One method I ſhall mention:—Take a ſlip of glaſs, place one end through the ſlit of the microſcope where the ſliders go; and on the other end, which extends to ſome diſtance, any object may be placed that cannot be placed in the ſliders; then place over the object, upon the ſlip of glaſs, a magnifying lens, ſcrewed into a braſs ring (having ſeveral lenſes fixed in braſs rings for that purpoſe, and in ſuch a manner that the ring of braſs may extend as far as the focal diſtance of the lens), and bring the ſpeculum to reflect the light up to the object.

Microſcopes of this ſort are the moſt eaſy and pleaſant for uſe, of any extant; and will ſerve to view very minute objects; ſuch as the animalcules and ſalts in fluids, the farina in vegetables, the circulation of the blood in ſmall inſects, &c. If the object have any degree of tranſparency, this microſcope is likely to make more diſcoveries in it than any other.

Of Double or Compound Microſcopes.

Double or compound microſcopes are thoſe which have two glaſſes, or lenſes: that next the object is called the object glaſs; and that next the eye, the eye glaſs.

The object a b (fig. 16) is placed at a little greater diſtance from the object glaſs than its focus; ſo that the rays flowing from the different points of the object, and paſſing through the object glaſs *c d*, will converge to *g* and *h*, where they form the image of the object. This image is viewed by the eye at *k*, through the eye glaſs *e f*, which is ſo placed, that the image *g h* is in its focus, and the eye in the focus alſo, but on the other ſide; thus, the rays of each pencil will converge at the eye *k*, where they croſs each other, and after paſſing through the humours of the eye, they will form the large inverted image A B on the retina.

This image, ſeen by ſuch a microſcope, will be inverted with regard to direct viſion: for the object *a b* lies in the ſame direction as the image A B on the retina, as ſeen in the figure; which is not the caſe when the object is ſeen by the naked eye, or by a ſingle microſcope.

To calculate the magnifying power of theſe double microſcopes, we reckon how many times the object is magnified by the object glaſs, and then how many times the image of this object is magnified by the eye glaſs: thus, if the object *a b* be magnified ſix times in length by the object glaſs *c d*, as it will when the image *g k* is ſix times the diſtance from the object glaſs, as the object *a b* is: and if the eye glaſs *e f* be only of half an inch focus, the image *g k* can be viewed at half an inch diſtance, which being only the twelfth part of ſix inches, the image *g h* will be magnified twelve times in length to what it will appear to the naked eye without the eye glaſs: and the image *g h* being ſix times the length of the object *a b*, the object *a b* is therefore magnified in length ſeventy-two times, and the ſuperficies and ſolidity in proportion.

The field of view is very ſmall in a microſcope of this ſort; therefore, there are generally two eye glaſſes uſed, which are placed ſometimes cloſe together, and ſometimes an inch or more aſunder: by which means the viſible area is much enlarged, and a larger field of view is obtained, though the object appears ſomewhat leſs magnified.

A Compound Microſcope.

A (fig. 18) is the body of the microſcope, which is moveable up or down in the external caſe C D, which is fixed on the foot of the inſtrument I, by three pillars. F is a plate fixed horizontally to the three pillars, and is called the ſtage; in the centre of which is a hole, on which a piece of glaſs, or any other implement is placed, with the object to be viewed. H is a reflecting ſpeculum, which reflects the light upwards, through the hole in the ſtage. K is a pipe, to which is ſcrewed the braſs button Q, which holds the magnifier, or the object glaſs.

There are generally three, and ſometimes four glaſſes uſed in this microſcope: firſt, the magnifying lens, or object glaſs at Q. Second, a large lens at B, called the body glaſs, by which a large field of view is obtained. And third, an eye glaſs at A, by which the magnified image of the object is magnified ſtill more.

From what has been ſaid, it is evident, that the magnifying power of all optical inſtruments depends upon this one principle; that the nearer the object can be brought to the eye, to be diſtinctly viſible, the more it will be magnified; and that in proportion as the diſtance of the magnified object from the eye, is leſs than the diſtance at which the object can be ſeen diſtinctly by the naked eye, which is in general from ſix to eight inches.

SECT. III.

OF REFRACTING, REFLECTING, AND SOLAR TELESCOPES.

Before the reader enters into the deſcription of teleſcopes, it will be neceſſary to explain the nature of concave lenſes, and their property in rendering rays divergent. Alſo the nature of reflection by plain and concave mirrors.

When parallel rays paſs through a concave glaſs, they will diverge after paſſing through it: thus, let *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, (fig. 1, plate 6,) be parallel rays: after paſſing through the glaſs, A B, they will diverge, as if they had come from the point C, which is the centre of the glaſs's concavity: this point C is called the negative, or virtual focus of the glaſs. Thus, the ray *a*, after paſſing through the glaſs, will proceed in the direction *k l*, and the ray *h*, in the direction *m n*, and the ray *c* in the direction *o p*, &c. The middle ray C, falling directly upon the middle of the glaſs, ſuffers no refraction.

This lens being equally concave on both ſides, the rays diverge, as if they came from the centre of the glaſs's concavity. But if the glaſs had been concave on one ſide only, and the other ſide plain, the rays would have diverged, as if they had come from a point at double the diſtance of C, or the centre of the glaſs's concavity.

If the rays, before they paſs through the glaſs, converge more than parallel rays diverge after they have paſſed through the glaſs, ſuch rays will continue to converge after paſſing through it; but will not meet ſo ſoon as if no glaſs had been in the way; and will incline towards the ſame ſide to which they would have diverged, if they had come parallel to the glaſs: thus, the rays *f h*, proceeding towards the glaſs in a converging direction, and converging more than the parallel rays diverge, after they have paſſed the glaſs; the rays *f h* will go on converging, after paſſing through the glaſs, though in a leſs degree; and will at length meet at I.

If parallel rays fall upon a concave mirror, they will be reflected back, and meet in a point at half the diſtance of the centre of the mirror's concavity. Thus, let *d f a*, C *m l*, *e l c*, (fig. 2,) be parallel rays falling upon the concave mirror A B: they will be reflected back, and meet in the point *m*, half the diſtance of the mirror from C, which is the centre of its concavity; and they will be reflected at as great an angle, from the perpendicular to the ſurface of the mirror, as they fall upon it, with regard to the perpendicular, but on the other ſide thereof. Thus, let C be the centre of the mirror's concavity. Draw the lines C *g* A, C *m* *l*, and C *h c*, from the centre C to the mirror. Theſe lines will be each perpendicular to the mirror, as they proceed therefrom like ſo many radii. Make the angle C A h equal to the angle *d* A C, and draw the line *a m h*, which will be the direction of the ray *d f* A after reflection; ſo that the angle of incidence *d* A C is equal to the angle of reflection C *a h*; the incident and reflecting rays making equal angles with the perpendicular C *g* A.

In the ſame manner the angle of reflection *m c* C may be proved equal to the angle of incidence C *c e*. The ray C *m l* paſſing through the centre of the mirror's concavity, and falling upon it at *l*, is therefore reflected back from it in the ſame line. All theſe reflected rays meet in the point *m*, and there paint the image of the body, from which the parallel rays proceed.

This point, as before obſerved, is half the diſtance of *l* C.

The rays which proceed from any celeſtial object may always be eſteemed parallel: and thereforee the images of ſuch objects, on a reflected ſpeculum, will be formed at half the diſtance of the centre of the mirror's concavity.

But the rays which proceed from any diſtant terreſtrial object do not come quite parallel to the mirror; but come diverging to it in ſeparate pencils; therefore they will not be converged to a point at the diſtance of half the radius of the concavity of the mirror, but at a little greater diſtance from the mirror, and in ſeparate points. And the nearer the object is to the mirror, the farther theſe points will be removed from it; and an inverted image of the object will there be formed, which will ſeem to hang pendulous in the air, by an eye placed beyond it.

Let A B be a mirror, whoſe centre of concavity is C (fig. 3). D E is an upright object placed beyond the centre C; from the upper end D, of which flows a pencil of diverging rays to every point of the concave ſurface of the mirror A B.

From the centre C, draw C A, C *c*, C B, touching the mirror in the ſame points where the pencils of rays touch it, which flow from D. Theſe three lines drawn from C to the mirror will be perpendicular to the ſurface of the mirror. Make the angle C A *d* equal to the angle D A C; and draw the right line A *d* for the reflection of the ray D A. Make the angle C B *d* equal to the angle D B C, and draw the right line *c d* for the courſe of the incident ray D *c*. Make the angle C B *d* equal to the angle D B C, and draw the right line B *d*, for the courſe of the incident ray D B; all theſe reflected rays will meet in the point *d*, where they will form the extremity D of the image D E.

If the pencils of rays be drawn from the other point of the object E, in the direction E *f*, E *g*, E *h*, and be continued to the ſurface of the mirror, they will be reflected back from the mirror to the other end of the object *d*; and their angles of reflection will be equal to their incident angles: thus, they form an inverted image of the object, at the point *d*.

Though I have only drawn three lines in the figure, to repreſent rays flowing from the point D; yet it muſt be obſerved, that this point, as well as every other point in the object, ſends forth a *pencil of rays* to every part of the mirror; and the rays of each pencil are reflected back, and meet in all the intermediate points of the object *d*, as has been ſhown, with regard to the point D.

When the object is farther from the mirror than its centre C, the image will be leſs than the object, and will be between the object and the mirror; but when the object is nearer than the centre of concavity, the image will be further off, and bigger than the object.

If the radius of the mirror's concavity, and the diſtance of the object from the mirror be known, the diſtance of the image from the mirror may be found by this rule:—

Divide the product of the diſtance, multiplied by the radius, by double the diſtance made leſs by the radius, and the quotient is the diſtance required.

If a man place himſelf directly before a concave mirror, but at a further diſtance from it than the centre, he will ſee an inverted image of himſelf in the air, between him and the mirror, and leſs than himſelf in ſize. And when he holds out his hand towards the mirror, the hand of the image will alſo approach towards his hand; and if his hand be in the centre of the mirror, the hand of the image will coincide with his hand; and an inexperienced obſerver will fancy that he may ſhake hands with his own image. If he extend his hand forward further towards the mirror, the hand of the image will paſs by his hand, and come between his hand and his body: if he move his hand to either ſide, the hand of the image will move the contrary way. But a perſon ſtanding at a diſtance, and on one ſide of the mirror, will ſee nothing of the image, becauſe none of the reflected rays enter his eye.

It is neceſſary that the reader perfectly underſtand what is here demonſtrated concerning reflection, that he may perceive how the image is formed in the large concave mirror of the reflecting teleſcope.

Again, if a concave mirror be placed againſt a wall oppoſite a fire in a chamber, and a mahogany table highly poliſhed placed in the focus of the mirror; a perſon ſtanding facing the mirror, but not directly between that and the fire, will ſee a large, bright, erect image of the fire upon the table; and to a caſual obſerver, juſt entering the room, it will appear in all reſpects like a real fire. But the room ſhould be made quite dark for this purpoſe, that the fire may appear more bright. But to a perſon who ſtands on one ſide the table, nothing will appear but a long beam of light.

If, inſtead of a fire, a large candle be placed oppoſite the mirror, a perſon ſtanding by the candle will ſee the appearance of a large bright ſtar over the table: thus, by fixing a large candle in a frame, with a ſmall wax taper to turn round it, in a circle, an appearance will be made in the focus of the glaſs, repreſenting a bright planet, and its ſatellite; the ſatellite performing a revolution round the planet.

The Refracting Teleſcope.

The firſt form of refracting teleſcopes that I ſhall mention, is that which conſiſts of two lenſes only. The object glaſs muſt be convex; but the eye glaſs may be either convex or concave.

Let *c d* (fig. 4) be a convex object glaſs, in a long tube, and have its focus at D. A B is a remote object: from the extremity A flows a pencil of rays *g b i*, which will be ſo refracted by paſſing through the glaſs *c d*, as to meet in the point *f*; and the pencil of rays *k l m*, flowing from the other point of the object B, by paſſing through the lens will be refracted ſo as to unite in the point *e*: thus the points A and B of the object will be formed by refraction in the points *f e*: and all the intermediate points of the object ſend forth pencils of rays, which form correſponding points in the ſmall figure *f e*, the inverted image of the object. But as this image is ſmall, a concave glaſs *n c* is placed at the end next the eye, ſo that its virtual focus may be at F. Then, as the rays of the pencils come to the concave glaſs converging, they will converge leſs after paſſing through it, as hath been demonſtrated: and they will proceed to *b* and *a* before they unite, which is on the retina, and there form the large image *a b*.

The only inconvenience attending the uſe of this teleſcope is, that the field of view is very ſmall, by reaſon of the rays diverging ſo much, after paſſing through the concave glaſs.

For, from a view of the figure, it is evident that when the teleſcope lies directly towards the object, none of the rays which flow from the extremities A and B can enter the pupil of the eye; but fall upon the iris, above and below the pupil. So that, to view the different parts of an object, the teleſcope muſt be moved upwards or downwards, unleſs the object be a very remote one; and then it is but indiſtinctly ſeen.

Therefore to remedy this inconvenience, a convex eye glaſs is uſed inſtead of a concave, as *a b* (fig. 5), which is placed ſo, that its focus may be coincident with the focus of the object glaſs *c d*, whoſe focus is at E. Then the rays of the pencils flowing from the object A B, and paſſing through the object glaſs *c d*, will meet in its focus, and form the ſmall inverted image *m p*. And as the image is formed in the focus of the eye glaſs *a b*, the rays of each pencil, after paſſing through this glaſs, will be parallel, but the pencils themſelves will converge to each other, and croſs each other in the focus of the glaſs at *e*: the pupil of the eye being in this focus, the image will be viewed through the eye glaſs, under the angle D *e* C.

This teleſcope alſo has an inconvenience, which is, that it inverts the object with reſpect to it, when ſeen by the naked eye: that is, the image is painted in the eye in the ſame poſition in which the object lies; we therefore judge it to be inverted. This teleſcope is, therefore, only fit for viewing celeſtial bodies, in many of which their poſition is immaterial. But it muſt be obſerved, in the uſe of this teleſcope, that whatever way the object *ſeems* to move, the teleſcope muſt be moved the contrary way; for as the object is inverted, ſo will be the motion.

The magnifying power in this teleſcope, is as the focal diſtance of the object glaſs is to the focal diſtance of the eye glaſs. Therefore, if the former be divided by the latter, the quotient will expreſs the magnifying power.

Therefore, this teleſcope can be conſtructed ſo, as to magnify in any proportion, provided it be of a ſufficient length: for the greater the focal diſtance of the object glaſs is, the leſs may the focal diſtance of the eye glaſs be. Thus, an object glaſs of ten feet focal diſtance will admit of an eye glaſs, whoſe focal diſtance is little above two inches and a half, which will magnify the length of an object near forty-eight times, and its ſurface and ſolidity will be magnified in proportion; as in the microſcope.

Thus, if the focal diſtance of the object glaſs be equal to the focal diſtance of the eye glaſs, the teleſcope would not magnify the object in the leaſt.

Again, if the focal diſtance of the object glaſs be twenty inches, and the focal diſtance of the eye glaſs one inch, the length of an object ſeen by ſuch a teleſcope will be magnified twenty times, its ſurface four hundred times, and its ſolidity eight thouſand times.

To remedy the inconveniences of both the former kinds of teleſcopes, the following one is generally uſed for terreſtrial objects, which gives a larger field of view than that with the concave eye glaſs, and ſhows the object in its natural poſture, as ſeen by the naked eye. In this teleſcope there is one object glaſs *c d* (fig. 6), and three eye glaſſes, *e f*, *g h*, *i k*. They are ſo placed, that the diſtance between every two glaſſes, which are next each other, may be equal to the ſum of their focal diſtances: thus, the focus of one glaſs is coincident with the focus of the next glaſs. The foci of the two glaſſes *c d* and *e f*, meet at F; thoſe of the two glaſſes *e f*, *g h*, meet at *l*; and thoſe of *g h* and *i k*, at *m*; and that of *i k* at *n*, where the pupil of the eye is ſituated. Then it is evident, from the figure, that the pencils of rays which flow from the object A B will paſs through the object glaſs *c d*, where they croſs each other, and meet, and form an inverted image, in the focus of the glaſs at C F D; this image being alſo in the focus of the next glaſs *e f*, the rays of the pencils, which flow from the image C F D, will

become

become parallel after paſſing through this glaſs, and croſs each other at *l*, which is the focus of the next glaſs *g h*; therefore, the rays proceeding from this focus and paſſing through the next glaſs *g h*, are converged to the focus of this glaſs, where they form the erect image E F of the object A B; and as this image is the focus of the eye glaſs *i k*, and the eye on the oppoſite ſide of the glaſs in the other focus, the image is viewed through the eye glaſs in this teleſcope, in the ſame manner as in the former one, but in a contrary poſition; that is, in the ſame poſition as when ſeen by the naked eye. The object alſo appears under the angle *i n k*.

The three eye glaſſes have generally all their focal diſtances equal. The magnifying power of this teleſcope is found in the ſame manner as that of the laſt; viz. by dividing the focal diſtance of the object glaſs by the focal diſtance of any one of the eye glaſſes, as the three latter are all equal.

Objects ſeen through ſuch a teleſcope appear coloured about the edges, the reaſon of which is, that the rays of light coming from the object are unequally refracted through the glaſs lens, and particularly thoſe rays which paſs through a convex glaſs near its edges. Theſe rays, being unequally refracted, do not all meet again in the ſame points exactly. Therefore, in this teleſcope, there muſt be a plate with a ſmall round hole in the middle, fixed at *m*, parallel to the glaſſes. By this plate, the wandering rays about the edges of the glaſſes will be intercepted, and prevented from coming to the eye; and none admitted, but thoſe which come through the middle of the glaſſes, or, at leaſt, at a good diſtance from the edges. But this ſomewhat leſſens the field of view, which would be much larger if the plate could be diſpenſed with.

The Binocular, or Double Teleſcope.

The binocular, or double teleſcope, is only two teleſcopes of an equal magnifying power, fixed in a frame, parallel to each other, and adjuſted by the means of ſcrews, ſo as to be at the ſame diſtance from each other as the pupils of the two eyes. Then by looking through them both at once, through one with each eye, the object will be ſeen by both eyes, and appear brighter and more diſtinct than through a ſingle teleſcope.

The Reflecting Teleſcope.

Refracting teleſcopes require to be of a conſiderable length, to magnify to any great degree, and the great length of ſome of them has rendered them very inconvenient. Sir Iſaac Newton, therefore, invented the firſt teleſcope of the reflecting kind, and which has received conſiderable improvements ſince his time. Since the invention of theſe, refracting teleſcopes are very little uſed for celeſtial objects; for a reflecting teleſcope, of only ſix feet in length, can be brought to magnify the object as much as one of the other ſort of a hundred feet in length.

They are generally conſtructed now in the following manner: T T T T (fig. 7) is a large tube, at the bottom of which is placed a large concave mirror, D U V F, whoſe principal focus is at *m*. In the middle of this ſpeculum is a round hole P, oppoſite which is placed the ſmall mirror L, concave towards the great mirror, and fixed on the wire M, by which it may be brought nearer to the great mirror, or moved further off, having a long ſcrew on the outſide of the tube for that purpoſe, always keeping its axis in the ſame line with that of the great mirror; viz. P *m n*. In uſing this teleſcope, we may ſuppoſe the rays of light to fall parallel to each other upon the great mirror; for no portion of a celeſtial object can be ſeen, which is leſs than the ſurface of the mirror.

Let A B be the diſtant object, from whoſe two extremities flow the pencils of rays C and E, which we here ſuppoſe parallel: the rays C, falling upon the great mirror at D, are from thence reflected in the direction D G, and by croſſing at I, the point of convergence, and the principal focus of the mirror, they form the upper extremity I, of the image I K, ſimilar to the lower extremity B, of the object A B; and then they paſs on to the ſmall concave mirror, whoſe focus is at *n*. They fall upon this ſmall concave mirror at *g*, and are from thence reflected, converging in the direction *g* N; becauſe *g m* is longer than *g n*; and paſſing through the hole P, in the large mirror, they would paſs on to *r* before they meet, and there form the lower extremity *d* of the erect image *c d*, ſimilar to the lower extremity B, of the object A B. But paſſing through the plane convex glaſs R, they form that extremity of the image at *b*. In the ſame manner the rays E, which come from the extremity of the object A, and fall parallel upon the great mirror at F, are from thence reflected to its focus, where they form the lower extremity K, on the image I K, ſimilar to the upper extremity A, of the object A B; and from thence they paſs on to the ſmall mirror L, and fall upon it at *k*; from whence they are reflected, converging in the direction *k* O, and paſſing through the hole P of the great mirror, they would meet at *q*, and form the upper extremity *e*, of the image *e d*, ſimilar to the extremity A, of the object A B; but paſſing through the convex glaſs R, they meet, and croſs ſooner, as at *a*, where that point of the image is formed. In the ſame manner pencils of rays flow from every intermediate part of the object A B, to the great mirror D U V F, and are from thence reflected to the focus of the mirror *m*, where they form an inverted image of the object. And laſtly, the rays paſſing from the image *a b*, through the eye glaſs S, and through a ſmall hole *e*, in the end of the leſſer tube *t t*, they enter the eye *f*, where they croſs each other in the pupil, and paint the image of the object on the retina in its natural poſition, and the image is ſeen by the eye under the large angle *c e d*.

To find the magnifying power of this teleſcope, the rule is, to multiply the focal diſtance of the great mirror, by the diſtance of the ſmall mirror from the image which is next the eye; and multiply the focal diſtance of the ſmall mirror, by the focal diſtance of the eye glaſs; then divide the product of the former multiplication, by the product of the latter, and the quotient will be the magnifying power.

When the firſt image I K is formed a little further from the great mirror than its principal focus, as at *n*; the focus of the great mirror is never coincident with that of the ſmall one: therefore the rays of the pencils reflected from the ſmall mirror will not be parallel, but rather converging, ſo as to meet in the points *q e r*, where they would form a larger erect image than *a b*, if the glaſs R was not in the way. This image might then be viewed by means of a ſingle eye glaſs, properly placed; but in that caſe, the field of view would be leſs; therefore, the glaſs R is uſed to enlarge the view.

The adjuſting ſcrew, to which the ſmall mirror L is fixed, renders this teleſcope applicable to every ſight, as the ſmall mirror may be brought either nearer to the eye, or removed farther from it; by which means the rays may be made to diverge a little, for ſhort-ſighted eyes; or to converge, for perſons of a long ſight.

In the uſe of this teleſcope, it muſt be obſerved, that the nearer an object is, the more the pencils of rays will diverge before they fall upon the great mirror; and conſequently they will be the longer before they meet after reflection; therefore, the firſt image I K will be formed at a greater diſtance from the large mirror. But this image muſt always be formed further from the ſmall mirror than its principal focus *n*; therefore, the ſmall mirror muſt be ſet at a greater diſtance from the large one in viewing a near object than when viewing an object more remote; this is eaſily performed by turning the adjuſting ſcrew.

The reflecting teleſcope will admit of an eye glaſs, of a much ſhorter focal diſtance (and conſequently of a greater magnifying power) than a refracting teleſcope will; therefore, thoſe teleſcopes magnify to a greater power. And the rays of light are not coloured by the reflections of the concave mirror, if it be ground to a true figure, as they are by paſſing through a convex glaſs, though it be ground ever ſo true.

In theſe reflecting teleſcopes, the obſerver never ſees the object itſelf, but only the image of it, which is formed next the eye. This will be demonſtrated by the following experiment:

If the obſerver fix a ſtick acroſs the mouth of the teleſcope before the object glaſs, it will not hide any part of the object from his view, unleſs it covers the whole mouth of the teleſcope; but it will only make the object appear dimmer, by intercepting part of the rays. But if he place only a piece of wire acroſs the inſide of the tube, between the eye glaſs and his eye, it will hide part of the object, which he thinks he ſees; which proves, that it is not the real object, but the image, which is perceived. This is evident, from the ſmall mirror L, which is made of opake metal, and ſtands directly between the eye and the object, and would hide the whole object from the eye, if the two glaſſes R and S were taken out of the tube.

The Solar Teleſcope.

The ſolar teleſcope is the moſt uſeful inſtrument for viewing the face of the ſun of any hitherto invented. It is formed of a ſcioptic ball and ſocket, which is faſtened againſt a hole in the window-ſhutter of a darkened room; in the cylindrical hole of which is placed the end of a common refracting teleſcope, which is to be drawn out to its proper length; then the ball and teleſcope are moved about till the rays of the ſun fall perpendicularly on the object glaſs, through the hole of the ball: the tube which contains the eye-glaſs is then to be adjuſted, by drawing it in, or out, till the image of the ſun be formed on a white paper, in the focus of the teleſcope, very diſtinctly and large.

By this teleſcope, the face of the ſun may be repreſented in almoſt any ſize, which affords the mathematician an opportunity of viewing all the phenomena to be ſeen in that planet; ſuch as the ſpots on the ſun's diſc, which are ſeldom ſeen when viewed through ſmall teleſcopes, in the common way. But they may here be ſeen in all their different appearances; as their increaſe, diviſion, union, beginning and end, &c.; alſo, the ſolar eclipſe may be ſeen to great advantage. And the tranſits of Mercury and Venus over the face of the ſun are moſt delightfully exhibited by this inſtrument; as the planets here appear truly round, and very black; their comparative diameters to that of the ſun may be taken, with the true direction of their motions, their ingreſs, egreſs, &c. The circle of the ſun's diſc may be divided by lines and circles, that the quantity and time of the eclipſe may be exactly determined.

The *helioſtata* was the invention of Dr. Graveſand: by this excellent invention, the motion of the teleſcope, in viewing the ſolar light, is greatly taken off. It conſiſts of two parts: firſt, a plain metallic ſpeculum, ſupported on a ſtand; and a clock, which directs the ſpeculum according to the motion of the earth, and thereby preſerves the image of the ſun in the ſame point of view.

The Acromatic Teleſcope.

This teleſcope has a double, and ſometimes treble object glaſs, to correct the aberration of the rays of light; for all rays of light paſſing through a ſingle lens will be ſomewhat decompounded, and divided into ſeparate colours, which renders the object coloured; to remedy which, the double object glaſs is uſed. This conſiſts of a double concave lens of white flint glaſs, and a double convex of crown glaſs. The two parts of the lenſes, which are on the ſame ſide of the centre, have the ſame effect on the rays of light as two triangular priſms, which refract the rays of light in a contrary direction to each other; therefore, if the exceſs of refraction in the crown glaſs deſtroy the divergency of colours occaſioned by the flint glaſs, the incident ray will be refracted without any foreign colour; thus, the image formed in the focus of this compound object glaſs, will be of the ſame colour as the object. For the rays flowing upon theſe glaſſes from diſtant radiant objects will paſs through them in ſuch a manner, that the aberration, cauſed by the firſt glaſs, is ſo far corrected by the ſecond, that the rays emerging from it are nearly parallel, and converging to one focus. This teleſcope may, therefore, have a much larger aperture, and a greater magnifying power, than the common refracting teleſcope can. And if the teleſcope be ſhort, the lenſes ſhould be very convex: and it ſhould then have three lenſes for the object glaſs; a concave one of white flint, between two convex ones of crown glaſs.

The Camera Obſcura.

The camera obſcura is only a convex glaſs C D (fig. 9) fixed in a hole of a window-ſhutter. In the focus of this glaſs will be ſeen, on a ſheet of white paper placed parallel to the glaſs, as at G H, the images of all the objects outſide of the window, as trees, men, cattle, &c. which affords a moſt beautiful piece of landſcape, or perſpective, particularly if the ſun ſhines upon the objects: but the images appear inverted. The room for this purpoſe ſhould be ſo darkened, that no light can enter, but what comes through the lens.

To repreſent the image horizontally, the convex glaſs C D muſt be placed in a tube in the ſide of a ſquare box, within which is the plane mirror E F, leaning backwards in an angle of forty-five degrees from the perpendicular I *g*; then the pencils of rays flowing from the outward objects, and paſſing through the convex glaſs to the plane mirror, will be reflected from it upwards, and meet in points, and form the image I K, of the object A B, which is at the ſame diſtance from the mirror E F, as the image G H, which image they would have formed if no mirror had been in the way. This image I K will be formed on an oiled paper, ſtretched horizontally in the direction I K. On this paper the outlines of the images may be drawn with a black lead pencil, and then copied on a clean ſheet; but it is uſual to place a plain unpoliſhed glaſs in the direction I K to receive the images of the outer objects; and their outlines may be traced upon this, as on the paper.

The tube in which the glaſs C D is fixed muſt be moveable, that it may be drawn out to adjuſt the diſtance of the glaſs lens, from the plane mirror, in proportion to the diſtance of the focus. For this purpoſe, the tube muſt be moved backward or forward, till the images appear diſtinctly on the horizontal glaſs I K.

If the mirror E F be reclined in the oppoſite direction, that is, leaning forwards towards the lens, in an angle of forty-five degrees, the images of the objects will then appear in an horizontal poſition, but below the box; and inverted with reſpect to the image I K.

To form an horizontal image, as I K, of an upright object A B, it is neceſſary that the mirror be reclined from a perpendicular, in an angle of forty-five degrees. Then the rays of the pencils *a b c* flowing from the point A of the object A B, will fall upon the mirror at the points *k f g*, and from thence will be reflected in the lines *k* I, f I, *g* I, and form the point A of the object A B, at the point I in the image I K. And the rays of the pencils *q r s*, flowing from the point B, and paſſing through the convex lens C D, will fall upon the mirror at the point *m l k*, from whence they will be reflected to the point K, where they form the image

of

of the point of the object B. The ſame may be demonſtrated of the pencils of rays which flow from every intermediate point of the object A B. This horizontal reflection depends upon the rays of incidence (which fall upon the mirror, after paſſing through the convex lens C D) making angles, with a perpendicular drawn to the ſurface of the mirror, equal to the angles formed by the reflected rays, and the ſaid perpendicular; thus, if we ſuppoſe a perpendicular drawn to the ſurface of the mirror at the point k, where the ray A *a* C *e* falls upon it, that ray will be reflected upwards in the line k *d* I, which will make an angle with the aforeſaid perpendicular, and on the other ſide thereof, equal to the angle formed by the perpendicular, and the incident ray C k; and if a perpendicular be drawn to the point *f* of the plane mirror, where the ray A *b f* falls upon it, that ray will be reflected upwards in the direction *f h* I, which will alſo make an angle with the laſt perpendicular, equal to the angle formed by the incident ray A *b f*, and the ſame perpendicular. The ſame may be demonſtrated of every other pencil of rays which flow from the object A B.

The flat plano-convex glaſs, vulgarly called the multiplying glaſs, is uſed to multiply an object. It is formed of a plano-convex glaſs, the convex ſide of which is ground into ſeveral flat ſurfaces. An object ſeen through ſuch a glaſs will appear multiplied into as many different objects as the glaſs contains plane ſurfaces. Let A D (fig. 8) be ſuch a glaſs, the flat ſurfaces of which are *h b*, *b l d*, *d k*: then the rays which flow from the object C to all parts of the glaſs will be refracted by the plane ſurfaces to the eye at H. Thus, the ray *g* H falling perpendicularly on the middle ſurface, will paſs through the glaſs without any refraction, and ſhow the object C in its true place; but the ray *a b*, flowing from the ſame object, and falling obliquely on the plane ſurface *h b*, will be refracted in the direction *c* H: by which the object will appear as at E. And the ray *c d*, flowing from the object C, and falling obliquely on the flat ſurface *d k*, will be refracted in the direction fH; by which the object will appear in the line fm, as if it were at D. If the glaſs be turned round the axis H lg, the object C will remain in the ſame place, but the other objects D and E will appear to go round C; becauſe the planes on which the oblique rays fall, will be turned round by the motion of the glaſs.

From what has been ſaid of the properties of different lenſes, the intelligent reader may be able to form a combination of them, for any ordinary optical purpoſe; but it may be neceſſary to ſay a few words more, concerning the reflecting teleſcope.

The great mirror of the reflecting teleſcope, as invented by Sir ISAAC NEWTON, was truly ſpherical; which teleſcopes are of eſſential ſervice at this day, particularly in the *minutiæ* of aſtronomy, when ſmall apertures and long foci are uſed. Dr. *Herſchell* uſes a reflector of this form, in his ſeven feet reflecting teleſcope. And the higheſt power that a teleſcope of a ſpherical metal will bear with perfect diſtinctneſs, may be found by multiplying the diameter of the great mirror by ſeventy-four. Thus, in Dr. Herſchell's ſeven feet reflector, the aperture or diameter of the great mirror is 6.25 inches, which, multiplied by 74, gives 462 the magnifying power; and the focal diſtance of the ſingle eye glaſs may be found, by dividing the focal diſtance of the great mirror by the magnifying power: thus, 7 feet, the focal diſtance, multiplied by 12, and divided by 462, quotes 0.182 of an inch, the focal diſtance of the eye glaſs.

But the concave ſurface of the great mirror, in common reflecting teleſcopes, is of a parabolic form; which form Sir Iſaac Newton was unable to give to his metal, though he recommended it; and imagined it might be effected by mechanical devices.

A TABLE

A TABLE
OF THE
Focal Diſtances, Apertures, and magnifying Powers
OF THE
NEWTONIAN REFLECTORS.

Focal Diſtance of the concave Metal	Aperture of the concave Metal	Sir Iſaac Newton's Numbers.	Focal Diſtance of the ſingle Eye Glaſs.	The magnifying Power.
Feet.	*In. Dec.*		*In. Dec.*	
½	0. 86	100	0. 167	36
1	1. 44	168	0. 199	62
2	2. 45	283	0. 236	102
3	3. 31	383	0. 261	138
4	4. 10	476	0. 281	171
5	4. 85	562	0. 297	202
6	5. 57	645	0. 311	232
7	6. 24	—	0. 323	266
8	6. 89	800	0. 334	287
9	7. 54	—	0. 344	314
10	8. 16	946	0. 353	340
11	8. 76	—	0. 362	365
12	9. 36	1084	0. 367	390
13	9. 94	—	0. 377	414
14	10. 49	—	0. 384	437
15	11. 04	—	0. 391	460
16	11. 59	1345	0. 397	483
17	12. 14	—	0. 403	506
18	12. 67	—	0. 409	528
19	13. 20	—	0. 414	550
20	13. 71	1591	0. 420	571
21	14. 23	—	0. 425	593
22	14. 73	—	0. 430	614
23	15. 21	—	0. 435	635
24	15. 73	1824	0. 439	656

A TABLE

A TABLE

OF THE

Dimenſions and magnifying Powers of the Gregorian Reflecting Teleſcopes,

As conſtructed by

MR. SHORT.

Magnifying Power.	Diſtance between the two Glaſſes.	Focal Diſtance of the Glaſs next the Eye.	Focal Diſtance of the Glaſs next the Metals.	Diſtance between the little Speculum, and the firſt Eye Glaſs.	Breadth of the Hole in the great Speculum.	Focus of the little Speculum.	Breadth of the great Mirror.	Focal Diſtance of the great Mirror.
	In. Dec.	*In. Dec.*	*In. Dec.*	*In. Dec.*	*In. Dec.*	*In. Dec.*	*In. Dec.*	*In. Dec.*
39	1.68	.81	2.44	8.54	.31	1.10	1.54	5.65
60	2.08	1.04	3.13	14.61	.39	1.50	2.30	9.60
86	2.93	1.31	3.94	23.81	.50	2.14	3.30	15.50
165	3.41	1.71	5.12	41.16	.65	3.43	6.26	36.00
241	4.28	2.48	6.43	68.17	.85	5.00	9.21	60.00

Since the invention of theſe reflecting teleſcopes, the refracting ones are very little uſed for celeſtial obſervations. For a refracting teleſcope, even of a thouſand feet focus (ſuppoſing it poſſible to be uſed), could not be made to magnify more than one thouſand times; whereas a reflecting teleſcope, not exceeding ten feet, will magnify twelve hundred times.

Mr. Short, of Edinburgh, was the firſt who brought the Gregorian reflectors to perfection, by giving them the true parabolic form, whereby they will admit of a much larger aperture.

To the ſame ingenious gentleman we are alſo indebted for that excellent contrivance, the equatorial teleſcope, or, as he himſelf calls it, a *portable obſervatory*; by which moſt of the ſtars of the firſt and ſecond magnitude, as alſo the planets Mercury, Venus, and Jupiter, may be ſeen at mid-day, even while the ſun ſhines bright. By this inſtrument, celeſtial objects may be viewed with very little trouble. It conſiſts of a piece of machinery, by the help of which the mounted teleſcope may be turned in any direction, and directed to any degree of right aſcenſion, or declination. And any object may be eaſily kept in view, or recovered when loſt from ſight, without moving the eye from its ſituation. The particular effect of this teleſcope depends upon its excluding almoſt all the light, except what comes from the object itſelf. But any other teleſcope of the ſame magnifying power, and in the ſame ſituation, will have the ſame effect. From a ſimilar cauſe, the ſtars are viſible in day-time, from the bottom of a deep pit.

Mr. Ramſden has alſo lately invented a portable obſervatory, which, it is expected, will excel Mr. Short's.

To render the fixed ſtars viſible in day-time, it is only neceſſary to exclude the extraneous light as much as poſſible.

A comet, or any other heavenly body, may be ſeen through this teleſcope; its machinery directing it to the proper place in the heavens.

The principal advantages of microſcopes and teleſcopes depend—firſt, upon their property of magnifying the minute part of the objects, ſo that they can be ſeen more diſtinctly than by the naked eye; and, ſecondly, upon their throwing more light into the pupil of the eye than what would flow from the object itſelf. The magnifying power of the glaſs would be of little ſervice, if this latter property of tranſmitting the light be wanting; for the ſame quantity of light, ſpread over a larger ſurface, becomes proportionably diminiſhed in force; and though the objects may appear bigger, they would appear proportionably dimmer, without an increaſe of light, as well as magnitude. Thus, though a glaſs ſhould magnify the ſurface of an object an hundred times, yet if the focal diſtance of the glaſs were only eight inches (if this be poſſible), and its diameter only the ſize of the pupil of the eye, the object will appear an hundred times more dim through the glaſs, than to the naked eye; even though the glaſs tranſmitted all the light which fell upon it, which no glaſs can do. But if the focal diſtance of the glaſs be four inches, and the diameter the ſame as before; the object will appear far more bright, becauſe the glaſs could be placed twice as near the object, as before, and conſequently would receive four times as many rays of light from the object, as in the former caſe. Thus, diminiſhing the focal diſtance of the glaſs, and keeping its diameter as large as poſſible, we ſhould perceive the object more and more magnified, and at the ſame time very diſtinct and bright.

With regard to the comparative merit of teleſcopes, for terreſtrial objects, thoſe of the refracting kind have evidently the advantage of all others, where the aperture is equal, and the aberrations of the rays are corrected according to Mr. Dollond's method; becauſe the image is more perfect; and a greater quantity of light is tranſmitted, than can be reflected from the beſt materials hitherto known. A teleſcope for terreſtrial objects ſhould never magnify above an hundred times; if it do, the vapours ariſing from the earth will be ſo magnified, as to render viſion very obſcure and imperfect.

However, the imperfections in every kind of glaſs very much retard the improvement of theſe teleſcopes, in which there are five eye glaſſes; ſo that they cannot be made above three feet and an half long.

Upon the whole, therefore, the reflecting teleſcopes are the beſt in this reſpect, that they may be made of greater dimenſions; by which means they can magnify to a greater degree, and, at the ſame time, throw more light upon the eye.

With regard to the form of reflecting teleſcopes, it is now generally agreed, that the Gregorian have the advantage of the Newtonian ones, if they be well conſtructed; except, as before obſerved, in the minutiæ of aſtronomy. One advantage the Gregorian teleſcope poſſeſſes, is, that an object is perceived, by looking directly through it, and ſo is formed with greater eaſe than in the Newtonian one, where we muſt look into the ſide. The unavoidable imperfections of the metal alſo give the Gregorian the advantage: for, notwithſtanding the greateſt care in poliſhing the ſpeculum, it is found impoſſible to give it either a perfectly ſpherical, or perfectly parabolical form. From hence ariſes indiſtinctneſs of the image, formed by the great ſpeculum, which is frequently corrected by the little ſpeculum, provided they are properly matched. But if they be not properly matched, the error will be made much worſe. There is no rule by which the artiſt can tell what ſpeculum will fit another, without making an actual experiment; therefore, he ſhould have ſeveral, ready made, of each, ſo that, in fitting up a teleſcope, he may chooſe thoſe which ſuit each other beſt.

SECT. IV.

OF DR. HERSCHELL'S GRAND TELESCOPE.

The chief conſideration, in the conſtruction of the mechanical parts of the reflecting teleſcope, is to have a ſteady view of the diſtant object. For this purpoſe, the axis of the teleſcope muſt be kept conſtantly directed to the ſame point of the object: for if the teleſcope be moved gently from its poſition, the object will ſeem to move in the ſame, or in the oppoſite direction, according as the teleſcope ſhows the image inverted, or erect.

If the inſtrument ſhake ever ſo little, it makes the object dance before our eyes; and cauſes an indiſtinctneſs in the viſion. In the Gregorian teleſcopes it is hardly poſſible to prevent this quivering motion of the ſmall ſpeculum, as it is fixed to the tube by one arm only. And though the tube of the inſtrument be fixed ever ſo motionleſs, this ſpeculum will retain a tremulous motion, particularly if the teleſcope has a great magnifying power. This motion is an imperceptible tremor, like that of an harpſichord wire, while it is ſounding, and produces a ſimilar effect to the ſight. A perſon walking in the room where the inſtrument is fixed, prevents us from ſeeing the object diſtinctly; or even the pulſation in the body of the obſerver agitates the ground enough to produce this effect. The more rapid this motion is, the greater is the optical indiſtinctneſs; or the object appears more confuſed. Therefore, the more firm, elaſtic, and well bound together, the frame-work and apertures of the teleſcope are, the more hurtful will be the conſequences. Juſt as it is to a rider on a wheel-carriage—the more firmly the carriage is fixed on the wheels, the more violent will be the motion to the rider. A teleſcope, therefore, mounted with lead (if it were practicable), would be preferable to one mounted on either wood, iron, or braſs. This is the principal cauſe of that indiſtinctneſs of viſion in the very beſt reflecting teleſcopes. Refracting teleſcopes have not this inconvenience.

From hence we may eaſily judge of the merits of any particular apparatus for the ſupport of a teleſcope: and in general, all forms where the tube is ſupported only in or near the middle, or where it remotely depends upon one joint, are very deſtructive of diſtinctneſs of viſion.

The

The beſt reflector I have ever ſeen, or heard of, had its ſmall ſpeculum fixed on a thick plate of lead, placed acroſs the tube, and led by an adjuſting ſcrew at each end. At each ſide of the ſmall ſpeculum, the lead was turned into a cylindrical form, that it might not intercept the rays of light flowing from the object. And the large *ſpeculum was fixed* to the upper end of the tube, by a ſtrong, ſoft cement; the tube of the teleſcope was fixed at each end upon a lump of ſoft clay. This teleſcope had very little tremor.

The tremor, inſeparable from all reflecting teleſcopes, led Dr. Herſchell, in the conſtructing of his teleſcopes to ſo great a magnifying power, to invent an apparatus for their ſupport, which would prevent the leaſt viſible motion in the inſtrument. His judgment and ingenuity ſoon contrived ſtands, which have every property that can be deſired. His tubes are all ſupported at each end. The motions, both vertical and horizontal, are performed with the utmoſt ſimplicity and firmneſs.—I ſhall, therefore, conclude this Section with a deſcription of his forty feet teleſcope, which does the higheſt honour to his optical abilities, as a philoſopher.

Plate 7 repreſents this inſtrument in a *meridional* ſituation, as it appears when ſeen by a perſon placed to the ſouth-weſt of it. The foundation of the whole apparatus reſts upon two concentric circular brick walls by twenty concentric rollers I I I, and the whole apparatus is moveable round upon a pivot, which gives an horizontal motion to the whole, with the teleſcope.

The two concentric walls are two feet ſix inches deep under ground; two feet three inches broad at the bottom; and one foot two inches at the top; and are capped with paving ſtone, about three inches thick, and twelve inches three quarters broad. The outermoſt wall is forty-two feet in diameter; and the inner one twenty-one feet.

The tube of the teleſcope A is of a plain cylindrical form, made of rolled or ſheet iron, joined together without rivets by a kind of ſeaming. Its length is thirty-nine feet four inches; and it meaſures four feet ten inches in diameter. It was formed of ſheet iron, in order to be of leſs weight, and of greater durability; for a wooden one muſt have exceeded this iron one, at leaſt three thouſand pounds, in weight.

Very great mechanical ſkill is diſplayed in the conſtruction of the apparatus by which the teleſcope is ſupported. In order to direct the teleſcope to every altitude, the point of ſupport is moveable, and its motion is effected by mechaniſm; ſo that the teleſcope may be moved from its moſt backward point of ſupport, to the moſt forward; and by means of the pullies G G ſuſpended from the great beam H, be ſet to any altitude, even up to the very zenith. The tube is alſo made to reſt, with the point of ſupport, in a pivot, which permits it to be turned ſideways.

The concave face of the great ſpeculum is forty-eight inches of poliſhed ſurface in diameter: the thickneſs now is about three inches and a half: it is of equal thickneſs in every part. Its weight, when it came from the caſt, was two thouſand one hundred and eighteen pounds, of which it muſt have loſt a ſmall quantity in poliſhing. When this ſpeculum is to be placed in the tube, it is ſuſpended vertically by a crane in the laboratory, and placed on a ſmall narrow carriage, which is drawn out, rolling upon planks, till it comes near the back of the tube: here it is again ſuſpended; and placed in the tube by a very curious contrivance.

The method of obſerving a celeſtial object by this teleſcope, is by the *front view*, as it is called by Dr. Herſchell. The obſerver is placed in the ſeat C, ſuſpended at the end of the tube, with his back towards the object he views: then looking down the tube, he ſees the image of the object reflected from the mirror. There is no ſmall ſpeculum in this teleſcope; but the magnifiers are applied to the firſt focal image.

At the opening of the teleſcope near the place of the eye glaſs, which the obſerver looks through, at the focal image, is a ſpeaking pipe, which runs down to the bottom of the tube, where it goes into a turning joint; and after ſeveral other inflections, it at length divides into two parts, one of which goes into the obſervatory D, and the other into the work-room E. By this ſpeaking-pipe, the obſerver conveys his obſervations to the aſſiſtant in the obſervatory: and the workmen are alſo directed by it to perform the neceſſary motions.

In the obſervatory there is a valuable ſidereal time-piece, made by Mr. Shelton. On one ſide of this is alſo a polar diſtance-piece, with a dial-plate of the ſame dimenſions with the time-piece: this piece may be made to ſhow the polar diſtance, zenith diſtance, declination or altitude, by ſetting it differently.

This noble inſtrument, with proper eye-glaſſes, magnifies above ſix thouſand times; and is the largeſt inſtrument, and has the greateſt magnifying power of any that has been made. By it Dr. Herſchell has been able to obſerve the lightning in the atmoſphere of the moon; and has diſcovered ſeveral celeſtial bodies, unknown to all preceding aſtronomers. Thoſe who wiſh for a fuller account of this inſtrument, and the admirable machinery which ſupports it, viz. the ſtairs, ladders, and platform B, may ſee the particulars of every part in the Second Part of the Tranſactions of the Royal Society, for 1795, where, by means of eighteen plates, and ſixty-three pages of letter-preſs, an ample detail is given of every circumſtance relating to joiner's, carpenter's, and ſmith's work. The whole was finiſhed on Auguſt the 28th, 1789, on which day the ſixth ſatellite of Saturn was diſcovered.

SECT.

SECT. V.

THE IMPROVED SOLAR AND LUCERNAL MICROSCOPES.

The Improved Solar Microſcope, which is conſtructed to view both tranſparent and opake Objects.

FIG. 4, plate 8, is the ſolar microſcope, mounted for exhibiting opake objects. Fig. 5 is the ſingle tooth and pinion microſcope, uſed for ſhowing tranſparent objects. The cylindrical tube Y being made to fit into the tube EF, fig. 4. Fig. 6 is the ſlider, which contains the magnifying lenſes: it fits into a dovetail ſlit, at the upper end of the microſcope, fig. 5. A D E F, fig. 4, is the body of the ſolar microſcope, one part whereof A D is conical, the other part C E F is cylindrical. The cylindrical part receives the tube G of the opake box H L, or the tube Y of the ſingle microſcope, fig. 5. At the large end A B, on the conical part, is fixed a lens, to receive the rays from the mirror O N, and to refract them towards the box H L. The mirror N O P is fixed to a moveable circular plate *a b c*; and may be moved into the moſt convenient poſition for reflecting the light, by means of the nuts Q and R. By the nut Q, it may be moved from eaſt to weſt, and by the nut R, it may be raiſed or depreſſed to any angle. *d e* are two ſcrews, to faſten the microſcope to a window-ſhutter. In the box H L is a plane mirror M, for reflecting the light, that it receives from the large lens, to the object, and thereby illuminating it. S is a ſcrew to adjuſt this mirror M to the proper diſtance and angle. V X are two tubes of braſs, ſliding one within the other, to carry the magnifying lenſes;

lenſes: the internal tube is ſometimes taken out, and the external one uſed by itſelf. At H there is a braſs plate, the back part of which is fixed to the tube *h*, in which tube there is a ſpiral wire to keep the braſs plate againſt the ſide H of the box; the ſlider, (fig. 6,) with the opake objects, is to be placed between this plate and the ſide of the box. To put the ſlider in, the plate is to be drawn back by means of the nut *g*. *i k* is a door on one ſide of the opake box.—Theſe are all the parts which conſtitute the ſolar microſcope for viewing opake objects. I ſhall therefore now deſcribe the ſingle microſcope uſed for tranſparent objects.

The ſingle Microſcope, for examining tranſparent Objects.

The box H L muſt be removed from the ſolar microſcope, (fig. 4:) and in its place the tube Y of the ſingle microſcope (fig. 5) muſt be inſerted. Within the body of this ſingle microſcope at *m*, (fig. 5,) are two thin plates, preſſed one againſt the other by a ſpiral ſpring, which are to be ſeparated, in order to let the ivory ſlider paſs between them. The ſlider (fig. 6) contains the magnifying lenſes as before obſerved. Any of the magnifiers may be placed before the object. When any one of the magnifying lenſes is at the centre of the hole P, a ſmall ſpring falls into one of the notches on the ſide of the ſlider to hold it faſt. Under the plate *m*, two lenſes are alſo placed for enlarging the field of view on the ſcreen: the ſmaller of the two is fixed on a piece of braſs, and is next the plate *m*; this is to be taken out when the magnifiers numbered 4, 5, or 6, are uſed, or when the megalaſcope lens is uſed, (fig. 7;) but it is to be replaced when the lenſes numbered 1, 2, 3, are uſed. This microſcope is adjuſted to the focus, by turning the nut O.

To uſe the Solar Microſcope, (Fig. 4.)

Make a round hole in the window-ſhutter, a little larger than the circular plate *a b c:* paſs the mirror O N P through this hole, and apply the ſquare plate to the ſhutter, ſcrewing it thereto by the two milled ſcrews *d e:* the ſcrews are to paſs through the ſhutter into their nuts in the microſcope, and thereby to hold the body of the microſcope firm. Screw the conical tube A B C D to the circular plate *a b c*; and then ſlide the tube G of the opake box into the cylindrical plate C D E F, if opake objects are to be viewed; but if tranſparent objects are to be viewed, place the tube Y (fig. 5) in the cylindricel tube (fig. 4). The room ſhould be made as dark as poſſible, and no light ſuffered to enter, but what comes through the body of the microſcope; for, on this circumſtance, together with the brightneſs of the ſun-ſhine, the diſtinctneſs of the object, in a great meaſure, depends. The mirror N O P is to be then ſo adjuſted by means of the two ſcrews Q and R, as to reflect all the ſun's light through the lens, at A B, in the body of the microſcope; the mirror is to be ſo adjuſted, till you have formed a clear round ſpot of light upon a ſcreen, or ſheet of white paper, placed at ſome diſtance.

In viewing an opake object, place the object between the plates at H: open the door *i k* of the opake box, and adjuſt the ſmall mirror M, till the object is ſtrongly illuminated. If this cannot be effected by the ſcrew S, you muſt move the two ſcrews Q and R, in order to get light reflected ſtrongly from the great mirror N O P, to the ſmall one M. The object being ſtrongly illuminated, ſhut the door *i k*, and a diſtinct image of the object will ſoon be obtained on the ſcreen, by adjuſting the tubes V and X. In the northern latitudes a clear large ſpot of light cannot always be obtained, nor when the ſun is perpendicular to the front of the room; as the light then falls upon the back of the mirror.

And *Note.* As the ſun is continually changing its place, it will be neceſſary to change the poſition of the mirror, by the two ſcrews Q and R, in order to keep the ſun's rays continually direct through the axis of the inſtrument.

To view tranſparent Objects.

Fix the ſingle microſcope (fig. 5) in the cylindrical tube E F (fig. 4); put the ſlider (fig. 6) into its place at *x* (fig. 5), and the ſlider with the object between the two plates at *m*; and adjuſt the large mirror N O P (fig. 4), as before directed in opake objects, and regulate the focus of the magnifier of the ſingle microſcope (fig. 5), by the ſcrew O. The moſt pleaſant magnifiers for common uſe, are thoſe marked number 4 and 5 in the ſlider (fig. 6). The ſize of the object may be increaſed or diminiſhed, by altering the diſtance of the ſcreen from the microſcope. That diſtance which ſhews the object moſt diſtinct, is about five or ſix feet.

To examine tranſparent Objects of a larger Size by the Megalaſcope.

Take out the ſlider (fig. 6) from its place in fig. 5, and ſcrew the button (fig. 7) into the hole at P (fig. 5); then remove the glaſs which is under the plate at *m*, and regulate the focus as before.

At the end of the tube G (fig. 4) there is a lens for increaſing the denſity of the rays, which ſerves to burn any combuſtible ſubſtance, or melt any fuſible one. In uſing this microſcope, this lens muſt generally be taken out, leſt the object ſhould be burnt by the intenſity of the heat.

This microſcope affords the greateſt entertainment of any, on account of its wonderful magnifying power, and the eaſe with which ſeveral perſons may view each ſingle object at the ſame time. It was formerly uſed only for tranſparent objects; but in the year 1774, Mr. *B. Martin* improved this inſtrument ſo far, as to render it applicable to opake objects. He ſpeaks of it thus: "With this inſtrument all opake objects, whether of the animal, vegetable, or mineral kingdom, may be exhibited in great perfection, in all their native beauty; the lights and ſhades, the prominences and cavities, and all the varieties of different hues, tints, and colours, heightened by the reflection of the ſolar rays condenſed upon them."

The improved lucernal Microſcope.

The lucernal microſcope is repreſented (fig. 1), where A B C D E is a large mahogany pyramidical box, which is the body of the microſcope; and ſupported on the braſs pillar F G, by the ſocket H, and the curved piece. I K M N are two tubes, one within the other: to the inner tube is fixed the vertical piece M L, which is a guide to the eye, to direct it to the axis of the inſtrument. This piece may be raiſed, or depreſſed, pulled further out, or puſhed further in, to adjuſt it to the focus of the glaſſes, and make it coincide with the centre of the field of view. The tubes M N are fixed to the box in a dovetailed piece of braſs. O P is a ſmall tube which carries the magnifiers; O is one of the magnifiers ſcrewed into the end of the tube. Q R S T V X is a long ſquare bar, paſſing through the ſockets Y, Z, and ſerves to hold the ſtage for the objects: this bar may be moved backward or forward, by means of the pinion *a*, and the handle *b c*, in order to adjuſt the ſtage to the focus. *d e* is a braſs bar to ſupport the curved piece I K, and keep the body of the microſcope firm and ſteady. *f g r s* is the ſtage to hold the ſliders with the opake objects; it ſlides upon the bar Q R S by means of the ſocket *h i*, by which means it is brought nearer to, or removed farther from, the magnifying lens. The objects are placed in the front ſide of the ſtage, next the tube of the microſcope, between four ſmall braſs plates (which cannot be ſhewn in the figure), the edges of two of the plates are ſeen at *k l*; the two upper plates are moveable, and are preſſed together by a ſpiral ſpring, to confine the ſlider with the objects; theſe two plates may be raiſed higher, or moved lower, by the ſmall nut *m*. The upper part of the ſtage *f g r s*, may be taken out, and the ſtage for tranſparent objects (fig. 3) inſerted in its place. At the lower part of the ſtage there is a ſemicircular glaſs lamp at *n*, to receive the light from the lighted lamp, and throw it upon the concave mirror *l*, from whence it is reflected on the object.

Fig. 3, is the ſtage for tranſparent objects: the two legs 5 and 6, fit into the top of the under part of the ſtage *r s* (fig. 1). 7 (fig. 3) is the part that holds the ſliders. 9 and 10, the tube that holds the magnifiers: within this tube, there is another, which may be placed at any diſtance from the object by the pin 11. When this ſtage is uſed by itſelf, as a ſingle microſcope, the magnifiers muſt be ſcrewed into the hole 12, and be adjuſted to the focus, by the nut 13. At the end of the wooden box A B C (fig. 1), is a ſlider A, repreſented as partly drawn out: when this ſlider is taken out, there will be perceived three grooves, one of which contains the board that forms the end of the box: the next contains a frame, with a gray glaſs; the third, or the innermoſt, contains two large convex lenſes.

To examine opake Objects, by this Microſcope.

The inſtrument (fig. 1) is ready mounted, for this purpoſe. Take out the wooden ſlider A, and lift out the cover and the gray glaſs from their groves, under the ſlider A: the guide for the eye L M N being put into its place, and the ſocket at the bottom of the opake ſtage on the bar Q X T, ſo that the concave mirror *l* may be next the end D E of the wooden body. Screw the tube P O into the end D E, with the magnifiers, and the handle *b*, or milled nut (fig. 2), placed at the end of the pinion *a*: place the lighted lamp before the glaſs lamp at *n*, and the object to be examined between the ſpring plates of the ſtage; and the inſtrument is ready for uſe.

To examine transparent Objects.

For this purpoſe, the upper part of the opake ſtage *f g r s* (fig. 1) muſt be removed; and the ſtage for tranſparent objects (fig. 3) inſerted in its place, with the end 9 and 10 next the lamp. Place the gray glaſs in its groove at the end A B (fig. 1), and the objects to be viewed in the ſlider-holder at the front of the ſtage; then tranſmit as ſtrong a light as poſſible to the object, by raiſing or lowering the lamp; and the object will be beautifully depictured on the gray glaſs, being regulated to the focus of the magnifier by turning the pinion *a*.

If the gray glaſs be taken out of the microſcope, the image of the object may be received on a paper ſkreen.

There is no inſtrument invented by the art of man capable of affording ſo great entertainment as the microſcope. By this inſtrument we can pry into the minuteſt receſſes of nature; and diſcover truths which have lain hidden from the creation of the world. It is from the uſe of this inſtrument that we can confidently aſſert, that there is hardly any part of nature which is not the abode of animal exiſtence. By it we diſcover thouſands of living creatures in a ſingle drop of water; and whole nations of animated beings in the bloom which ſurrounds a plum, a pear, and moſt other fruit; not to mention numberleſs inſects in every part of nature, ſo ſmall, that a thouſand of them might be cruſhed by a ſingle grain of ſand.

From the diſcovery of theſe truths, the philoſophic mind is led to make ſtill further improvements: thus, in the formation of a mite, we can ſee the ſame proportion and ſymmetry in the ſtructure of its limbs, as in thoſe of any of the larger quadrupeds; as free a circulation of blood in the one as in the other; as many members adapted to different purpoſes; as numerous veins, muſcles, arteries, ſinews, &c.&c. with a proboſcis to take in its food, like that of the elephant.

And if we deſcend to inſects of a yet ſmaller ſize, we ſhall always find in each, all the properties and attributes eſſential to animal life: if we deſcend to thoſe inſects, whoſe bulk, compared to that of a mite, is as that of the mite to the elephant.

Others are of ſo ſmall a ſize, as to be *beyond the reach of* any magnifying power; and whoſe whole exiſtence is circumſcribed within the ſpace of a few hours, or a ſummer's day at the moſt. Myriads there are in the vegetable world, to whom the morning gives birth, and who expire with the ſetting ſun, beſides numberleſs others who have their habitations on the bodies of other animals, ſuch as the ichneumon, whoſe peculiar habitation is on the body of the caterpillar.

In ſhort, there is no part of nature which does not afford to the microſcopical obſerver an abundant ſource of wonder and admiration, and diſplay the attributes of that Supreme Being, whoſe wiſdom is as conſpicuous in the conſtruction of the ſmalleſt atom, to be diſcovered by our aſſiſted ſight, as his power is evident in the formation of the planetary ſyſtems.

But there are ſome objects which have more immediately engaged the attention of the curious obſerver: ſuch as, what is called the ſilver tree; and the cryſtallization of ſalts. The ſilver tree is formed by diſſolving a little ſilver in a ſmall quantity of aqua fortis; and then adding to it twice the quantity of common water. Drop a little of this upon a piece of plain glaſs, and put upon it a ſmall piece of braſs wire; and immediately the wire is put upon the mixture, trees will appear to grow, and diſplay their branches, which will be extended, as far as the liquor extends on the glaſs; and has all the appearance of real trees, but of a ſilver caſt. To obſerve the cryſtallization of ſalts, diſſolve a little ſal-ammoniac in common water, drop a little of this mixture on a piece of glaſs, as in the former caſe, and when viewing it through the microſcope, hold a hot iron near it, to make it evaporate the quicker. As ſoon as the evaporation takes place, it wil

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